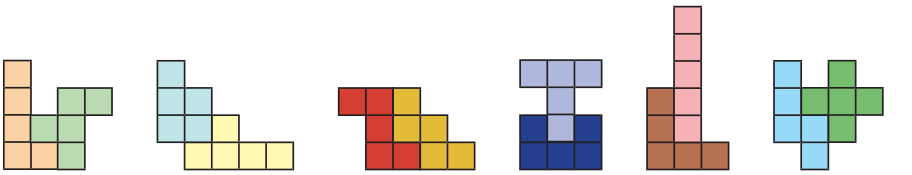


0674CH06

6 PERIMETER AND AREA

6.1 Perimeter

Do you remember what the perimeter of a closed plane figure is? Let us refresh our understanding!

The perimeter of any closed plane figure is the distance covered along its boundary when you go around it once. For a **polygon**, i.e., a closed plane figure made up of line segments, the perimeter is simply the sum of the lengths of its all sides, i.e., the total distance along its outer boundary.

The perimeter of a polygon = the sum of the lengths of its all sides.

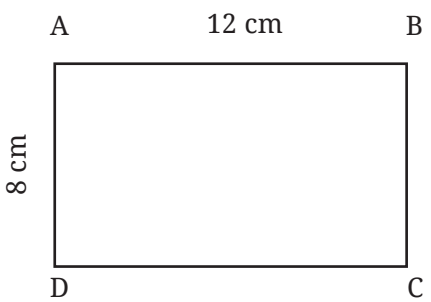
Let us revise the formulas for the perimeter of rectangles, squares, and triangles.

Perimeter of a Rectangle

Consider a rectangle ABCD whose length and breadth are 12 cm and 8 cm, respectively. What is its perimeter?

Perimeter of the rectangle = Sum of the lengths of its four sides

= AB + BC + CD + DA

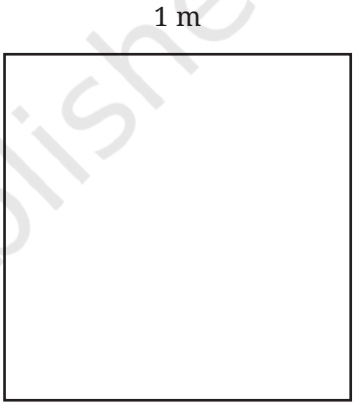


= AB + BC + AB + BC
= 2 × AB + 2 × BC
= 2 × (AB + BC)
= 2 × (12 cm + 8 cm)
= 2 × (20 cm)
= 40 cm.

Opposite sides of a rectangle are always equal. So, AB = CD and AD = BC

From this example, we see that—
Perimeter of a rectangle = length + breadth + length + breadth.
Perimeter of a rectangle = 2 × (length + breadth).
The perimeter of a rectangle is twice the sum of its length and breadth.

Perimeter of a Square



Debojeet wants to put coloured tape all around a square photo frame of side 1m as shown. What will be the length of the coloured tape he requires? Since Debojeet wants to put the coloured tape all around the square photo frame, he needs to find the perimeter of the photo frame.

Thus, the length of the tape required = perimeter of the square
= sum of the lengths of all four sides of the square
= 1 m + 1 m + 1 m + 1 m = 4 m.

Now, we know that all four sides of a square are equal in length. Therefore, in place of adding the lengths of each side, we can simply multiply the length of one side by 4.

Thus, the length of the tape required = 4 × 1 m = 4 m.

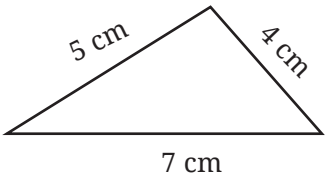
From this example, we see that

Perimeter of a square = 4 × length of a side.

The perimeter of a square is quadruple the length of its side.

Perimeter of a Triangle

Consider a triangle having three given sides of lengths 4 cm, 5 cm and 7 cm. Find its perimeter.



Perimeter of the triangle = 4 cm + 5 cm + 7 cm
= 16 cm.

Perimeter of a triangle = sum of the lengths of its three sides.

Example: Akshi wants to put lace all around a rectangular tablecloth that is 3 m long and 2 m wide. Find the length of the lace required.

Solution

Length of the rectangular table cover = 3 m.

Breadth of the rectangular table cover = 2 m.

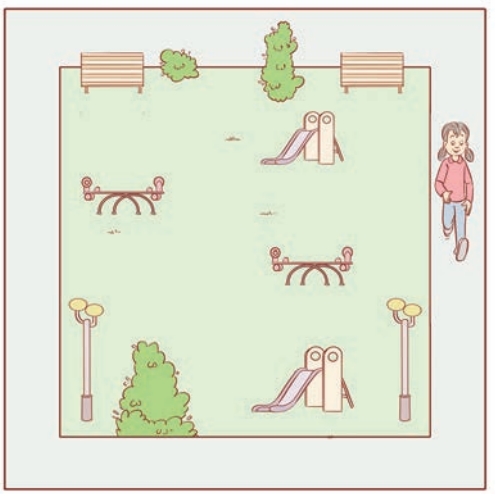
Akshi wants to put lace all around the tablecloth.

Therefore, the length of the lace required will be the perimeter of the rectangular tablecloth.

Now, the perimeter of the rectangular tablecloth = 2 × (length + breadth)
= 2 × (3 m + 2 m) = 2 × 5 m = 10 m.

Hence, the length of the lace required is 10 m.

Example: Find the distance travelled by Usha if she takes three rounds of a square park of side 75 m.



Solution

Perimeter of the square park = 4 × length of a side = 4 × 75 m = 300 m.

