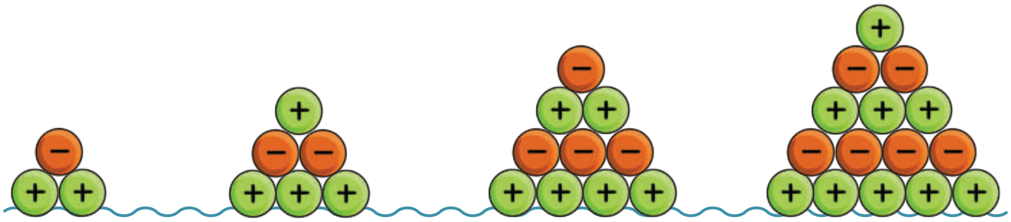


10 THE OTHER SIDE OF ZERO

Integers

0674CH10

More and More Numbers!

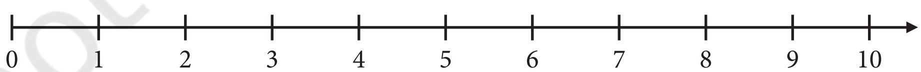
Recall that the very first numbers we learned about in the study of mathematics were the counting numbers $1, 2, 3, 4, \ldots$

Then we learned that there are even more numbers! For example, there is the number 0 (zero), representing nothing, which comes before 1. The number 0 has a very important history in India and now in the world. For example, around the world we learn to write numbers in the Indian number system using the digits 0 to 9, allowing us to write numbers however large or however small using just these 10 digits.

We then learned about more numbers that exist between the numbers $0, 1, 2, 3, 4, \ldots$, such as $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{13}{6}$. These are called *fractions*.

But are there still more numbers? Well, 0 is an additional number that we didn't know about earlier, and it comes before 1 and is less than 1. Are there perhaps more numbers that come before 0 and are less than 0?

Phrased another way, we have seen the number line:



However, this is actually only a number '*ray*', in the language we learned earlier in geometry; this ray starts at 0 and goes forever to the right. Do there exist numbers to the left of 0, so that this number ray can be completed to a true number line?

That is what we will investigate in this chapter!

Can there be a number less than 0? Can you think of any ways to have less than 0 of something?

10.1 Bela's Building of Fun

Children flock to Bela's ice cream factory to see and taste her tasty ice cream. To make it even more fun for them, Bela purchased a multi-storied building and filled it with attractions. She named it *Bela's Building of Fun*.

But this was no ordinary building!

Observe that some of the floors in the 'Building of Fun' are below the ground. What are the shops that you find on these floors? What is there on the ground floor?

A lift is used to go up and down between the floors. It has two buttons: '+' to go up and '–' to go down. Can you spot the lift?

To go to the Art Centre from the Welcome Hall, you must press the '+' button twice.

We say that the button press is + + or + 2.

To go down two floors, you must press the '–' button twice, which we write as – – or – 2.

So if you press + 1 (i.e., if you press the '+' button once), then you will go up one floor and if you press – 1 (i.e., if you press the '–' button once), then you will go down 1 floor.

> Lift button presses and numbers:
> +++ is written as + 3
> – – – – is written as – 4

What do you press to go four floors up? What do you press to go three floors down?

Numbering the Floors in the Building of Fun

Entry to the 'Building of Fun' is at the ground floor level and is called the 'Welcome Hall'. Starting from the ground floor, you can reach the Food Court by pressing +1 and can reach the Art Centre by pressing +2. So, we can say that the Food Court is on Floor +1 and that the Art Centre is on Floor +2.

Starting from the ground floor, you must press –1 to reach the Toy Store. So, the Toy Store is on Floor –1 similarly starting from the ground floor, you must press –2 to reach the Video Games shop. So, the Video Games shop is on Floor –2.

The Ground floor is called Floor 0. Can you see why?

Number all the floors in the Building of Fun.

Did you notice that +3 is the floor number of the Book Store, but it is also the number of floors you move when you press +3? Similarly, –3 is the floor number but it is also the number of floors you go down when you press –3, i.e., when you press – – – .

A number with a '+' sign in front is called a **positive number**. A number with a '–' sign in front is called a **negative number**.

In the 'Building of Fun', the floors are numbered using the ground floor, Floor 0, as a reference or starting point. The floors above the ground floor are numbered with positive numbers. To get to them from the ground floor, one must press the + button some number of times. The floors below the ground are numbered with negative numbers. To get to them from the ground floor, one must press the – button some number of times.

Zero is neither a positive nor a negative number. We do not put a '+' or '–' sign in front of it.

Addition to Keep Track of Movement

Start from the Food Court and press $+2$ in the lift. Where will you reach? ___________

We can describe this using an expression:

Starting floor + Movement = Target floor.

The starting floor is $+1$ (Food Court) and the number of button presses is $+2$. Therefore, you reach the target floor $\mathbf{(+1) + (+2) = +3}$ (Book Store).

Figure it Out

1. You start from Floor $+2$ and press -3 in the lift. Where will you reach? Write an expression for this movement.
2. Evaluate these expressions (you may think of them as Starting Floor + Movement by referring to the Building of Fun).
   a. $(+1) + (+4) =$ ______ b. $(+4) + (+1) =$ ______
   c. $(+4) + (-3) =$ ______ d. $(-1) + (+2) =$ ______
   e. $(-1) + (+1) =$ ______ f. $0 + (+2) =$ ________
   g. $0 + (-2) =$ ________
3. Starting from different floors, find the movements required to reach Floor -5. For example, if I start at Floor $+2$, I must press -7 to reach Floor -5. The expression is $(+2) + (-7) = -5$.
   Find more such starting positions and the movements needed to reach Floor -5 and write the expressions.

Combining Button Presses is also Addition

Gurmit was in the Toy Store and wanted to go down two floors. But by mistake he pressed the '+' button two times. He realised his mistake and quickly pressed the '–' button three times. How many floors below or above the Toy Store will Gurmit reach?

Gurmit will go one floor down. We can show the movement resulting from combining button presses as an expression: $(+2) + (-3) = -1$.

Figure it out

Evaluate these expressions by thinking of them as the resulting movement of combining button presses:

a. $(+1) + (+4) =$ ____________ b. $(+4) + (+1) =$ ____________

c. $(+4) + (-3) + (-2) =$ ______ d. $(-1) + (+2) + (-3) =$ ______

Back to Zero!

On the ground floor, Basant is in a great hurry and by mistake he presses +3. What can he do to cancel it and stay on the ground floor? He can cancel it by pressing –3. That is, (+3) + (–3) = 0.

We call –3 the inverse of +3. Similarly, the inverse of –3 is +3.

If Basant now presses +4 and then presses –4 in the lift, where will he reach?

Here is another way to think of the concept of inverse. If you are at Floor +4 and you press its inverse –4, then you are back to zero, the ground floor! If you are at Floor –2 and press its inverse +2, then you go to (–2) + (+2) = 0, again the ground floor!

- Write the inverses of these numbers:

  +4, –4, –3, 0, +2, –1.

- Connect the inverses by drawing lines.

+5 –7 –8 +9

–9 +8 –5 +7

Comparing Numbers using Floors

- Who is on the lowest floor?
  1. Jay is in the Art Centre. So, he is on Floor +2.
  2. Asin is in the Sports Centre. So, she is on Floor ___.
  3. Binnu is in the Cinema Centre. So, she is on Floor ____.
  4. Aman is in the Toys Shop. So, he is on Floor ____.

Floor +3 is lower than Floor +4. So, we write $+3 < +4$. We also write $+4 > +3$.

Should we write $-3 < -4$ or $-4 < -3$?

