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PRACTICE PAPER 2 (2023-24)
CHAPTER-11 SURFACE AREAS AND VOLUMES (ANSWERS)

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**
CLASS : IX — **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The curved surface area of the cone of slant height x/2 is 2πx. Then area of its base is
(a) 4π sq. units (b) $4\pi x^2$ sq. units (c) πx^2 sq. units (d) 16π sq. units
Ans: (d) 16π sq. units
Let R be the radius of base of the cone.
∴ Curved surface area of cone, $\pi R\left(\frac{x}{2}\right) = 2\pi x \Rightarrow R = 4$
∴ Area of its base = $\pi R^2 = \pi(4)^2 = 16\pi$ sq. units

2. The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4, then the ratio of their curved surface areas is
(a) 4 : 5 (b) 25 : 16 (c) 16 : 25 (d) 5 : 4
Ans: (d) 5 : 4
Let r be the radius of two cones and 5x and 4x be their slant heights.
∴ The ratio of their curved surface areas = $\frac{\pi r(5x)}{\pi r(4x)} = \frac{5}{4}$

3. The ratio of the radii of two spheres is 4 : 5. The ratio of their surface areas is
(a) 4 : 5 (b) 2 : √5 (c) 5 : 4 (d) 16 : 25
Ans: (d) 16 : 25
Let r_1 and r_2 be the radii and S_1 and S_2 be the surface areas of two spheres respectively.
$\frac{r_1}{r_2} = \frac{4}{5} \Rightarrow \frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. The ratio of the surface areas of the balloon in the two cases is
(a) 1 : 4 (b) 1 : 3 (c) 2 : 3 (d) 2 : 1
Ans: (a) 1 : 4
Let S_1 and S_2 be the surface areas of the spherical balloons with radii 7 cm and 14 cm respectively.
∴ $S_1 = 4\pi(7)^2 = 196\pi$ cm^2 and
$S_2 = 4\pi(14)^2 = 784\pi$ cm^2
∴ $S_1 : S_2 = 196\pi : 784\pi = 196 : 784 = 1 : 4$

5. If the radius and height of a cone are both increased by 10%, then the volume of the cone is approximately increased by
(a) 10% (b) 21% (c) 33% (d) 100%

Ans: (c) 33%

Let V_1 and V_2 be the volumes of original and new cones respectively. r and h are radius and height of original cone.

$$\therefore \quad V_1 = \frac{1}{3}\pi r^2 h, V_2 = \frac{1}{3}\pi\left(r + \frac{10r}{100}\right)^2\left(h + \frac{10h}{100}\right) = \frac{1331}{1000}\left(\frac{1}{3}\pi r^2 h\right)$$

$\therefore$ Required increased percentage in volume

$$= \left(\frac{\text{New volume} - \text{Original volume}}{\text{Original volume}}\right) \times 100\%$$

$$= \frac{V_2 - V_1}{V_1} \times 100\% = \left(\frac{1331}{1000} - 1\right) \times 100\% = 33.1\% = 33\% \text{ (approx.)}$$

6. If h, S and V denote respectively the height, curved surface area and volume of a right circular cone, then $3\pi Vh^3 - S^2h^2 + 9V^2$ is equal to

(a) 8 (b) 0 (c) 4π (d) $32\pi^2$

Ans: (b) 0

Let *r* and *l* be the radius and slant height of the cone.

$$\therefore \quad S = \pi r l = \pi r\sqrt{r^2 + h^2} \text{ and } V = \frac{1}{3}\pi r^2 h$$

Now, $3\pi Vh^3 - S^2h^2 + 9V^2$

$$= 3\pi\left(\frac{1}{3}\pi r^2 h\right)h^3 - (\pi r\sqrt{r^2 + h^2})^2 h^2 + 9\left(\frac{1}{3}\pi r^2 h\right)^2$$

$$= \pi^2 r^2 h^4 - \pi^2 r^2 h^2(r^2 + h^2) + \pi^2 r^4 h^2$$

$$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0$$

7. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is

(a) 1 : 2 : 3 (b) 2 : 1 : 3 (c) 2 : 3 : 1 (d) 3 : 2 : 1

Ans: (a) 1 : 2 : 3

Let *r* be the radius of cone, hemisphere and cylinder.