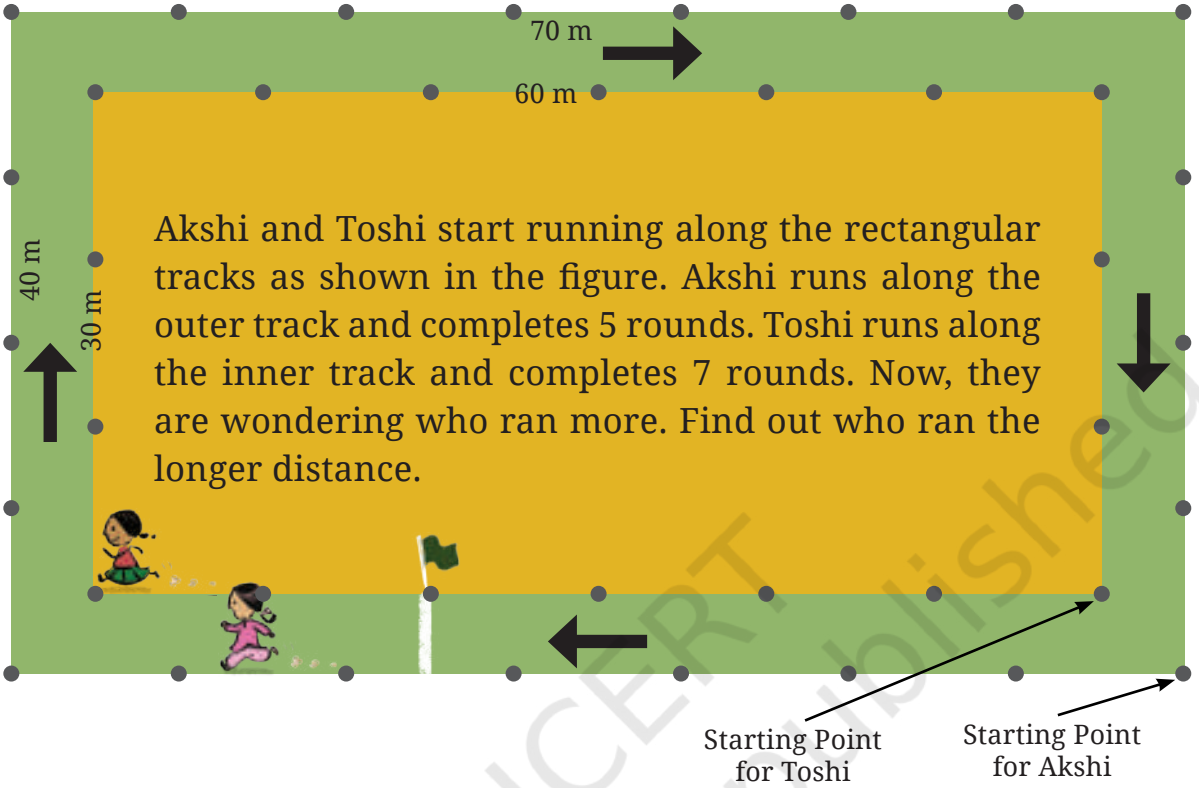
Distance covered by Usha in one round = 300 m.

Therefore, the total distance travelled by Usha in three rounds = 3 × 300 m = 900 m.

Figure it Out

1. Find the missing terms:
   a. Perimeter of a rectangle = 14 cm; breadth = 2 cm; length = ?.
   b. Perimeter of a square = 20 cm; side of a length = ?.
   c. Perimeter of a rectangle = 12 m; length = 3 m; breadth = ?.
2. A rectangle having sidelengths 5 cm and 3 cm is made using a piece of wire. If the wire is straightened and then bent to form a square, what will be the length of a side of the square?
3. Find the length of the third side of a triangle having a perimeter of 55 cm and having two sides of length 20 cm and 14 cm, respectively.
4. What would be the cost of fencing a rectangular park whose length is 150 m and breadth is 120 m, if the fence costs ₹40 per metre?
5. A piece of string is 36 cm long. What will be the length of each side, if it is used to form:
   a. A square,
   b. A triangle with all sides of equal length, and
   c. A hexagon (a six sided closed figure) with sides of equal length?
6. A farmer has a rectangular field having length 230 m and breadth 160 m. He wants to fence it with 3 rounds of rope as shown. What is the total length of rope needed?

Matha Pachchi!



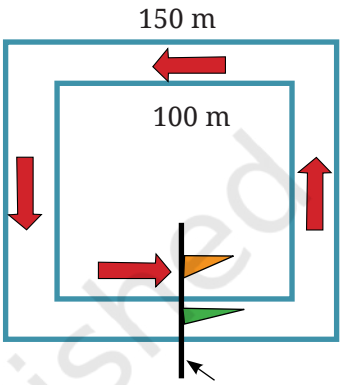
Each track is a rectangle. Akshi's track has length 70 m and breadth 40 m. Running one complete round on this track would cover 220 m, i.e., 2 × (70 + 40) m = 220 m. This is the distance covered by Akshi in one round.

Figure it Out

1. Find out the total distance Akshi has covered in 5 rounds.
2. Find out the total distance Toshi has covered in 7 rounds. Who ran a longer distance?
3. Think and mark the positions as directed—
   a. Mark 'A' at the point where Akshi will be after she ran 250 m.
   b. Mark 'B' at the point where Akshi will be after she ran 500 m.
   c. Now, Akshi ran 1000 m. How many full rounds has she finished running around her track? Mark her position as 'C'.
   d. Mark 'X' at the point where Toshi will be after she ran 250 m.
   e. Mark 'Y' at the point where Toshi will be after she ran 500 m.

f. Now, Toshi ran 1000 m. How many full rounds has she finished running around her track? Mark her position as 'Z'.

Deep Dive: In races, usually there is a common finish line for all the runners. Here are two square running tracks with the inner track of 100 m each side and outer track of 150 m each side. The common finishing line for both runners is shown by the flags in the figure which are in the center of one of the sides of the tracks.

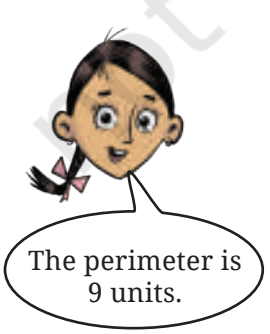


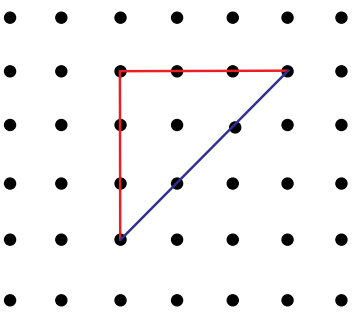
If the total race is of 350 m, then we have to find out where the starting positions of the two runners should be on these two tracks so that they both have a common finishing line after they run for 350 m. Mark the starting points of the runner on the inner track as 'A' and the runner on the outer track as 'B'.