Floor −4 is lower than Floor −3. So, $-4 < -3$. It is also correct to write $-3 > -4$

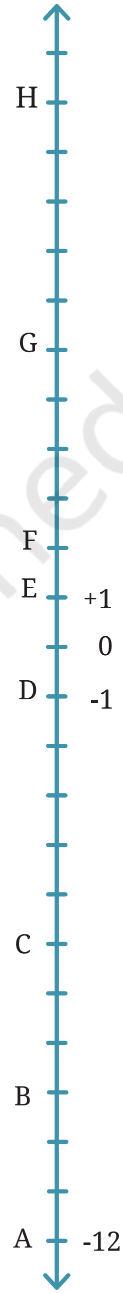


Figure it Out

1. Compare the following numbers using the Building of Fun and fill in the boxes with < or >.

   a. $-2 \ \square\ +5$ b. $-5 \ \square\ +4$ c. $-5 \ \square\ -3$

   d. $+6 \ \square\ -6$ e. $0 \ \square\ -4$ f. $0 \ \square\ +4$

> Notice that all negative number floors are below Floor 0. So, all negative numbers are less than 0. All the positive number floors are above Floor 0. So, all positive numbers are greater than 0.

2. Imagine the Building of Fun with more floors. Compare the numbers and fill in the boxes with < or >:

   a. $-10 \ \square\ -12$ b. $+17 \ \square\ -10$

   c. $0 \ \square\ -20$ d. $+9 \ \square\ -9$

   e. $-25 \ \square\ -7$ f. $+15 \ \square\ -17$

3. If Floor A = −12, Floor D = −1 and Floor E = +1 in the building shown on the right as a line, find the numbers of Floors B, C, F, G and H.

4. Mark the following floors of the building shown on the right.

   a. −7 b. − 4 c. + 3 d. − 10

Subtraction to Find which Button to Press

In earlier classes, we understood the meaning of subtraction as 'take away'. For example, "There are 10 books on the shelf. I take away 4 books. How many are left on the shelf?"

We can express the answer using subtraction: 10 – 4 = 6. Or 'Ten take away four is six.'

You may also be familiar with another meaning of subtraction which is related to comparison or making quantities equal. For example, consider this situation: "I have ₹10 with me and my sister has ₹6."

Now, I can ask the question: 'How much more money should my sister get in order to have the same amount as me?'

We can write this in two ways: $6 + ? = 10$ Or $10 - 6 = ?$.

Here, we see the connection between 'finding the missing number to be added' and subtraction.

For subtraction of positive and negative numbers, we will use this meaning of subtraction as 'making equal' or 'finding the missing number to be added'.

Evaluate $15-5$, $100-10$ and $74-34$ from this perspective.

Teachers' Note

In general, when there are two unequal quantities, subtraction can indicate the change needed to make the quantities equal. Subtraction shows how much the starting quantity should change in order to become the target quantity. In the context of different floor levels, what is the change required to reach the Target Floor from the Starting Floor? Notice that the change needed may be positive (for an increase) or negative (for a decrease).

Your starting floor is the Art Centre and your target floor is the Sports Centre. What should be your button press?

You need to go three floors up, so you should press $+3$. We can write this as an expression using subtraction:

Target floor – Starting floor = Movement needed.

In the above example, the starting floor is $+2$ (Art Centre) and the target floor is $+5$. The button press to get to $+5$ from $+2$ is $+3$. Therefore,

$$\mathbf{(+5) - (+2) = +3.}$$

Explanation:

Recall the connection between addition and subtraction. For $3 + ? = 5$, we can find the missing number using subtraction: $5 - 3 = 2$. That is, subtraction is the same as finding the missing number to be added.

We know that

Starting floor + Movement needed = Target Floor.

If the movement needed is to be found, then,

Starting floor + ? = Target Floor.

So

Target floor – Starting floor = ? = Movement needed.

More examples:

a. If the Target Floor is -1 and Starting Floor is -2, what button should you press?
   You need to go one floor up, so, you should press $+1$.
   Expression: $\mathbf{(-1) - (-2) = (+1)}$.

b. If the Target Floor is -1 and Starting floor is $+3$, what button should you press?
   You need to go four floors down, so, you should press -4.
   Expression: $\mathbf{(-1) - (+3) = (-4)}$.

c. If the Target Floor is $+2$ and Starting Floor is -2, what button should you press?
   You need to go four floors up, so, you should press $+4$.
   Expression: $\mathbf{(+2) - (-2) = (+4)}$.

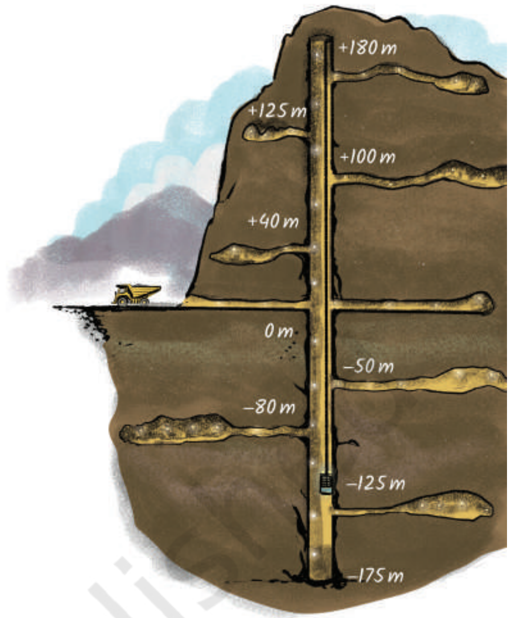
Figure it Out

Complete these expressions. You may think of them as finding the movement needed to reach the Target Floor from the Starting Floor.

a. $(+1) - (+4) =$ _______ b. $(0) - (+2) =$ _______
c. $(+4) - (+1) =$ _______ d. $(0) - (-2) =$ _______
e. $(+4) - (-3) =$ _______ f. $(-4) - (-3) =$ _______
g. $(-1) - (+2) =$ _______ h. $(-2) - (-2) =$ _______
i. $(-1) - (+1) =$ _______ j. $(+3) - (-3) =$ _______

Adding and Subtracting Larger Numbers

The picture shows a mine, a place where minerals are extracted by digging into the rock. The truck is at the ground level, but the minerals are present both above and below the ground level. There is a fast moving lift which moves up and down in a mineshaft carrying people and ore.



Some of the levels are marked in the picture. The ground level is marked 0. Levels above the ground are marked by positive numbers and levels below the ground are marked by negative numbers. The number indicates how many meters above or below the ground level it is.

In the mine, just like in the Building of Fun:

Starting level + Movement = Target level.

For example:

$(+40) + (+60) = +100$ $\qquad$ $(–90) + (–55) = –145$

Target level – Starting level = Movement needed.

For example:

$(+40) – (–50) = +90$ $\qquad$ $(–90) – (+40) = –130$

How many negative numbers are there?

Bela's Building of Fun had only six floors above and five floors below. That is numbers –5 to +6. In the mine above, we have numbers from –200 to +180. But we can imagine larger buildings or mineshafts. Just as positive numbers +1, +2, +3, ... keep going up without an end, similarly, negative numbers –1, –2, –3, ... keep going down. Positive and negative numbers, with zero, are called **integers**. They go both ways from 0: ... –4, –3, –2, –1, 0, 1, 2, 3, 4, ...

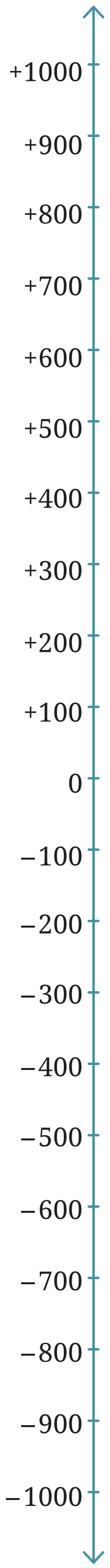


Figure it Out

Complete these expressions.

a. $(+40) + \_\_\_\_\_\_ = +200$
b. $(+40) + \_\_\_\_\_\_ = -200$
c. $(-50) + \_\_\_\_\_\_ = +200$
d. $(-50) + \_\_\_\_\_\_ = -200$
e. $(-200) - (-40) = \_\_\_\_\_\_$
f. $(+200) - (+40) = \_\_\_\_\_\_$
g. $(-200) - (+40) = \_\_\_\_\_\_$

Check your answers by thinking about the movement in the mineshaft.