Height of hemisphere = Radius of hemisphere = *r*

$\therefore$ Height of cylinder, h = Height of hemisphere = *r*

Similarly, height of cone = *r*

Now, volume of cone = $\frac{1}{3}\pi r^2(r) = \frac{1}{3}\pi r^3$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

Volume of cylinder = $\pi r^2(r) = \pi r^3$

$\therefore$ Required ratio = $\frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = \frac{1}{3} : \frac{2}{3} : 1 = 1 : 2 : 3$

8. Volume of a hemisphere is 19404 cubic cm. The total surface area is

(a) 2772 sq. cm (b) 4158 sq. cm (c) 5544 sq. cm (d) 1386 sq. cm

Ans: (b) 4158 sq. cm

Volume of the hemisphere = 19404 cm^3

$$\Rightarrow \quad \frac{2}{3}\pi r^3 = 19404 \Rightarrow r^3 = \frac{3 \times 19404 \times 7}{22 \times 2}$$

$$\Rightarrow \quad r^3 = (21)^3 \Rightarrow r = 21 \text{ cm}$$

$$\therefore \quad \text{Total surface area} = 3\pi r^2 = 3 \times \frac{22}{7} \times (21)^2 = 4158 \text{ sq. cm.}$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. Assertion (A): If a cone and a hemisphere have equal base and volume, then the ratio of their heights is 2 : 1.
Reason (R): Volume of a cylinder of height h and base radius r is $\pi r^2 h$.
Ans: (b) Both A and R are true but R is not the correct explanation of A.
Let r be the radius of base of both cone and hemisphere and h be the height of cone.
Since, Volume of cone = Volume of hemisphere

$$\therefore \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3 \Rightarrow \frac{h}{r} = \frac{2}{1} \text{ or } 2:1$$

$\therefore$ Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

10. Assertion (A): If the radius of a sphere is tripled, then the ratio of the volume of the original sphere to that of the new is 1 : 27.
Reason (R): Volume of a sphere with radius r is $4\pi r^3$.
Ans: (c) A is true but R is false.
Let r be the radius of original sphere.
$\therefore$ Radius of new sphere = 3r

$$\therefore \quad \text{Required ratio} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(3r)^3} = \frac{1}{27} \text{ or } 1:27.$$

Here, Reason is clearly wrong.
$\therefore$ Assertion is correct but Reason is wrong.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. The diameters of two cones are equal. If their slant heights are in the ratio 7:4, find the ratio of their curved surface area.
Ans: Let diameter of both cones be *d*.

Let radius = $\frac{d}{2} = r$ (say)

$\therefore$ Let slant height of first cone be $7x$ and slant height of second cone be $4x$.
Let C_1 and C_2 be curved surface area of first and second cone respectively.

$$\therefore \frac{C_1}{C_2} = \frac{\pi r(7x)}{\pi r(4x)} = \frac{7}{4}$$

$\Rightarrow C_1 : C_2 = 7 : 4$
$\therefore$ Ratio of their curved surface area = 7 : 4

12. A solid sphere of radius 3 cm is melted and then recast into small spherical balls each of diameter 0.6 cm. Find the number of small balls thus obtained.
Ans: Radius of solid sphere = 3 cm

Volume of solid sphere = $\frac{4}{3}\pi(3)^3$

Diameter of small spherical ball = 0.6 cm

Radius of small spherical ball = $\frac{0.6}{2} = 0.3$ cm

Volume of small spherical ball = $\frac{4}{3}\pi(0.3)^3$

Number of small spherical balls =

$$\frac{\frac{4}{3}\pi(3)^3}{\frac{4}{3}\pi(0.3)} = \frac{3\times3\times3}{0.3\times0.3\times0.3} = 1000$$

13. The diameter of a sphere is 42 cm. It is melted and drawn into a cylindrical wire of 28 cm diameter. Find the length of the wire.

Ans: Diameter of sphere = 42 cm

Radius of sphere = $\frac{42}{2}$ = 21 cm

Diameter of cylindrical wire = 28 cm

Radius of cylindrical wire = $\frac{28}{2}$ = 14 cm

Let h be the length of cylindrical wire

Now, volume of sphere = volume of cylindrical wire

$\frac{4}{3}\pi(21)^3 = \pi(14)^2 \times h$

$\Rightarrow h = \frac{4}{3}\times\frac{21\times21\times21}{14\times14} = 63cm$

14. There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii.

Ans: Let radii of Ist cone be r_1 cm and radii of 2nd cone be r_2 cm.