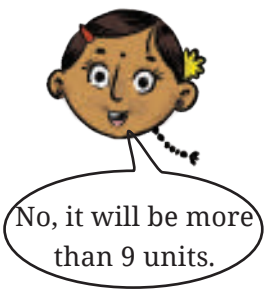
Estimate and Verify

Take a rough sheet of paper or a sheet of newspaper. Make a few random shapes by cutting the paper in different ways. Estimate the total length of the boundaries of each shape then use a scale or measuring tape to measure and verify the perimeter for each shape.

Akshi says that the perimeter of this triangle shape is 9 units. Toshi says it can't be 9 units and the perimeter will be more than 9 units. What do you think?

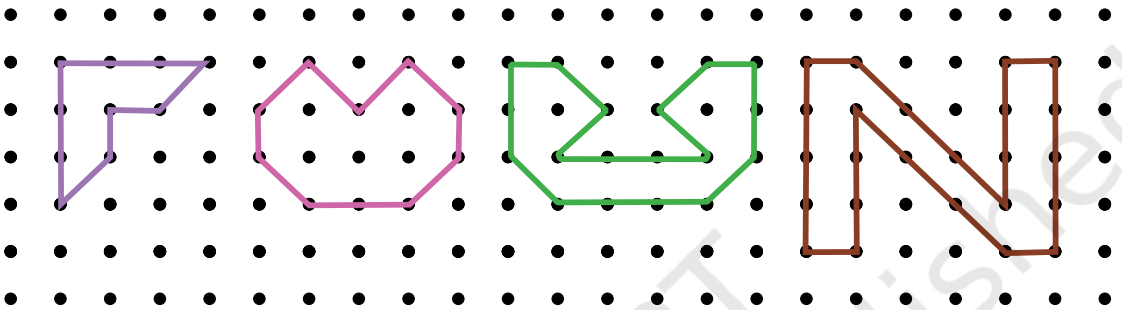






This figure has lines of two different unit lengths. Measure the lengths of a red line and a blue line; are they same? We will call the red lines–straight lines and the blue lines–diagonal lines. So, the perimeter of this triangle is 6 straight units + 3 diagonal units. We can write this in a short form as: 6s + 3d units.

Write the perimeters of the figures below in terms of straight and diagonal units.

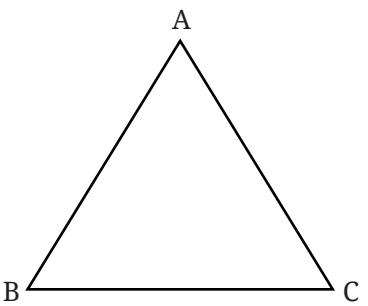


Perimeter of a Regular Polygon

Like squares, closed figures that have all sides and all angles equal are called **regular polygons**. We studied the sequence of regular polygons as 'Shape Sequence' #1 in Chapter 1. Examples of regular polygons are the equilateral triangle (where all three sides and all three angles are equal), regular pentagon (where all five sides and all five angles are equal), etc.

Perimeter of an equilateral triangle

We know that for any triangle its perimeter is sum of all three sides.



Using this understanding, we can find the perimeter of an equilateral triangle.

Perimeter of an equilateral triangle
= AB + BC + AC = AB + AB + AB
= 3 times length of one side.

Perimeter of an equilateral triangle = 3 × length of a side.

What is a similarity between a square and an equilateral triangle?

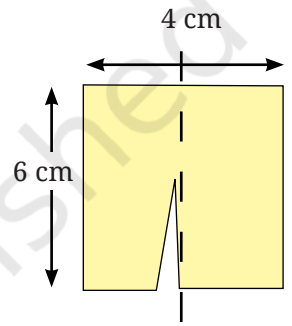
Find various objects from your surroundings that have regular shapes and find their perimeters. Also, generalise your understanding for the perimeter of other regular polygons.

Teacher's Note

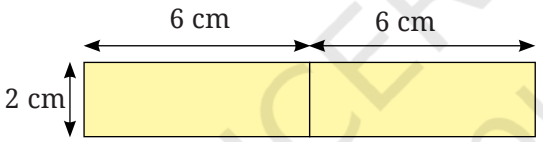
Discuss more about regular polygons and encourage students to come up with a general formula for the perimeter of a regular polygon.

Split and Rejoin

A rectangular paper chit of dimension 6 cm × 4 cm is cut as shown into two equal pieces. These two pieces are joined in different ways.



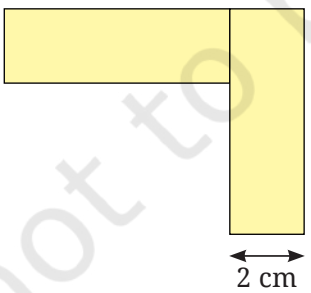
a.



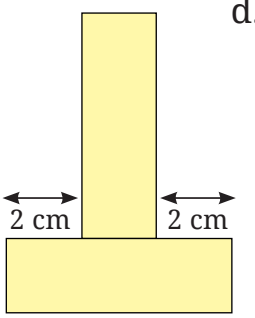
For example, the arrangement a. has a perimeter of 28 cm.

Find out the length of the boundary (i.e., the perimeter) of each of the other arrangements below.

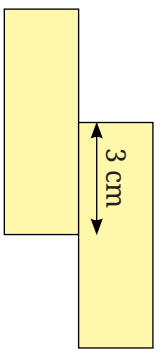
b.



c.



d.



Arrange the two pieces to form a figure with a perimeter of 22 cm.

6.2 Area

We have studied the areas of closed figures (regular and irregular) in previous grades. Let us recall some key points.

The amount of region enclosed by a closed figure is called its **area**.

In previous grades, we arrived at the formula for the area of a rectangle and a square using square grid paper. Do you remember?

Area of a square = _____

Area of a rectangle = _____

Teacher's Note

Help students in recalling the method of finding the area of a rectangle and a square using grid papers. Provide square grid papers to students and let them come up with the formula.

Let's see some real-life problems related to these ideas.

Example: A floor is 5 m long and 4 m wide. A square carpet of sides 3 m is laid on the floor. Find the area of the floor that is not carpeted.

Solution

Length of the floor = 5 m.

Width of the floor = 4 m.

