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PRACTICE PAPER - 2 (2023-24)
CHAPTER-9 CIRCLES (ANSWERS)

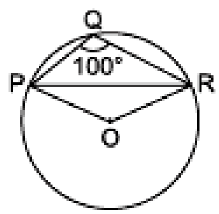
SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : IX **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

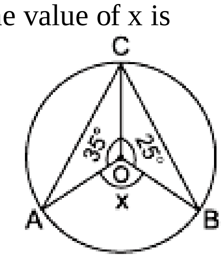
1. Diagonals of a cyclic quadrilateral are the diameters of that circle, then quadrilateral is a
(a) parallelogram (b) square (c) rectangle (d) trapezium
Ans: (c) rectangle

2. In the given figure, the value of ∠OPR is



(a) 65° (b) 10° (c) 20° (d) 50°
Ans: (b) 10°

3. In figure, O is the centre of the circle. The value of x is



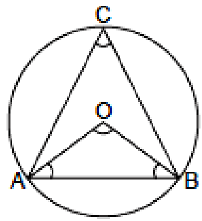
(a) 140° (b) 60° (c) 120° (d) 300°
Ans: (d) 300°
We have ∠AOC + ∠BOC + ∠AOB = 360°
(∵ Angle at the centre of a circle)
⇒ 35° + 25° + x = 360° ⇒ x = 300°

4. In a circle with centre O, chords AB and CD are of lengths 5 cm and 6 cm respectively and subtend angles x° and y° at the centre of the circle respectively. Then
(a) x = y (b) x < y (c) x > y (d) cannot say
Ans: (b) x < y

5. Given a circle of radius r and with centre O. A point P lies in a plane such that OP > r, then point P lies
(a) in the interior of the circle (b) on the circle

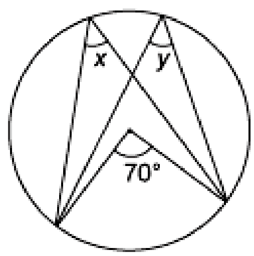
(c) in the exterior of the circle (d) cannot say
Ans: (c) in the exterior of the circle

6. A chord of a circle is equal to the radius of the circle. Then the angle subtended by the chord at the point of major arc is
(a) 90° (b) 30° (c) 150° (d) 60°
Ans: (b) 30°
Given, a chord of a circle is equal to the radius of the circle.



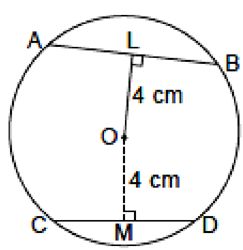
⇒ OA = AB = OB
So, Δ OAB is an equilateral triangle.
⇒ ∠AOB = 60°
Thus, $\angle ACB = \frac{1}{2} \times \angle AOB = \frac{1}{2} \times 60° = 30°$

7. In the given figure, value of y is



(a) 35° (b) 140° (c) 70° + x (d) 70°
Ans: (a) 35°
The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. So,
$y = \frac{1}{2} \times 70° = 35°$

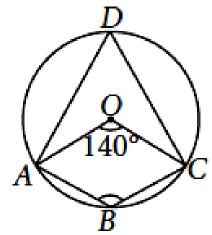
8. Given a chord AB of length 5 cm, of a circle with centre O. OL is perpendicular to chord AB and OL = 4 cm. OM is perpendicular to chord CD such that OM = 4 cm. Then CM is equal to
(a) 4 cm (b) 5 cm (c) 2.5 cm (d) 3 cm
Ans: (c) 2.5 cm
Since OL = OM = 4 cm, so
AB = CD (∵ Chords equidistant from the centre of a circle are equal in length)
⇒ CD = 5 cm



Since the perpendicular drawn from the centre of a circle to a chord bisects the chord, so
$CM = MD = \frac{1}{2} CD \Rightarrow CM = 2.5$ cm

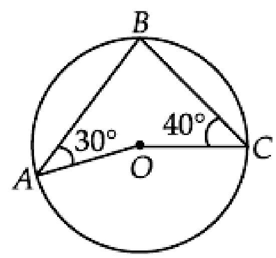
In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. Assertion (A): In the given figure, O is the centre of circle. If ∠AOC = 140°, then ∠ABC =110°.