Let l_1 and l_2 be the slant height of two cones.

$\therefore$ Curved surface area of 1st cone = 2 × Curved surface area of 2nd cone

$\Rightarrow \pi r_1 l_1 = 2(\pi r_2 l_2)$

$\Rightarrow \pi r_1 l_1 = 2\pi r_2(2l_1)$ (as $l_2 = 2l_1$)

$\Rightarrow \pi r_1 l_1 = 4\pi r_2 l_1$

$\Rightarrow \frac{r_1}{r_2} = \frac{4\pi l_1}{\pi l_1}$

$\Rightarrow \frac{r_1}{r_2} = \frac{4}{1}$

$\therefore$ Required ratio of the radii of two cones = 4 : 1.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A corn cob, shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.

Ans: Since the grains of corn are found only on the curved surface of the corn cob, we would need to know the curved surface area of the corn cob to find the total number of grains on it. In this question, we are given the height of the cone, so we need to find its slant height.

Here, $l = \sqrt{r^2+h^2} = \sqrt{(2.1)^2+20^2} = \sqrt{404.41}$ = 20.11 cm

Therefore, the curved surface area of the corn cob = πrl

= $\frac{22}{7}$ × 2.1 × 20.11 cm = 132.726 cm^2 = 132.73 cm^2 (approx.)

Number of grains of corn on 1 cm^2 of the surface of the corn cob = 4

Therefore, number of grains on the entire curved surface of the cob

$= 132.73 \times 4 = 530.92 = 531$ (approx.)
So, there would be approximately 531 grains of corn on the cob.

16. The outer curved surface areas of hemisphere and sphere are in the ratio 2 : 9. Find the ratio of their radii.
Ans: Let radius of sphere be R cm and radius of hemisphere be r cm.

$$\frac{\text{Outer curved surface area of hemisphere}}{\text{Surface area of sphere}} = \frac{2\pi r^2}{4\pi R^2}$$

$$\frac{2}{9} = \frac{2r^2}{4R^2}$$

$$\Rightarrow \frac{r^2}{R^2} = \frac{4}{9}$$

$$\Rightarrow \frac{r}{R} = \frac{2}{3}$$

$\therefore$ Required ratio of their radii = 2 : 3

17. A hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is ₹ 5 per 100 cm^2.
Ans: Let radius of the base of hemispherical dome = r m
Circumference of the base of hemispherical dome = 17.6 m

$\Rightarrow 2\pi r = 17.6$

$\Rightarrow 2 \times \frac{22}{7} \times r = 17.6$

$\Rightarrow r = \frac{17.6 \times 7}{2 \times 22} = 2.8$ m

Inner surface area of hemispherical dome = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 2.8 \times 2.8\ m^2$

$= 49.28\ m^2 = 492800\ cm^2$

Cost of painting per 100 cm^2 = ₹ 5
Total cost of painting inner surface area of hemispherical dome

$=$ ₹ $5 \times \frac{492800}{100} =$ ₹ 24640

SECTION – D

Questions 18 carry 5 marks.

18. (a) The circumference of the base of 10 m high conical tent is 44 m. Calculate the length of canvas used in making the tent, if width of canvas is 2 m. (3)
(b) Into a conical tent of radius 8.4 m and vertical height 3.5 m, how many full bags of wheat can be emptied, if space for the wheat in each bag is 1.96 m^3? (2)
Ans: (a) Let r m be the radius of the base of conical tent.
Circumference of base of conical tent = 44 m

$\Rightarrow 2\pi r = 44$

$\Rightarrow 2 \times \frac{22}{7} \times r = 44$

$\Rightarrow r = 7$ m

Height of conical tent = h = 10 m

$\therefore$ Slant height of conical tent = $l = \sqrt{r^2 + h^2}$

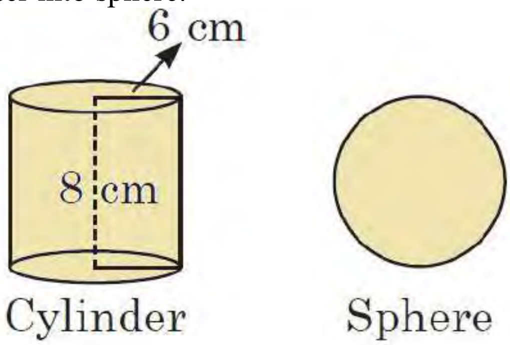
$\Rightarrow l = \sqrt{7^2 + 10^2} = \sqrt{49 + 100} = \sqrt{149} = 12.21$ m