Area of the floor = length × width = 5 m × 4 m = 20 sq m.

Length of the square carpet = 3 m.

Area of the carpet = length × length = 3 m × 3 m = 9 sq m.

Hence, the area of the floor laid with carpet is 9 sq m.

Therefore, the area of the floor that is not carpeted is: area of the floor minus the area of the floor laid with carpet = 20 sq m – 9 sq m = 11 sq m.

Example: Four square flower beds each of side 4 m are in four corners on a piece of land 12 m long and 10 m wide. Find the area of the remaining part of the land.

Solution

Length of the land (l) = 12 m.

Width of land (w) = 10 m.

Area of the whole land = $l \times w$ = 12 m × 10 m = 120 sq m.

The sidelength of each of the four square flower beds is (s) = 4 m.

Area of one flower bed = $s \times s$ = 4 m × 4 m = 16 sq m.

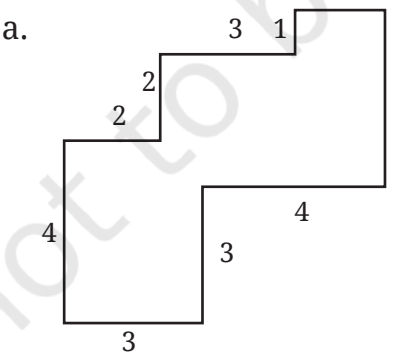
Hence, the area of the four flower beds = 4 × 16 sq m = 64 sq m.

Therefore, the area of the remaining part of the land is: area of the complete land minus the area of all four flower beds = 120 sq m – 64 sq m = 56 sq m.

Figure it Out

1. The area of a rectangular garden 25 m long is 300 sq m. What is the width of the garden?
2. What is the cost of tiling a rectangular plot of land 500 m long and 200 m wide at the rate of ₹8 per hundred sq m?
3. A rectangular coconut grove is 100 m long and 50 m wide. If each coconut tree requires 25 sq m, what is the maximum number of trees that can be planted in this grove?
4. By splitting the following figures into rectangles, find their areas (all measures are given in metres):

a.



b.

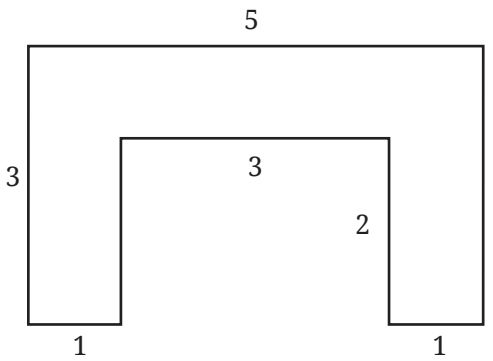
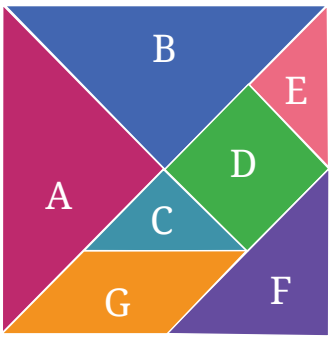


Figure it Out

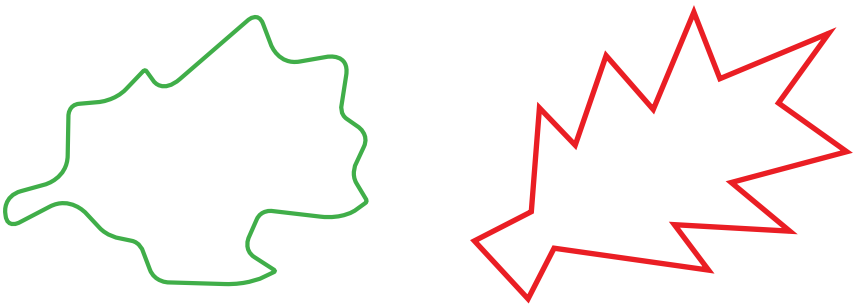
Cut out the tangram pieces given at the end of your textbook.



1. Explore and figure out how many pieces have the same area.
2. How many times bigger is Shape D as compared to Shape C? What is the relationship between Shapes C, D and E?
3. Which shape has more area: Shape D or F? Give reasons for your answer.
4. Which shape has more area: Shape F or G? Give reasons for your answer.
5. What is the area of Shape A as compared to Shape G? Is it twice as big? Four times as big?

   Hint: In the tangram pieces, by placing the shapes over each other, we can find out that Shapes A and B have the same area, Shapes C and E have the same area. You would have also figured out that Shape D can be exactly covered using Shapes C and E, which means Shape D has twice the area of Shape C or shape E, etc.
6. Can you now figure out the area of the big square formed with all seven pieces in terms of the area of Shape C?
7. Arrange these 7 pieces to form a rectangle. What will be the area of this rectangle in terms of the area of Shape C now? Give reasons for your answer.
8. Are the perimeters of the square and the rectangle formed from these 7 pieces different or the same? Give an explanation for your answer.

Look at the figures below and guess which one of them has a larger area.



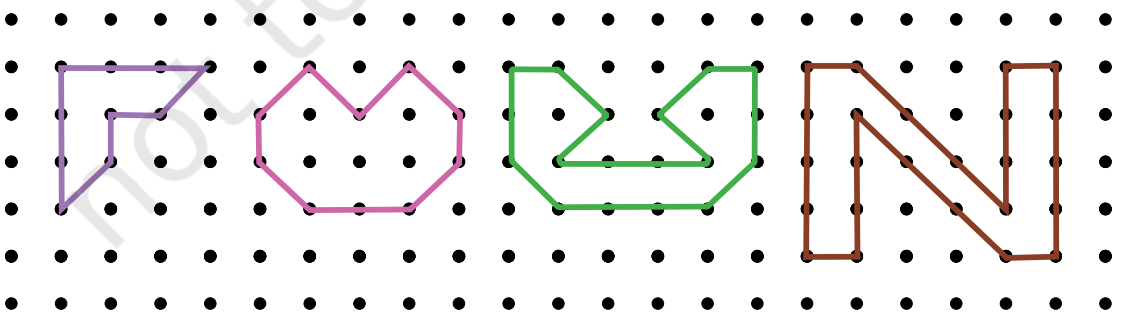
a. b.