Reason (R): In cyclic quadrilateral, opposite angles are supplementary.
Ans: (a) Both A and R are true and R is the correct explanation of A.

10. Assertion (A): In the given figure, ∠BAO = 30° and ∠BCO = 40°. Then the measure of ∠AOC= 70°.

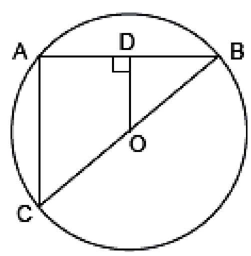


Reason (R): Angle subtended by an arc of a circle at the centre of the circle is twice the angle subtended by that arc on the remaining part of the circle.
Ans: (d) A is false but R is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If BC is a diameter of a circle of centre O and OD is perpendicular to the chord AB of a circle, show that CA = 2OD.
Ans: Since OD ⊥ AB

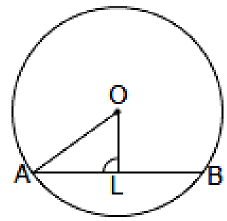


∴ D is the mid-point of AB (perpendicular drawn from the centre to a chord bisects the chord)
O is centre ⇒ O is the mid-point of BC.
In ΔABC, O and D are the mid-points of BC and AB, respectively.

∴ OD || AC and OD = $\frac{1}{2}$AC (mid-point theorem)

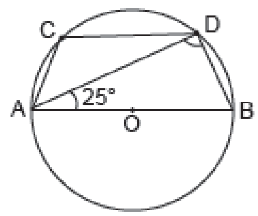
∴ CA = 2OD

12. In a circle of radius 5 cm having centre O, OL is drawn perpendicular to the chord AB. If OL = 3 cm, find the length of AB.
Ans: Let AB be a chord of circle having centre O.



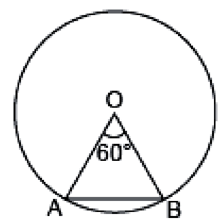
OL ⊥ AB and OA is the radius of the circle.
So, OA = 5 cm
In ΔOAL, Applying Pythagoras theorem, we have
$OA^2 = AL^2 + OL^2$
$\Rightarrow (5)^2 = AL^2 + (3)^2 \Rightarrow 25 = AL^2 + 9 \Rightarrow AL^2 = 25 - 9 = 16 \Rightarrow AL = \sqrt{16}$ cm = 4 cm
As we know that perpendicular drawn from the centre to the chord bisects the chord.
$\therefore$ AB = 2 (AL) = 2 (4) = 8 cm

13. In the given figure, AB is diameter of the circle with centre O and CD || AB. If ∠DAB = 25°, then find the measure of ∠CAD.



Ans: ∠ADB = 90° [Angle in a semicircle]
∠BAD = ∠ADC = 25° [Alternate interior angles]
$\therefore$ ∠BDC = 90° + 25° = 115°
Now, ∠BDC + ∠BAC = 180° [opp. ∠s of cyclic quadrilateral]
$\Rightarrow$ 115° + ∠BAC = 180°
$\Rightarrow$ ∠BAC = 180° – 115° = 65°
Now, ∠BAC = ∠BAD + ∠CAD
$\Rightarrow$ 65° = 25° + ∠CAD
$\therefore$ ∠CAD = 65° – 25° = 40°

14. In the given figure, chord AB subtends ∠AOB equal to 60° at the centre O of the circle. If OA =5 cm. then find the length of AB.