Let x be the length of canvas used in making the tent.
$\therefore$ Area of canvas used = $x \times 2$ m^2
Also, $x \times 2 = \pi rl$
$\Rightarrow 2x = \frac{22}{7} \times 7 \times 12.21$
$\Rightarrow x = 11 \times 12.21 = 134.31$ m
$\therefore$ Required length of canvas = 134.2 m.
(b) Radius of conical tent = 8.4 m
Height of conical tent = 3.5 m
$\therefore$ Capacity of the conical tent = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (8.4)^2 \times 3.5 = 258.72$ m^3
Space of occupied by each bag of wheat = 1.96 m^3
$\therefore$ Number of bags =
$$\frac{\text{Capacity of the conical tent}}{\text{Space occupied by each bag of wheat}} = \frac{258.72}{1.96}$$
= 132

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. In Class IX-B, a Maths teacher is teaching conversion concepts. He brings some clay in the classroom to teach the topic mensuration. First he forms a cylinder of radius 6 cm and height 8 cm and then he moulds that cylinder into sphere.



Read the above passage and answer the questions
(a) Find the volume of the cylindrical shape in terms of π. (1)
(b) Find the radius of the sphere (2)
(c) Find the volume of sphere, the teacher made. (1)
Ans: (a) Volume of cylinder = $\pi r^2 h = \pi(6)^2 \times 8 = 288\pi$ cm^3
(b) Let R be the radius of the sphere
Since volume of sphere = volume of cylinder
$$\Rightarrow \quad \frac{4}{3}\pi R^3 = \pi r^2 h$$
$$\Rightarrow \quad R^3 = \frac{3}{4}(6)^2 \times 8 = 6^3 \Rightarrow R = 6 \text{ cm}$$
(c) Volume of sphere = volume of cylinder = 288π cm3

20. Aditya was doing an experiment to find the radius r of a ball. For this he took a cylindrical container with radius R = 7 cm and height 10 cm. He filled the container almost half by water as shown in the figure-1. Now he dropped the ball into the container as in figure-2.
He observed that in figure-2, the water level in the container raised from P to Q i.e, to 3.4 cm.

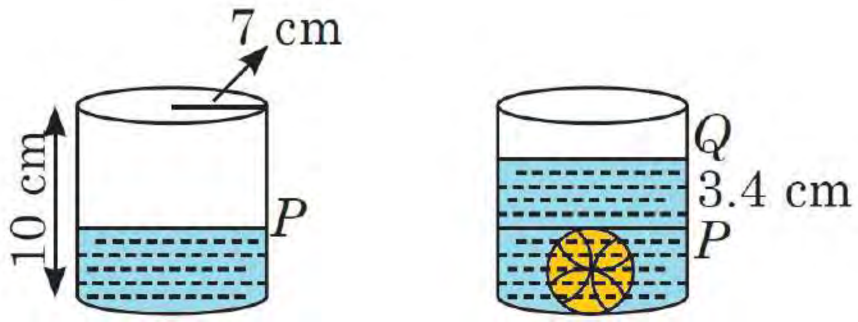


Figure - 1 Figure - 2

Read the above passage and answer the questions

(a) What is the approximate radius of the ball? (2)

(b) What is the volume of the cylinder? (1)

(c) What is the volume of the spherical ball? (1)

Ans: (a) Volume of ball = Increased volume of water

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi(7)^2 \times 3.4$$

$$\Rightarrow \quad r^3 = \frac{7 \times 7 \times 3.4 \times 3}{4} = 124.95 \Rightarrow r \approx 5 \text{ cm.}$$

(b) Volume of cylinder = $\pi R^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 10 = 1540 \text{ cm}^3$$

(c) Volume of spherical ball = $4/3 \, \pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 5 \times 5 \times 5 = \frac{11000}{21} = 523.81 \text{ cm}^3$$