We can estimate the area of any simple closed shape by using a sheet of squared paper or graph paper where every square measures 1 unit × 1 unit or 1 square unit.

To estimate the area, we can trace the shape onto a piece of transparent paper and place the same on a piece of squared or graph paper and then follow the below conventions—

1. The area of one full small square of the squared or graph paper is taken as 1 sq unit.
2. Ignore portions of the area that are less than half a square.
3. If more than half of a square is in a region, just count it as 1 sq unit.
4. If exactly half the square is counted, take its area as $\frac{1}{2}$ sq unit.

Find the area of the following figures.

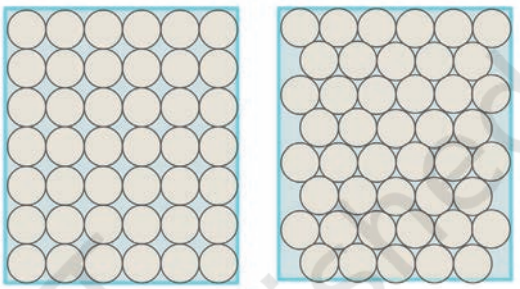


Let's Explore!

Why is area generally measured using squares?

Why can't we use circles instead of squares to find the area?

Draw a circle on a graph sheet with diameter (breadth) of length 3. Count the squares and use them to estimate the area of the circular region.



As you can see, circles can't be packed tightly without gaps in between. So, it is difficult to get an accurate measurement of area using circles as units. Here, the same rectangle is packed in two different ways with circles—the first one has 42 circles and the second one has 44 circles.

Try using different shapes (triangle and rectangle) to fill the given space (without overlaps and gaps) and find out the merits associated with using a square shape to find the area rather than another shape. List out the points that make a square the best shape to use to measure area.

1. Find the area (in square metres) of the floor outside of the corridor.
2. Find the area (in square metres) occupied by your school playground.

Let's Explore!

On a squared grid paper (1 square = 1 square unit), make as many rectangles as you can whose lengths and widths are a whole number of units such that the area of the rectangle is 24 square units.

Math Talk

a. Which rectangle has the greatest perimeter?

b. Which rectangle has the least perimeter?

c. If you take a rectangle of area 32 sq cm, what will your answers be? Given any area, is it possible to predict the shape of the rectangle with the greatest perimeter as well as the least perimeter? Give examples and reasons for your answer.

6.3 Area of a Triangle

Draw a rectangle on a piece of paper and draw one of its diagonals. Cut the rectangle along that diagonal and get two triangles.

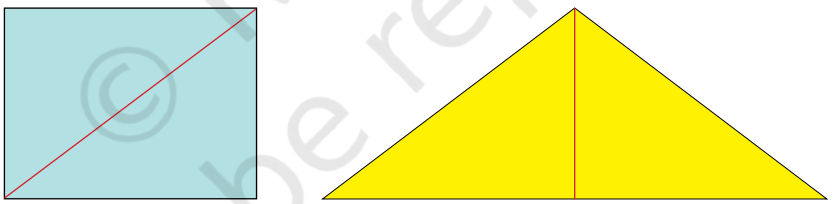
Check! whether the two triangles overlap each other exactly. Do they have the same area?

Try this with more rectangles having different dimensions. You can check this for a square as well.

Can you draw any inferences from this exercise? Please write it here. _____

_____

Now, see the figures below. Is the area of the blue rectangle more or less than the area of the yellow triangle? Or is it the same? Why?



Can you see some relationship between the blue rectangle and the yellow triangle and their areas? Write the relationship here.

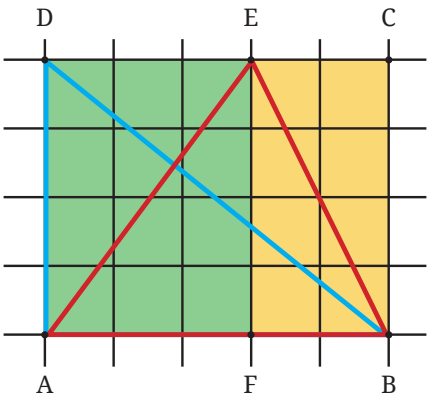
_____

Teacher's Note

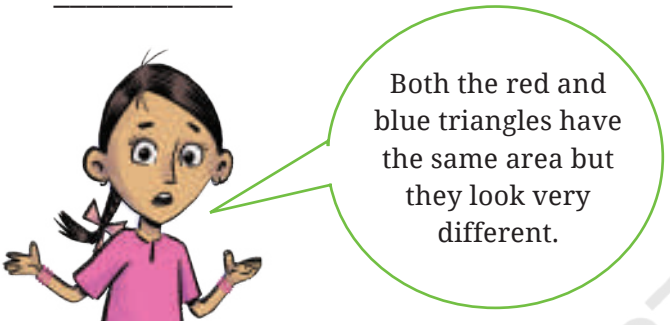
Help students in articulating their inferences and in defining the relationships they have observed in their own words, gradually leading to a common statement for whole classroom. Recall the definition of a diagonal in the classroom.

Draw suitable triangles on grid paper to verify your inferences and relationships observed in the above exercises.

Use your understanding from previous grades to calculate the area of any closed figure using grid paper and—

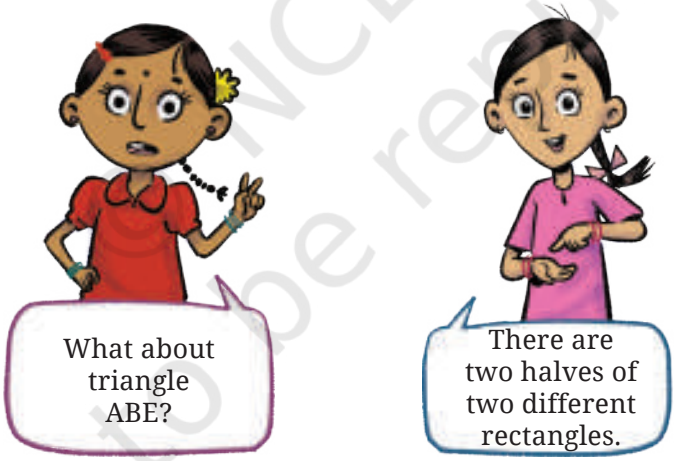


1. Find the area of blue triangle BAD. _____
2. Find the area of red triangle ABE. _____



Area of rectangle ABCD = _____

So, the area of triangle BAD is half of the area of the rectangle ABCD.