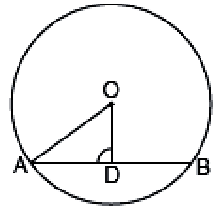
Ans: In ΔAOB
∠AOB = 60° (Given)
OA = OB (Equal radii)
$\therefore$ ∠OAB = ∠OBA (Angles opposite to equal sides OA and OB) ...(i)
In ΔAOB ∠OAB + ∠AOB + ∠OBA = 180° (Angle sum property of triangle)
60° + ∠OAB + ∠OAB = 180° ($\because$ ∠OAB = ∠OBA, using (i))
$\Rightarrow$ 2∠OAB = 180° – 60°
$\Rightarrow$ ∠OAB = 60° $\Rightarrow$ ∠OBA = 60°
$\therefore$ ΔAOB is an equilateral triangle
Hence OA = OB = AB
$\Rightarrow$ AB = 5 cm (as OA = 5 cm)

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Distance of a chord AB of a circle from the centre is 12 cm and length of the chord AB is 10 cm. Find the diameter of the circle.

Ans: Let in a circle having centre O, AB be the chord of length 10 cm.



Distance of chord AB from centre is OD

$\therefore$ OD = 12 cm

OA = radius of circle

In ΔAOD, OD $\perp$ AB

As we know perpendicular drawn from the centre to the chord bisects the chord.

$\therefore$ AD = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 10 = 5 cm

Now, in ΔAOD

$OA^2 = AD^2 + OD^2$ (Applying Pythagoras theorem)

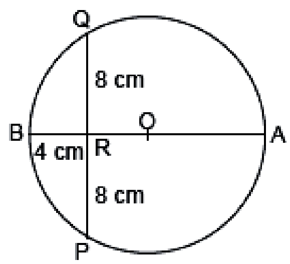
$\Rightarrow OA^2 = (5)^2 + (12)^2 = 25 + 144$

$\Rightarrow OA^2 = 169$

$\therefore OA = \sqrt{169} = 13$ cm

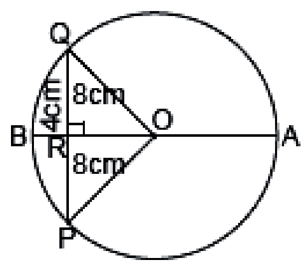
$\therefore$ diameter = 2 (OA) = 2 × 13 cm = 26 cm

16. In the given figure, diameter AB of circle with centre O bisects the chord PQ. If PR = QR = 8 cm and RB = 4 cm, find the radius of the circle.



Ans:

Here AB is the diameter of the circle and AB bisects PQ.



Also, PR = RQ = 8 cm

OB, OP and OQ are radii of the circle.

$\Rightarrow$ OB = OQ = OP = r (say)

Consider OR = x cm

OB = OR + BR = $(x + 4)$ cm

$\Rightarrow x = (OB - 4) = (r - 4)$ cm

As R is the mid point of PQ.

Also, RO is a line segment passing through centre O.

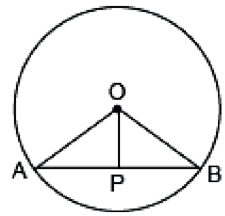
$\therefore$ OR $\perp$ PQ (line segment joining mid pt. of chord to the centre of circle is perpendicular to the chord)

In right-angled ΔOQR

$OQ^2 = OR^2 + QR^2$ (by Pythagoras theorem)

$= x^2 + (8)^2$
$\Rightarrow$ $OQ^2 = x^2 + 64$
$r^2 = (r - 4)^2 + 64$ [Put $x = (r - 4)$]
$\Rightarrow$ $r^2 = r^2 + 16 - 8r + 64$
$\Rightarrow$ $8r = 80$
$\Rightarrow$ $r = 10$ cm

17. Prove that the line drawn through the centre of a circle to the mid point of a chord is perpendicular to the chord.
Ans:
In circle, O is the centre and OP bisects the chord AB.
Join OA and OB (radius of circle)



In ΔAPO and ΔBPO
OA = OB (radius of circle)
OP = OP (common)
AP = BP (given)
$\therefore$ $\Delta APO \cong \Delta BPO$ (by SSS)
$\therefore$ $\angle OPA = \angle OPB$ ...(i) (by CPCT)
Also $\angle OPA + \angle OPB = 180°$ (Linear pair angles)
$\Rightarrow$ $\angle OPA + \angle OPA = 180°$ [Using (i)]
$2\angle OPA = 180°$
$\Rightarrow$ $\angle OPA = 90° \Rightarrow OP \perp AB$.