Adding, Subtracting, and Comparing any Numbers

To add and subtract even larger integers, we can imagine even larger lifts! In fact, we can imagine a lift that can extend forever upwards and forever downwards, starting from Level 0. There does not even have to be any building or mine around – just an 'infinite lift'!

We can use this imagination to add and subtract any integers we like.

For example, suppose we want to carry out the subtraction $+2000 - (-200)$. We can imagine a lift with 2000 levels above the ground and 200 below the ground. Recall that

Target level – Starting level = Movement needed.

To go from the Starting Floor –200 to the Target Floor +2000, we must press +2200 (+200 to get to zero, and then +2000 more after that to get to +2200). Therefore, $(+2000) - (-200) = +2200$.

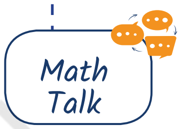
Notice that $(+2000) + (+200)$ is also +2200.

Try evaluating the following expressions by similarly drawing or imagining a suitable lift:

a. $-125 + (-30)$
b. $+105 - (-55)$
c. $+105 + (+55)$
d. $+80 - (-150)$
e. $+80 + (+150)$
f. $-99 - (-200)$
g. $-99 + (+200)$
h. $+1500 - (-1500)$

In the above example, we saw that $+2000 - (-200) = +2000 + (+200) = +2200$. In other words, subtracting a negative number is the same as adding the corresponding positive number. That is, we can replace subtraction of a negative number by addition of a positive number!

In the other exercises that you did above, did you notice that subtracting a negative number was the same as adding the corresponding positive number?



Take a look at the 'infinite lift' above. Does it remind you of a number line? In what ways?

Back to the Number Line

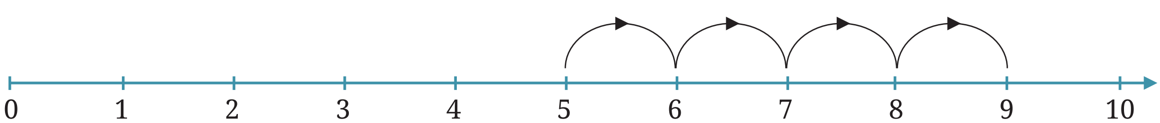
The 'infinite lift' we saw above looked very much like a number line, didn't it? In fact, if we rotate it by 90°, it basically becomes a number line. It also tells us how to complete the number *ray* to a number line, answering the question that we had asked at the beginning of the chapter. To the left of 0 are the negative numbers $-1, -2, -3, \ldots$

Usually we drop the + signs on positive numbers, and simply write them as 1, 2, 3, …

Instead of traveling along the number line using a lift, we can simply imagine walking on it. To the right is the positive (forward) direction, and to the left is the negative (backward) direction.

Smaller numbers are now to the left of bigger numbers, and bigger numbers are to the right of smaller numbers. So $2 < 5$; $-3 < 2$; and $-5 < -3$.

If, from 5 you wish to go over to 9, how far must you travel along the number line?



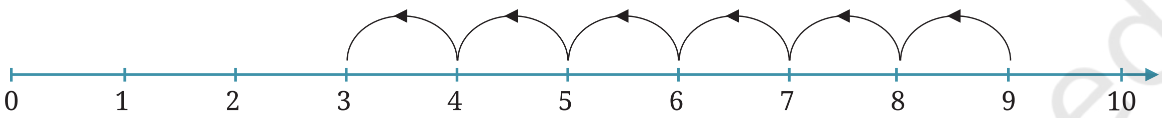
You must travel 4 steps. That is why $5 + 4 = 9$.

(Remember: **Starting Number** + **Movement** = **Target Number.**)

The corresponding subtraction statement is $9 - 5 = 4$.

(Remember: **Target Number** – **Starting Number** = **Movement Needed.**)

Now, from 9, if you wish to go to 3, how much must you travel along the number line?



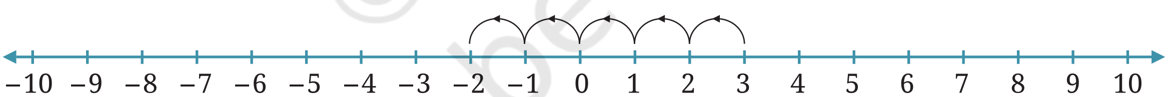
You must move 6 steps backward, i.e., you must move –6. Hence, we write $9 + (–6) = 3$.

(Remember again : **Starting number** + **Movement** = **Target number.**)

The corresponding subtraction statement is $3 - 9 = -6$.

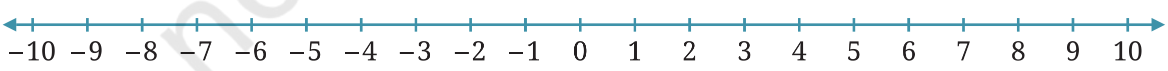
(Remember again: **Target number** – **Starting number** = **Movement needed.**)

Now, from 3, if you wish to go to –2, how far must you travel?



You must travel –5 steps, i.e., 5 steps backward. Thus $3 + (-5) = -2$. The corresponding subtraction statement is: $-2 - 3 = -5$.

Figure it Out



1. Mark 3 positive numbers and 3 negative numbers on the number line above.
2. Write down the above 3 marked negative numbers in the following boxes: ☐ ☐ ☐

3. Is $2 > -3$? Why? Is $-2 < 3$? Why?
4. What are (i) $-5+0$ (ii) $7+(-7)$ (iii) $-10+20$ (iv) $10-20$ (v) $7-(-7)$ (vi) $-8-(-10)$?

Using the Unmarked Number Line to Add and Subtract

Just as you can do additions, subtractions and comparisons with small numbers using the number line above, you can also do them with large numbers by imagining an 'infinite number line', or drawing an 'unmarked number line' as follows:

This line shows only the position of zero. Other numbers are not marked. It can be convenient to use this **u**nmarked **n**umber **l**ine to add and subtract integers. You can show, or simply imagine, the scale of the number line and the positions of numbers on it.

For example, this unmarked number line (UNL) shows the addition problem: $85 + (-60) = ?$:

We then can visualise that $85 + (-60) = 25$

The following UNL shows a subtraction problem which can also be written as a missing addend problem: $(-100)-(+250) = ?$ or $250 + ? = -100$.

We can then visualise that $? = -350$ in this problem.

In this way, you can carry out addition and subtraction problems, with positive and negative numbers, on paper or in your head using an unmarked number line.

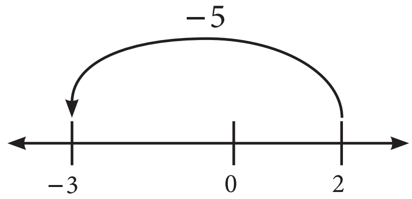
Use unmarked number lines to evaluate these expressions:

a. $-125 + (-30) =$ _______

b. $+105 - (-55) =$ _______

c. $+80 - (-150) =$ _______

d. $-99 - (-200) =$ _______



Converting subtraction to addition and addition to subtraction

Recall that **Target floor – Starting floor = Movement needed**

or

Target floor = Starting floor + Movement needed

If we start at 2 and wish to go to –3, what is the movement needed?

First method: Looking at the number line, we see we need to move –5 (i.e., 5 in the backward direction). Therefore, **$-3 - 2 = -5$**. The movement needed is –5.

Second method: Break the journey from 2 to –3 into two parts.

a. From 2 to 0, the movement is $0 - 2 = -2$.

b. From 0 to –3, the movement is $-3 - 0 = -3$.

The total movement is the sum of the two movements: **$-3 + (-2) = -5$**.

Look at the two coloured expressions. There is no subtraction in the second one!

In this way, we can always convert subtraction to addition. **The number that is being subtracted can be replaced by its inverse and then added instead.**

Similarly, a number that is being added can be replaced by its inverse and then subtracted. In this way, we can also always convert addition to subtraction.