Area of triangle ABE = Area of triangle AEF + Area of triangle BEF.

Here, the area of triangle AEF = half of the area of rectangle AFED.

Similarly, the area of triangle BEF = half of the area of rectangle BFEC.

Thus, the area of triangle ABE = half of the area of rectangle AFED + half of the area of rectangle BFEC

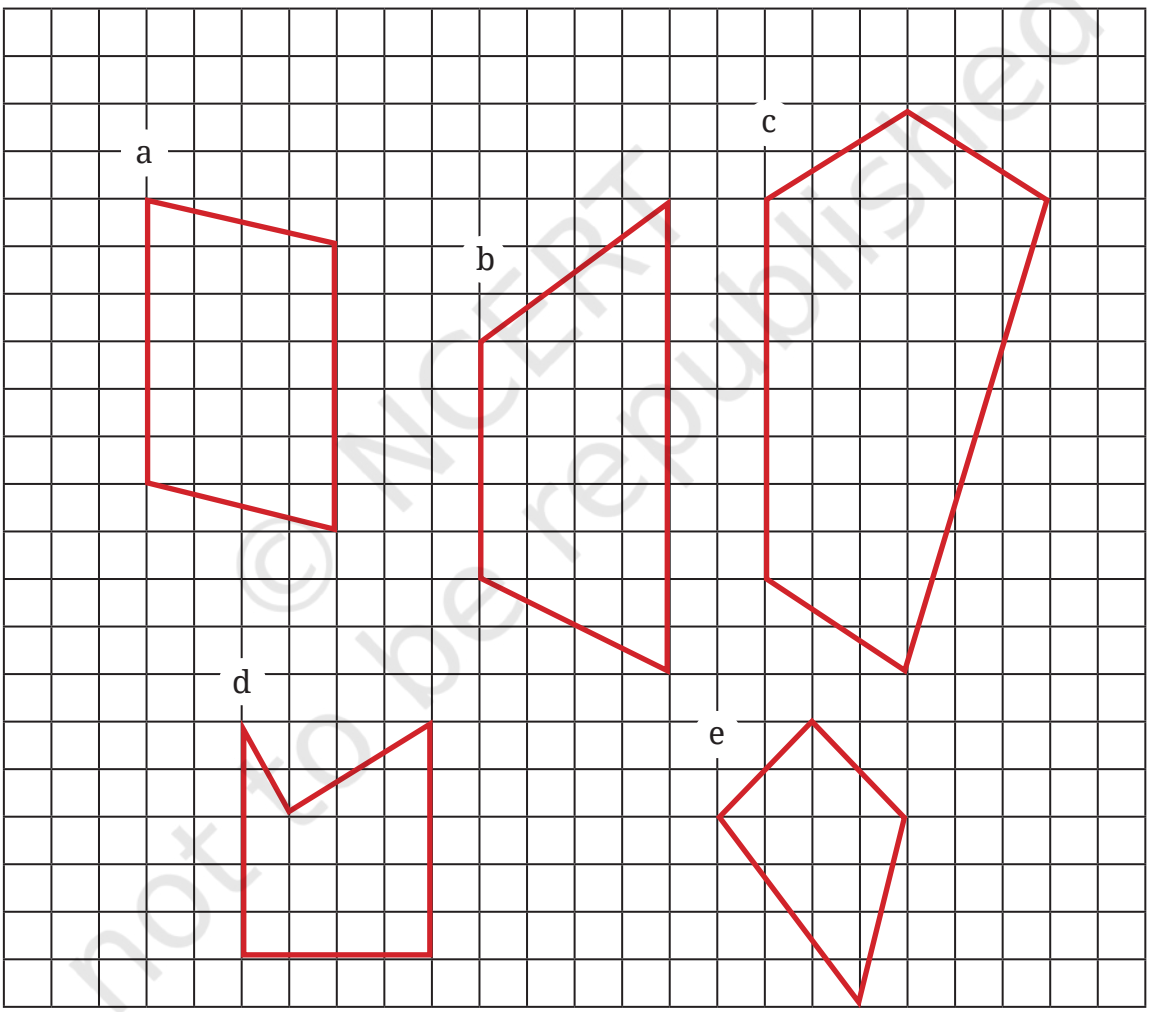
= half of the sum of the areas of the rectangles AFED and BFEC
= half of the area of rectangle ABCD.

Conclusion _____

_____

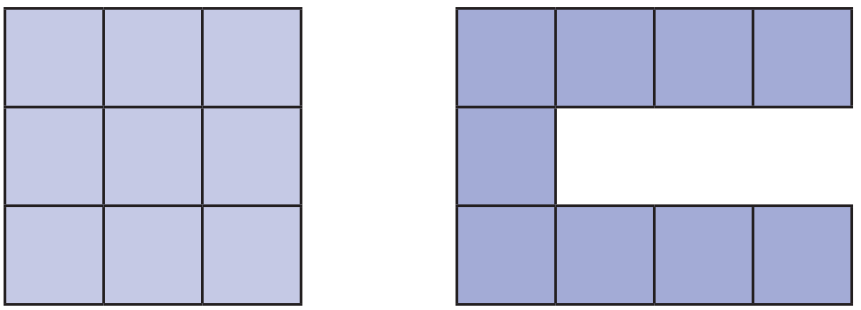
Figure it Out

1. Find the areas of the figures below by dividing them into rectangles and triangles.



Making it 'More' or 'Less'

Observe these two figures. Is there any similarity or difference between the two?



Using 9 unit squares (having an area of 9 sq units), we have made figures with two different perimeters—the first figure has a perimeter of 12 units and the second has a perimeter of 20 units.

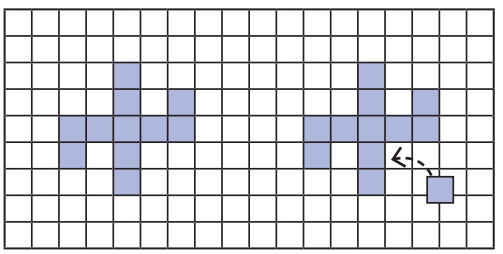
Arrange or draw different figures with 9 sq units to get other perimeters. Each square should align with at least one other square on at least one side completely and together all squares should form a single connected figure with no holes.