SECTION – D

Questions 18 carry 5 marks.

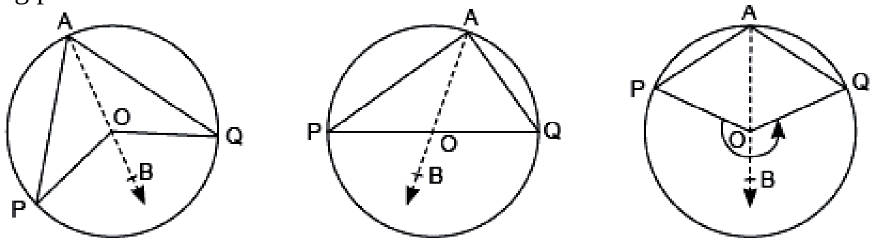
18. Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.
Ans: Given: ABCD is a quadrilateral, AH, BF, CF and DH are the angle bisectors of internal angles A, B, C and D.
These bisectors form a quadrilateral EFGH.
To Prove: EFGH is cyclic
Proof: In AEB.
$\angle EAB + \angle ABE + \angle AEB = 180°$ (Sum of angles of ΔABC)
$\Rightarrow$ $\angle AEB = 180° - (\angle EAB + \angle ABE)$ ...(i)
Also $\angle AEB = \angle FEH$ ...(ii) (Vertically opposite angles)
By equating (i) and (ii) $\angle FEH = 180° - (\angle EAB + \angle ABE)$ ...(iii)
Similarly, in ΔGDC $\angle FGH = 180° - (\angle GDC + \angle GCD)$ ...(iv)
Adding (iii) and (iv)
$\angle FEH + \angle FGH = 360° - (\angle EAB + \angle ABE + \angle GDC + \angle GCD)$
$= 360° - \frac{1}{2}(\angle BAD + \angle ABC + \angle ADC + \angle BCD)$ (As AH, BF, CF and HD are bisectors of $\angle$A, $\angle$B, $\angle$C, $\angle$D)
$= 360° - \frac{1}{2} \times 360°$ (Sum of angles of quadrilateral, ABCD)
$\angle FEH + \angle FGH = 360° - 180° = 180°$
$\Rightarrow$ FEHG is a cyclic quadrilateral. (If the sum of opposite angles of quadrilateral is 180°, then it is cyclic)

OR

Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Ans:

Given : Given an arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle.



case (*i*) case (*ii*) case (*iii*)

To Prove: POQ = 2 PAQ

Construction: Join AO and extends it to B.

Proof: Consider three cases

case (i): When arc PQ is a minor arc.

case (ii): When arc PQ is a semicircle.

case (iii): When arc PQ is a major arc.

In all the three cases

Taking ΔAOQ

$\angle BOQ = \angle OAQ + \angle OQA$ (Exterior angle of Δ is equal to the sum of interior opposite angles)

Also OA = OQ (radii of circle)

$\Rightarrow \angle OAQ = \angle OQA$ (Angles opposite to equal sides)

$\Rightarrow \angle BOQ = \angle OAQ + \angle OAQ$

$\Rightarrow \angle BOQ = 2\angle OAQ$ ...(i)

Similarly $\angle BOP = 2\angle OAP$ ...(ii)

Adding (i) and (ii) we have

$\angle BOQ + \angle BOP = 2\angle OAQ + 2\angle OAP = 2(\angle OAQ + \angle OAP)$