Examples:

a. $(+7) - (+5) = (+7) + (-5)$

b. $(-3) - (+8) = (-3) + (-8)$

c. $(+8) - (-2) = (+8) + (+2)$

d. $(+6) - (-9) = (+6) + (+9)$

10.2 The Token Model

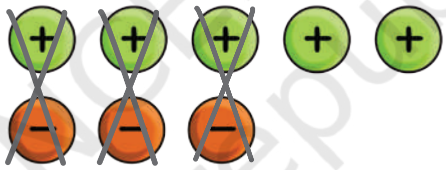
Using Tokens for Addition

In Bela's Building of Fun, the lift attendant is bored. To entertain himself, he keeps a box containing lots of positive (red) and negative (black) tokens. Each time he presses the '+' button, he takes a positive token from the box and puts it in his pocket. Similarly, each time he presses the '–' button, he takes a negative token and puts it in his pocket.

He starts on the ground floor (Floor 0) with an empty pocket. After one hour, he checks his pocket and finds 5 positive and 3 negative tokens. On which floor is he now?

He must have pressed '+' five times and '–' 3 times and $(+5)+(-3)=+2$. So he is at Floor $+2$ now.

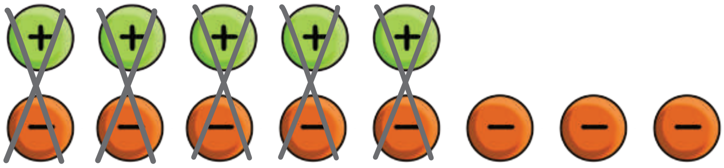
Here is another way to do the calculation.



A positive token and a negative token cancel each other, because the value of this pair of tokens together is zero. These two tokens in his pocket meant that he pressed '+' once and '–' once, respectively, and these cancel each other. We say that a positive and a negative token make a 'zero pair'. When you remove all the zero pairs, you are left with two positive tokens, so $(+5) + (-3) = +2$.

We can perform any such addition using tokens!

Example: Add $+5$ and -8.



From the picture, we see that we can remove five zero pairs, and we are then left with –3. Therefore (+5) + (–8) = –3.

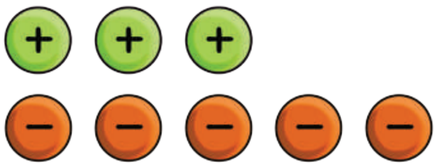
Figure it Out

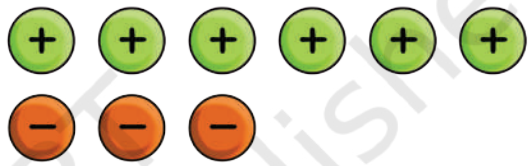
1. Complete the additions using tokens.

a. (+6) + (+4) b. (–3) + (–2)

c. (+5) + (–7) d. (–2) + (+6)

2. Cancel the zero pairs in the following two sets of tokens. On what floor is the lift attendant in each case? What is the corresponding addition statement in each case?

a. 

b. 

Using Tokens for Subtraction

We have seen how to perform addition of integers with positive tokens and negative tokens. We can also perform subtraction using tokens!

Example: Let us subtract: (+5) – (+4).

(+5) – (+4) = +1

This is easy to do. From 5 positives take away 4 positives to see the result.

Example: Let us subtract: (–7) – (–5).

(–7) – (–5) = –2

Is (–7) – (–5) the same as (–7) + (+5)?

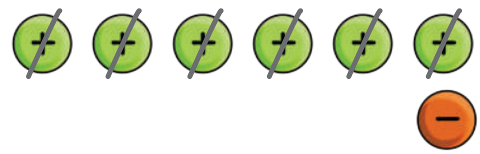
Example: Let us subtract: (+5) – (+6).

Put down 5 positives.

But there are not enough tokens to take out 6 positives!

To get around this issue, we can put out an extra zero pair (a positive and a negative), knowing that this does not change the value of the set of tokens.

Now we can take out 6 positives! See what is left:



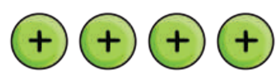
We conclude that $(+5) - (+6) = -1$.

Figure it Out

1. Evaluate the following differences using tokens. Check that you get the same result as with other methods you now know:
   a. $(+10) - (+7)$ b. $(-8) - (-4)$ c. $(-9) - (-4)$
   d. $(+9) - (+12)$ e. $(-5) - (-7)$ f. $(-2) - (-6)$
2. Complete the subtractions:
   a. $(-5) - (-7)$ b. $(+10) - (+13)$ c. $(-7) - (-9)$
   d. $(+3) - (+8)$ e. $(-2) - (-7)$ f. $(+3) - (+15)$

Example: $+4 - (-6)$.

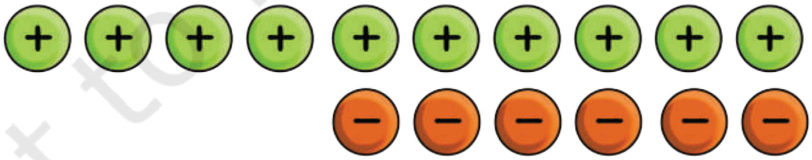
Start with 4 positives.



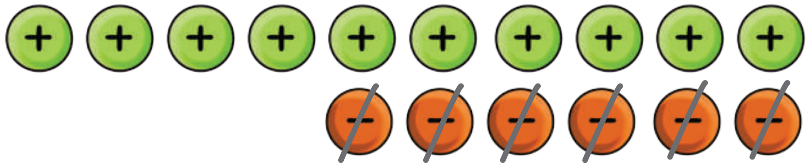
We have to take out 6 negatives from these. But there are not enough negatives.

This is not a problem. We add some zero pairs as this does not change the value of the set of tokens.

But how many zero pairs? We have to take away 6 negatives so we put down 6 zero pairs:



Now we can take away 6 negatives:



Therefore, $+4 - (-6) = +10$.

Figure it Out

1. Try to subtract: $-3-(+5)$.
   How many zero pairs will you have to put in? What is the result?
2. Evaluate the following using tokens.
   a. $(-3)-(+10)$ b. $(+8)-(-7)$ c. $(-5)-(+9)$
   d. $(-9)-(+10)$ e. $(+6)-(-4)$ f. $(-2)-(+7)$

10.3 Integers in Other Places

Credits and Debits

Suppose you open a bank account at your local bank with the ₹100 that you had been saving over the last month. Your bank balance therefore starts at ₹100.

Then you make ₹60 at your job the next day and you deposit it in your account. This is shown in your bank passbook as a 'credit'.

- Your new bank balance is ______.

The next day you pay your electric bill of ₹30 using your bank account. This is shown in your bank passbook as a 'debit'.

- Your bank balance is now ______.

The next day you make a major purchase for your business of ₹150. Again this is shown as a debit.

- What is your bank balance now? ______

Is this possible?

(Yes, some banks do allow your account balance to become negative, temporarily! Some banks also charge you an additional amount if your balance becomes negative, in the form of 'interest' or a 'fee'.)

Your strategic large purchase the previous day allows you to make 200 rupees at your business the next day.

- What is your balance now? ______

You can think of 'credits' as positive numbers and 'debits' as negative numbers. The total of all your credits (positive numbers) and debits (negative numbers) is your total bank account balance. This can be positive or negative!

In general, it is better to try to keep a positive balance in your bank account!

Figure it Out

1. Suppose you start with 0 rupees in your bank account, and then you have credits of ₹30, ₹40, and ₹50, and debits of ₹40, ₹50, and ₹60. What is your bank account balance now?
2. Suppose you start with 0 rupees in your bank account, and then you have debits of ₹1, 2, 4, 8, 16, 32, 64, and 128, and then a single credit of ₹256. What is your bank account balance now?
3. Why is it generally better to try and maintain a positive balance in your bank account? What are circumstances under which it may be worthwhile to temporarily have a negative balance?

As you can see, positive and negative numbers along with zero are extremely useful in the world of banking and accounting.