Using 9 unit squares, solve the following.

1. What is the smallest perimeter possible?
2. What is the largest perimeter possible?
3. Make a figure with a perimeter of 18 units.
4. Can you make other shaped figures for each of the above three perimeters, or is there only one shape with that perimeter? What is your reasoning?

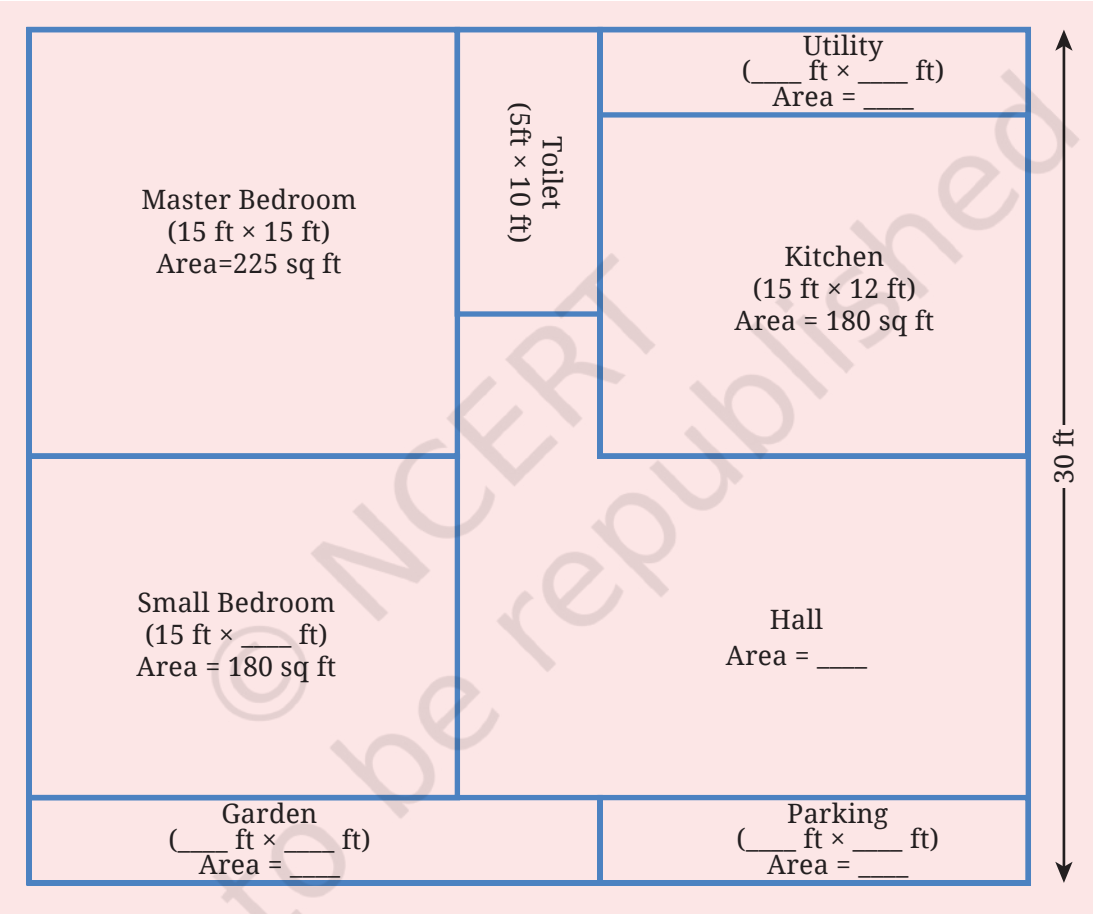
Let's do something tricky now! We have a figure below having perimeter 24 units.

Without calculating all over again, observe, think and find out what will be the change in the perimeter if a new square is attached as shown on the right.



Experiment placing this new square at different places and think what the change in perimeter will be. Can you place the square so that the perimeter: a) increases; b) decreases; c) stays the same?

Below is the house plan of Charan. It is in a rectangular plot. Look at the plan. What do you notice?

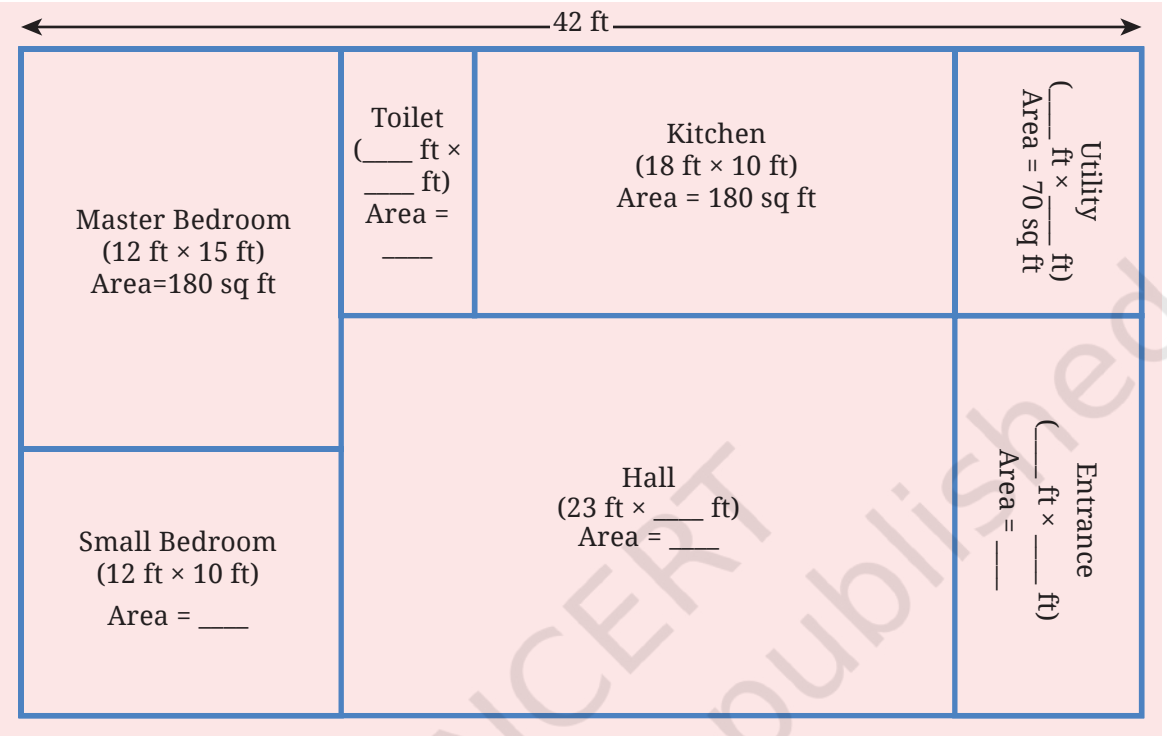


Some of the measurements are given.

a. Find the missing measurements.

b. Find out the area of his house.