$\Rightarrow \angle POQ = 2\angle PAQ$

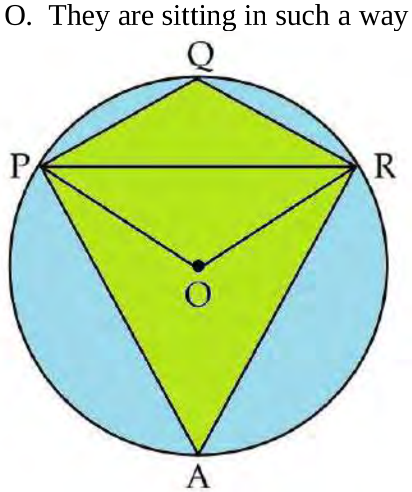
Specially for case (iii) we can write reflex $\angle POQ = 2\angle PAQ$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Four Friends Rima, Mohan, Sohan and Sita are sitting on the circumference of a circular park full of water. Their locations are marked by points A, P, Q and R such that the APQR is a quadrilateral with greenery.

Rohit joins them and sits at the centre of the circular park, so he is equidistant from all the other friends. His position is marked as O. They are sitting in such a way that $\angle PQR = 110°$.



(i) What is measure of reflex $\angle POR$? (1)

(ii) What is the measure of $\angle PAR$? (2)

OR

(ii) Find ∠OPR? (2)

(iii) What is measure of ∠POR? (1)

Ans: (i) Reflex ∠POR = 2∠PQR (∵ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.)

∴ Reflex ∠POR = 2 × 110° = 220°

(ii) ∠PAR + ∠PQR = 180° (∵ Sum of opposite angles of cyclic quadrilateral is 180°)

∴ ∠PAR = 180° – 110° = 70°

OR

In ∆OPR, OP = OR (equal radii)

∴ ∠OPR = ∠ORP = x° (Angle opposite to equal sides are equal.) [1]

∠OPR + ∠ORP + ∠POR = 180°

⇒ x + x + 140° = 180° (∵ ∠POR = 2∠PAR)

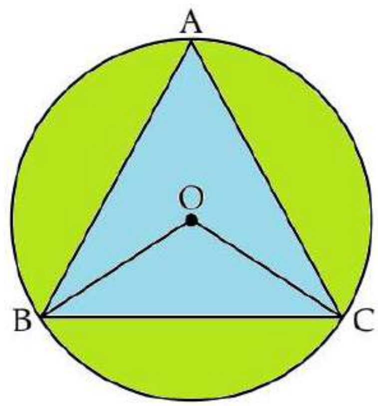
⇒ 2x = 40° ⇒ x = 20°

Hence, ∠OPR = 20°

(iii) ∠POR = 2∠PAR (∵ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.)

∴ ∠POR = 2 × 70° = 140°

20. One triangular shaped pond is there in a park marked by ABC. Three friends are sitting positions at A, B and C. They are studying in Class IX in an International. A, B and C are equidistant from each other as shown in figure given below.



(i) What is the value of ∠BAC? (1)

(ii) What will be the value of ∠BOC? (2)

OR

(ii) What will be the value of ∠OBC? (2)

(iii) Which angle will be equal to ∠OBC?

Ans: (i) AB = BC = AC as per the given statement

∴ ∆ABC is an equilateral triangle

∴ ∠BAC = 60^0 (Angles of a equilateral triangle)

(ii) ∠BOC = 2∠BAC (∵ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.)

∴ ∠BOC = 2 × 60^0 = 120^0

OR

(ii) In ∆BOC, OB = OC (radii)

⇒ ∠OBC = ∠OCB (∠s opposite to equal sides are equal)

⇒ ∠OBC + ∠OCB + ∠BOC = 180^0 (Angle sum property of triangle)

⇒ ∠OBC + ∠OBC + 120^0 = 180^0

⇒ 2∠OBC = 60^0 ⇒ ∠OBC = 30^0

(iii) In ∆BOC, OB = OC (radii)

⇒ ∠OCB = ∠OBC (∠s opposite to equal sides are equal)