Geographical Cross-sections

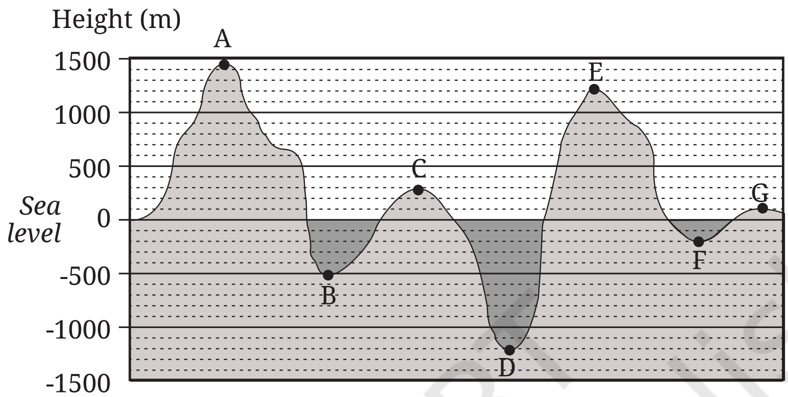
We measure the height of geographical features like mountains, plateaus, and deserts from 'sea level'. The height at sea level is 0m. Heights above sea level are represented using positive numbers and heights below sea level are represented using negative numbers.

Figure it Out

1. Looking at the geographical cross section fill in the respective heights:

   A ____ B ____ C ____ D ____

   E ____ F: ____ G ____



Teachers' Note

Ask what a geographical cross-section is by showing the figure in this page. It is like imagining a vertical slice taken out at some location on the earth. This is what would be seen from a side view. Discuss the notion of "sea level" for measuring heights and depths in geography.

2. Which is the highest point in this geographical cross-section? Which is the lowest point?
3. Can you write the points A, B, ..., G in a sequence of decreasing order of heights? Can you write the points in a sequence of increasing order of heights?
4. What is the highest point above sea level on Earth? What is its height?
5. What is the lowest point with respect to sea level on land or on the ocean floor? What is its height? (This height should be negative).

Temperature

During summertime you would have heard in the news that there is a 'heat wave'. What do you think will be the temperature during the summer when you feel very hot? In winter we have cooler or colder temperatures.

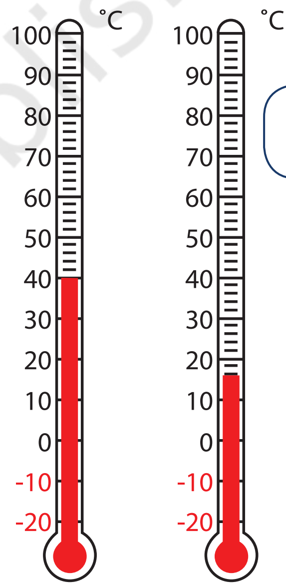
What has been the maximum temperature during the summer and the minimum temperature during the winter last year in your area? Find out.

When we measure temperature, we use Celsius as the unit of measure (°C). The thermometers below are showing 40°C and 15°C temperatures.

Figure it Out

Math Talk

1. Do you know that there are some places in India where temperatures can go below 0°C? Find out the places in India where temperatures sometimes go below 0°C. What is common among these places? Why does it become colder there and not in other places?



2. Leh in Ladakh gets very cold during winter. The following is a table of temperature readings taken during different times of the day/night in Leh on a day in November. Match the temperature with the appropriate time of the day/night.

Temperature
14°C
8°C
-2°C
-4°C

Time
02:00 am
11:00 pm
02:00 pm
11:00 am

Teachers' Note

Talk about thermometers and how they are used to measure temperature. Bring a laboratory thermometer to the class and measure the temperature of hot water and cold water. Point out to children that there are markings in the thermometer that are below 0°C. Have a discussion on what 0°C indicates, namely, the freezing point of water.

10.4 Explorations with Integers

A Hollow Integer Grid

4	–1	–3
–3		1
–1	–1	2

5	–3	–5
0		–5
–8	–2	7

There is something special about the numbers in these two grids. Let us explore what that is.

Top row:	$4 + (-1) + (-3) = 0$	$5 + (-3) + (-5) =$ ____
Bottom row:	$(-1) + (-1) + 2 = 0$	$(-8) + (-2) + 7 =$ ____
Left column:	$4 + (-3) + (-1) = 0$	$5 + 0 + (-8) =$ ____
Right column:	$(-3) + 1 + 2 = 0$	$(-5) + (-5) + 7 =$ ____

In each grid, the numbers in each of the two rows (the top row and the bottom row) and the numbers in each of the two columns (the leftmost column and the rightmost column) add up to give the same number. We shall call this sum as the 'border sum'. The border sum of the first grid is '0'.

Figure it Out

1. Do the calculations for the second grid above and find the border sum.

2. Complete the grids to make the required border sum:

–10		
		–5
9		

Border sum is +4

6	8	
		–5
	–2	

Border sum is –2

7		
		–5

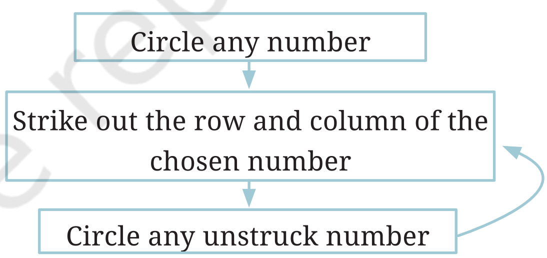
Border sum is –4

3. For the last grid above, find more than one way of filling the numbers to get border sum –4.
4. Which other grids can be filled in multiple ways? What could be the reason?
5. Make a border integer square puzzle and challenge your classmates.

An Amazing Grid of Numbers!

Below is a grid having some numbers. Follow the steps as shown until no number is left.

3	4	0	9
–2	–1	–5	4
1	2	–2	7
–7	–6	–10	–1



When there are no more unstruck numbers, STOP. Add the circled numbers.

In the example below, the circled numbers are –1, 9, –7, –2. If you add them, you get –1.

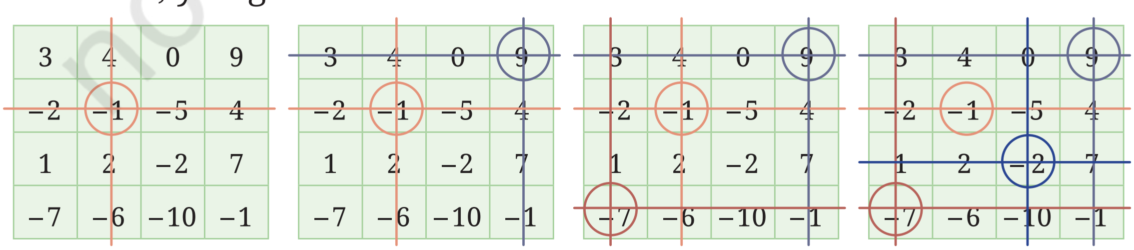


Figure it Out

1. Try afresh, choose different numbers this time. What sum did you get? Was it different from the first time? Try a few more times!
2. Play the same game with the grids below. What answer did you get?

7	10	13	16
–2	1	4	7
–11	–8	–5	–2
–20	–7	–14	–11

–11	–10	–9	–8
–7	–6	–5	–4
–3	–2	–1	0
1	2	3	4

3. What could be so special about these grids? Is the magic in the numbers or the way they are arranged or both? Can you make more such grids?