Now, find out the missing dimensions and area of Sharan's home. Below is the plan:



Some of the measurements are given.

a. Find the missing measurements.

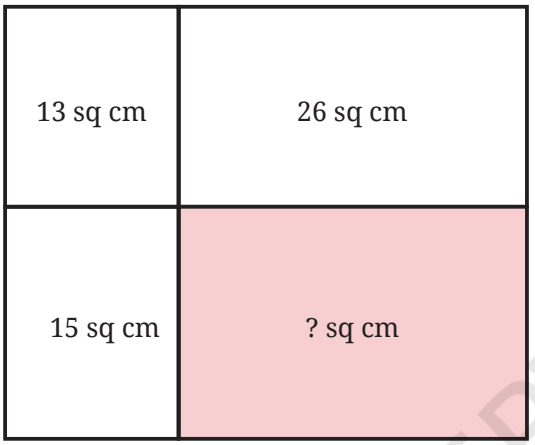
b. Find out the area of his house.

What are the dimensions of all the different rooms in Sharan's house? Compare the areas and perimeters of Sharan's house and Charan's house.

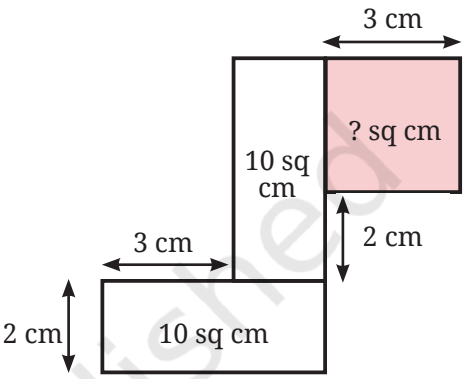
Area Maze Puzzles

In each figure, find the missing value of either the length of a side or the area of a region.

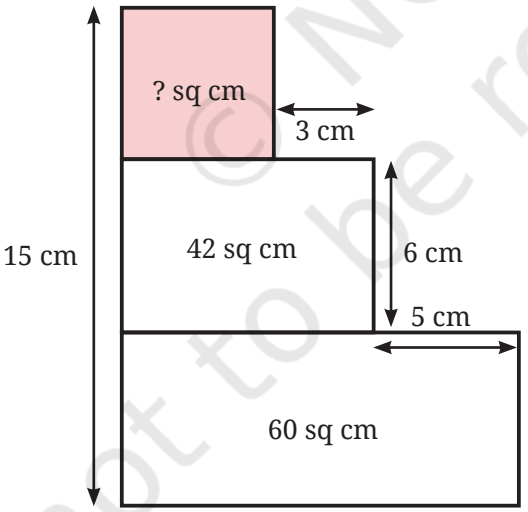
a.



b.



c.



d.

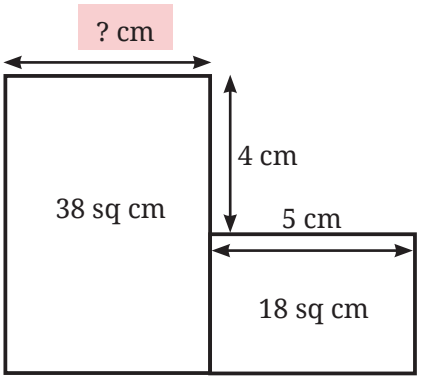


Figure it Out

1. Give the dimensions of a rectangle whose area is the sum of the areas of these two rectangles having measurements: 5 m × 10 m and 2 m × 7 m.
2. The area of a rectangular garden that is 50 m long is 1000 sq m. Find the width of the garden.
3. The floor of a room is 5 m long and 4 m wide. A square carpet whose sides are 3 m in length is laid on the floor. Find the area that is not carpeted.
4. Four flower beds having sides 2 m long and 1 m wide are dug at the four corners of a garden that is 15 m long and 12 m wide. How much area is now available for laying down a lawn?
5. Shape A has an area of 18 square units and Shape B has an area of 20 square units. Shape A has a longer perimeter than Shape B. Draw two such shapes satisfying the given conditions.
6. On a page in your book, draw a rectangular border that is 1 cm from the top and bottom and 1.5 cm from the left and right sides. What is the perimeter of the border?
7. Draw a rectangle of size 12 units × 8 units. Draw another rectangle inside it, without touching the outer rectangle that occupies exactly half the area.
8. A square piece of paper is folded in half. The square is then cut into two rectangles along the fold. Regardless of the size of the square, one of the following statements is always true. Which statement is true here?

a. The area of each rectangle is larger than the area of the square.

b. The perimeter of the square is greater than the perimeters of both the rectangles added together.

c. The perimeters of both the rectangles added together is always $1\frac{1}{2}$ times the perimeter of the square.

d. The area of the square is always three times as large as the areas of both rectangles added together.

SUMMARY

- The perimeter of a polygon is the sum of the lengths of all its sides.
  a. The perimeter of a rectangle is twice the sum of its length and width.
  b. The perimeter of a square is four times the length of any one of its sides.
- The area of a closed figure is the measure of the region enclosed by the figure.
- Area is generally measured in square units.
- The area of a rectangle is its length times its width. The area of a square is the length of any one of its sides multiplied by itself.
- Two closed figures can have the same area with different perimeters, or the same perimeter with different areas.
- Areas of regions can be estimated (or even determined exactly) by breaking up such regions into unit squares, or into more general-shaped rectangles and triangles whose areas can be calculated.