Try This

Figure it Out

1. Write all the integers between the given pairs, in increasing order.
   a. 0 and –7 b. –4 and 4
   c. –8 and –15 d. –30 and –23
2. Give three numbers such that their sum is –8.
3. There are two dice whose faces have these numbers: –1, 2, –3, 4, –5, 6. The smallest possible sum upon rolling these dice is –10 = (–5) + (–5) and the largest possible sum is 12 = (6) + (6). Some numbers between (–10) and (+ 12) are not possible to get by adding numbers on these two dice. Find those numbers.
4. Solve these:

8 – 13	(–8) – (13)	(–13) – (–8)	(–13) + (–8)
8 + (–13)	(–8) – (–13)	(13) – 8	13 – (–8)

5. Find the years below.
   a. From the present year, which year was it 150 years ago? ________
   b. From the present year, which year was it 2200 years ago? _______
   (**Hint**: Recall that there was no year 0.)

c. What will be the year 320 years after 680 BCE? ________

6. Complete the following sequences:

   a. (–40), (–34), (–28), (–22), _____, ______, ______

   b. 3, 4, 2, 5, 1, 6, 0, 7, _____, _____, _____

   c. _____, ______, 12, 6, 1, (–3), (–6), _____, ______, ______

7. Here are six integer cards: (+1), (+7), (+18), (–5), (–2), (–9).
   You can pick any of these and make an expression using addition(s) and subtraction(s).
   Here is an expression: (+18) + (+1) – (+7) – (–2) which gives a value (+14). Now, pick cards and make an expression such that its value is closer to (– 30).

8. The sum of two positive integers is always positive but a (positive integer) – (positive integer) can be positive or negative. What about

   a. (positive) – (negative)
   b. (positive) + (negative)
   c. (negative) + (negative)
   d. (negative) – (negative)
   e. (negative) – (positive)
   f. (negative) + (positive)

9. This string has a total of 100 tokens arranged in a particular pattern. What is the value of the string?

10.5 A Pinch of History

Like general fractions, general integers (including zero and the negative numbers) were first conceived of and used in Asia, thousands of years ago, before they eventually spread across the world in more modern times.

The first known instances of the use of negative numbers occurred in the context of accounting. In one of China's most important mathematical works, *The Nine Chapters on Mathematical Art (Jiuzhang Suanshu)*—which was completed by the first or second century CE—positive and negative numbers were represented using red and black rods, much like the way we represented them using red and black tokens!

There was a strong culture of accountancy also in India in ancient times. The concept of credit and debit was written about extensively by *Kautilya* in his Arthaśhāstra (c. 300 BCE), including the recognition that an account balance could be negative. The explicit use of negative numbers in the context of accounting is seen in a number of ancient Indian works, including in the Bakśhālī Manuscript from around the year 300, where a negative number was written using a special symbol placed after the number (rather than before the number as we do today).

The first general treatment of positive numbers, negative numbers, and *zero* - all on an equal footing as equally-valid numbers on which one can perform the basic operations of addition, subtraction, multiplication and even division–was given by Brahmagupta in his *Brāhma-sphuṭa-siddhānta* in the year 628 CE. Brahmagupta gave clear and explicit rules for operations on all numbers–positive, negative, and zero–that essentially formed the modern way of understanding these numbers that we still use today!

Some of Brahmagupta's key rules for addition and subtraction of positive numbers, negative numbers, and zero are given below:

Brahmagupta's Rules for Addition (*Brāhma-sphuṭa-siddhānta* **18.30, 628 CE):**

1. The sum of two positives is positive (e.g., 2 + 3 = 5).
2. The sum of two negatives is negative. To add two negatives, add the numbers (without the signs), and then place a minus sign to obtain the result (e.g., (–2) + (–3) = –5).
3. To add a positive number and a negative number, subtract the smaller number (without the sign) from the greater number (without the sign), and place the sign of the greater number to obtain the result (e.g., – 5 + 3 = –2, 2 + (–3) = –1 and –3 + 5 = 2).
4. The sum of a number and its inverse is zero (e.g., 2 + (–2) = 0).
5. The sum of any number and zero is the same number (e.g., –2 + 0 = –2 and 0 + 0 = 0).

Brahmagupta's Rules for Subtraction (Brāhma-sphuṭa-siddhānta 18.31-18.32):

1. If a smaller positive is subtracted from a larger positive, the result is positive (e.g., $3 - 2 = 1$).
2. If a larger positive is subtracted from a smaller positive, the result is negative (e.g., $2-3 = -1$).
3. Subtracting a negative number is the same as adding the corresponding positive number (e.g., $2 - (-3) = 2 + 3$).
4. Subtracting a number from itself gives zero (e.g., $2 - 2 = 0$ and $-2 - (-2) = 0$).
5. Subtracting zero from a number gives the same number (e.g., $-2 - 0 = -2$ and $0 - 0 = 0$). Subtracting a number from zero gives the number's inverse (e.g., $0 - (-2) = 2$).

Once you understand Brahmagupta's rules, you can do addition and subtraction with any numbers whatsoever - positive, negative, and zero!

Figure it Out

1. Can you explain each of Brahmagupta's rules in terms of Bela's Building of Fun, or in terms of a number line?
2. Give your own examples of each rule.

Brahmagupta was the first to describe zero as a number on an equal footing with positive numbers as well as with negative numbers, and the first to give explicit rules for performing arithmetic operations on all such numbers, positive, negative, and zero—forming what is now called a *ring*. It would change the way the world does mathematics.

However, it took many centuries for the rest world to adopt zero and negative numbers as numbers. These numbers were transmitted to, accepted, by and further studied by the Arab world by the 9th century, before making their way to Europe by the 13th century.

Surprisingly, negative numbers were still not accepted by many European mathematicians even in the 18th century. Lazare Carnot, a French mathematician in the 18th century, called negative numbers 'absurd'. But over time, zero as well as negative numbers proved to be indispensable in global mathematics and science, and are now considered to be critical numbers on an equal footing with and as important as positive numbers—just as Brahmagupta had recommended and explicitly described way back in the year 628 CE! This abstraction of arithmetic rules on all numbers paved the way for the modern development of algebra, which we will learn about in future classes.

SUMMARY

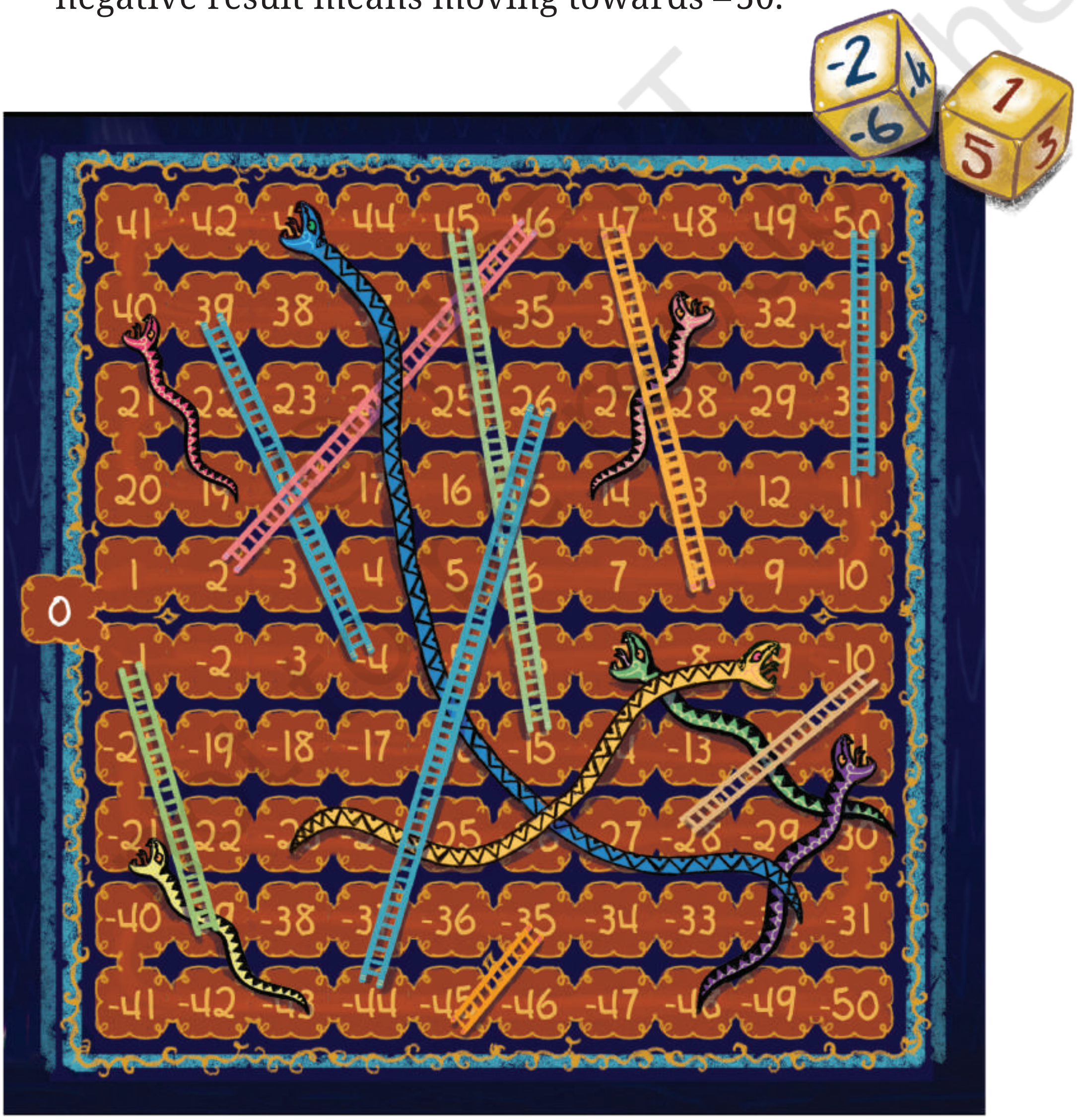
- There are numbers that are less than zero. They are written with a '–' sign in front of them (e.g., –2), and are called **negative numbers**. They lie to the left of zero on the number line.
- The numbers ..., –4, –3, –2, –1, 0, 1, 2, 3, 4, ... are called **integers**. The numbers 1, 2, 3, 4, ... are called **positive integers** and the numbers ..., –4, –3, –2, –1 are called **negative integers**. Zero (0) is neither positive nor negative.
- Every given number has another number associated to it which when added to the given number gives zero. This is called the **additive inverse** of the number. For example, the additive inverse of 7 is –7 and the additive inverse of –543 is 543.
- Addition can be interpreted as **Starting Position** + **Movement** = **Target Position**.
- Addition can also be interpreted as the combination of movements or increases/decreases: **Movement 1** + **Movement 2** = **Total Movement**.
- Subtraction can be interpreted as **Target Position – Starting Position** = **Movement**.

- In general, we can add two numbers by following Brahmagupta's Rules for Addition:
  a. If both numbers are positive, add the numbers and the result is a positive number (e.g., 2 + 3 = 5).
  b. If both numbers are negative, add the numbers (without the signs), and then place a minus sign to obtain the result (–2 + (–3) = –5).
  c. If one number is positive and the other is negative, subtract the smaller number (without the sign) from the greater number (without the sign), and place the sign of the greater number to obtain the result (e.g., –5 + 3 = –2).
  d. A number plus its additive inverse is zero (e.g., 2 + (–2) = 0).
  e. A number plus zero gives back the same number (e.g., –2 + 0 = –2).
- We can subtract two integers by converting the problem into an addition problem and then following the rules of addition. Subtraction of an integer is the same as the addition of its additive inverse.
- Integers can be compared: ... –3 < –2 < –1 < 0 < +1 < +2 < +3 <... Smaller numbers are to the left of larger numbers on the number line.
- We can give meaning to positive and negative numbers by interpreting them as credits and debits. We can also interpret positive numbers as distances above a reference point like the ground level. Similarly, negative numbers can be interpreted as distances below the ground level. When measuring temperatures in degrees Celsius, positive temperatures are those above the freezing point of water, and negative temperatures are those below the freezing point of water.

Integers: Snakes and Ladders

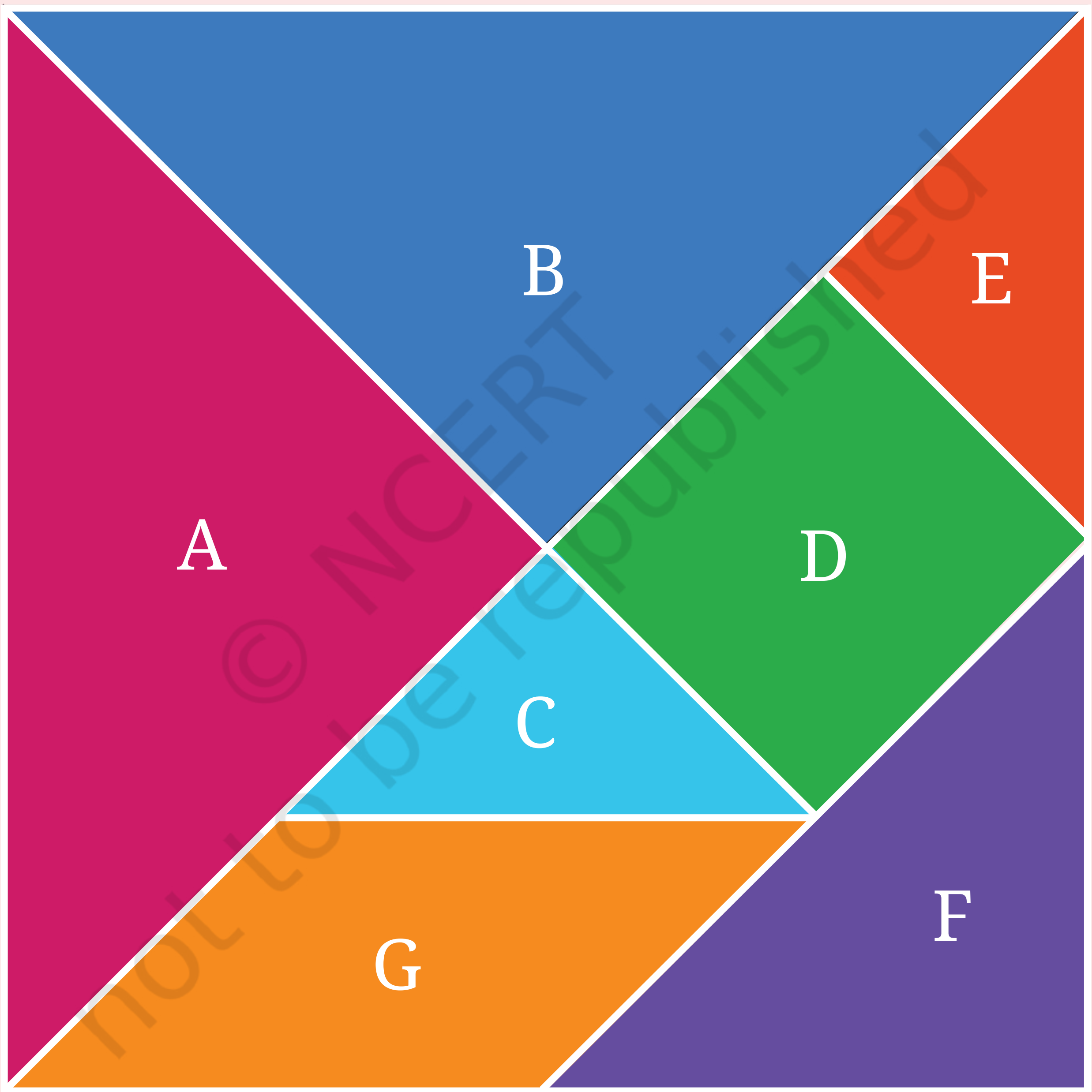
Rules

- This is a two player game. Each player has 1 pawn. Both players start at 0. Players can reach either –50 or +50 to win but need not decide or fix this before or during play.
- Each player rolls two dice at a time. One dice has numbers from +1 to +6 and the other dice has numbers from –1 to –6.
- After each roll of the two dice, the player can add or subtract them in any order and then move the steps that indicate the result. A positive result means moving towards +50 and a negative result means moving towards –50.



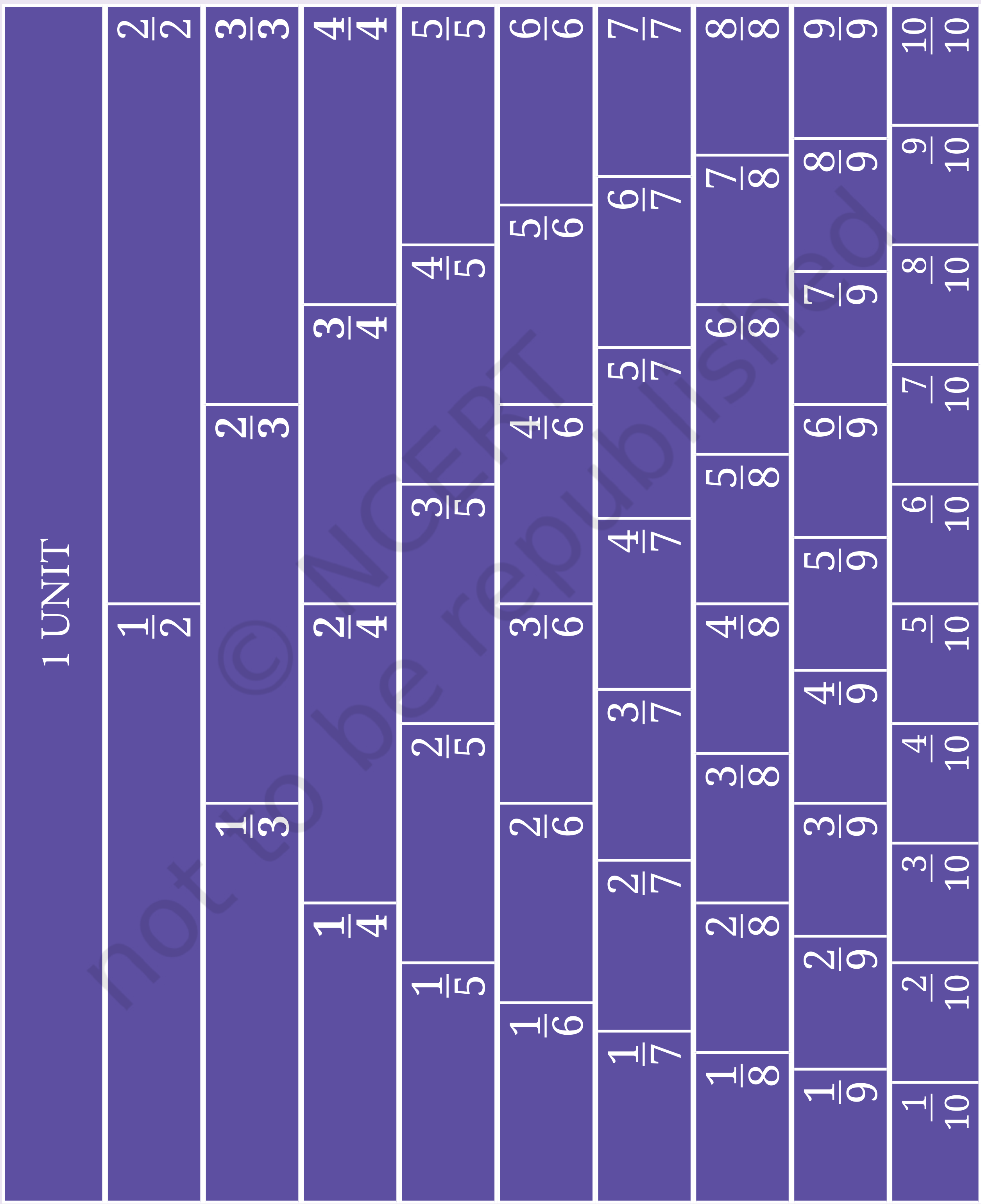
TANGRAM

Note: Cut each shape along the white border.

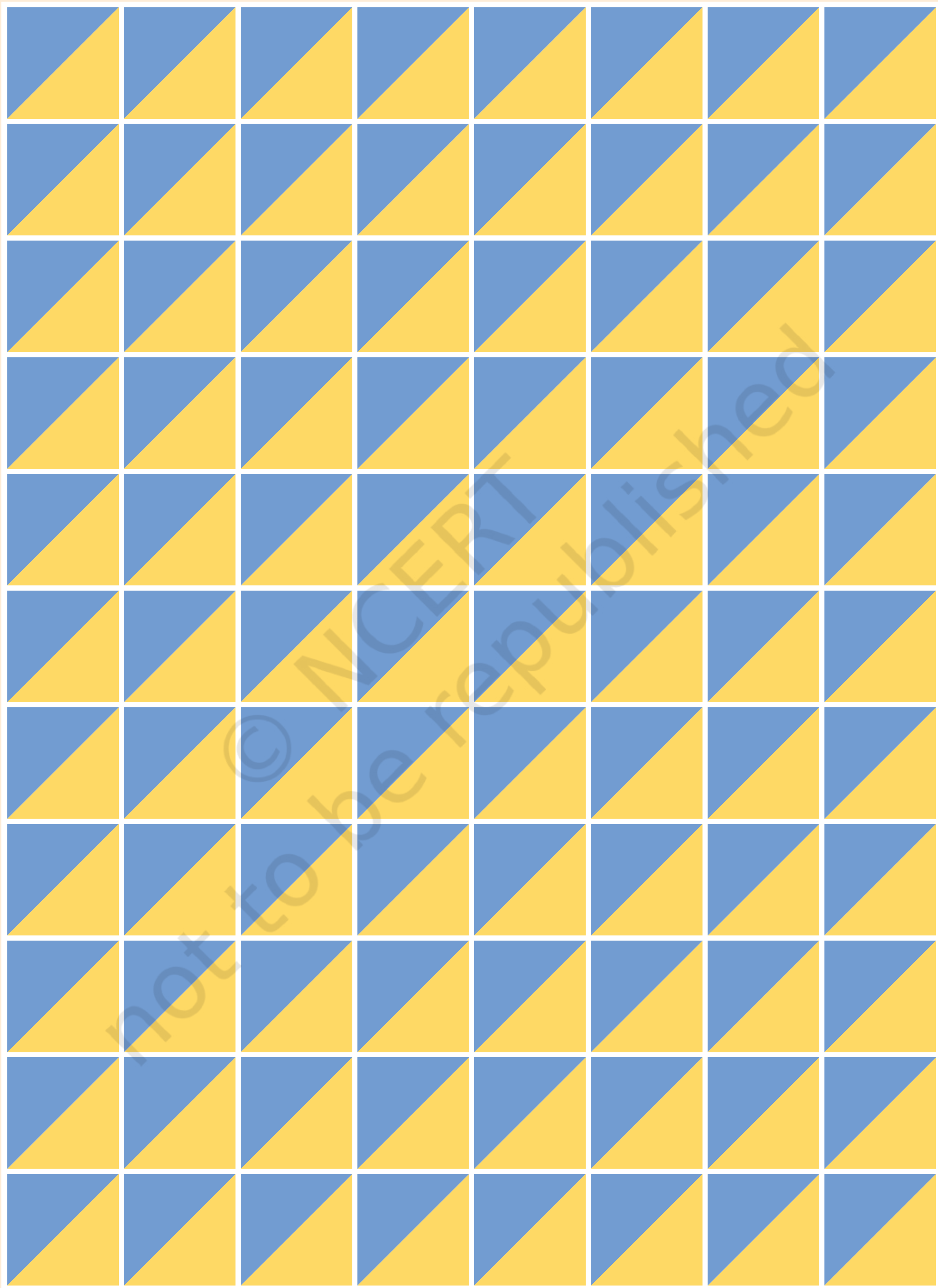


FRACTION WALL

Note: Cut each shape along the white border.



Note: Cut the tiles along the white border.



Notes

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