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PRACTICE PAPER 09 (2023-24)
CHAPTER 09 DIFFERENTIAL EQUATIONS (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**
CLASS : XII | **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m + n.

(a) 1 (b) 2 (c) 3 (d) 4

Ans:
$m + n = 4$
$\because\ m = 2$ (second order derivative)
$\therefore\ n = 2$ (degree of the highest order derivative)

2. If $\frac{dy}{dx} = y\sin 2x$, y(0) = 1, then solution is

(a) y = $e^{\sin^2 x}$ (b) y = $\sin^2$x (c) y = $\cos^2$x (d) y = $e^{\cos^2 x}$

Ans:
We have $\frac{dy}{dx} = y\sin 2x$
$\Rightarrow\ \frac{dy}{y} = \sin 2x\,dx \Rightarrow \log y = -\frac{\cos 2x}{2} + c$
Since $y(0) = 1 \Rightarrow x = 0, y = 1 \Rightarrow c = 1/2$
Now, $\log y = \frac{1}{2}(1 - \cos 2x) \Rightarrow \log y = \sin^2 x \Rightarrow y = e^{\sin^2 x}$

3. If m and n are the order and degree, respectively of the differential equation $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$, then write the value of m + n.

(a) 1 (b) 2 (c) 3 (d) 4

Ans: (c) 3
Here, m = 2 and n = 1 then m + n = 2 + 1 = 3

4. The order and the degree of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ are:

(a) 1, 1 (b) 2, 1 (c) 1, 2 (d) 3, 1

Ans: (b) 2, 1
The highest order is 2 and the degree of the highest order is 1.

Hence, the order is 2 and the degree is 1.

5. The general solution of the differential equation $e^x\,dy + (y\,e^x + 2x)\,dx = 0$ is
(a) $x\,e^y + x^2 = C$ (b) $x\,e^y + y^2 = C$ (c) $y\,e^x + x^2 = C$ (d) $y\,e^y + x^2 = C$
Ans: (c) $y\,e^x + x^2 = C$

6. The general solution of the differential equation $\frac{dy}{dx} = 2^{-y}$ is:
(a) 2y = x log 2 + C log 2 (b) 2y = x log 3 – C log 3
(c) y = x log 2 – C log 2 (d) None of these
Ans: (a) 2y = x log 2 + C log 2

Given, $\frac{dy}{dx} = 2^{-y}$

$$\Rightarrow \frac{dy}{2^{-y}} = dx \Rightarrow \int 2^y dx = \int dx$$

$$\Rightarrow \frac{2^y}{\log 2} = x + C \Rightarrow 2y = x\log 2 + C\log 2$$

7. The integrated factor of the differential equation: $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ is
(a) $\frac{1}{e^{\tan^{-1}x}}$ (b) $2\,e^{\tan^{-1}x}$ (c) $3e^{\tan^{-1}x}$ (d) $e^{\tan^{-1}x}$
Ans: (d) $e^{\tan^{-1}x}$

8. Solution of the differential equation $x\frac{dy}{dx} + y = xe^x$ is
(a) $xy = e^x(1 - x) + C$ (b) $xy = e^x(x + 1) + C$
(c) $xy = e^y(y - 1) + C$ (d) $xy = e^x(x - 1) + C$
Ans: (d) $xy = e^x(x - 1) + C$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** Solution of the differential equation $(1+x^2)\frac{dy}{dx} + y = \tan^{-1}x$ is

$$ye^{\tan^{-1}x} = (\tan^{-1}x - 1)e^{\tan^{-1}x} + C$$

Reason (R) : The differential equation of the form $\frac{dy}{dx} + Py = Q$, where P, Q be the functions of x or constant, is a linear type differential equation.
Ans: (b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).

10. Assertion (A): Solution of the differential equation $e^{dy/dx} = x^2$ is $y = 2(x\log x - x) + C$.

Reason (R): The differential equation $\frac{d^2y}{dx^2} + y = 0$ has degree 1 and order 2.

Ans: (b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the general solution of the differential equation $\frac{dy}{dx} = x - 1 + xy - y$.

Ans: $\frac{dy}{dx} = x - 1 + xy - y = (x-1)(y+1)$

$\Rightarrow \int \frac{dy}{y+1} = \int (x-1)dx$

$\Rightarrow \log|y+1| = \frac{x^2}{2} - x + C$ is the required solution.

12. Solve the following differential equation: $\frac{dy}{dx} = x^3 \cos ec y$, given that y(0) = 0.

Ans:

Given differential equation is $\frac{dy}{dx} = x^3.\text{cosec } y$

$\therefore \quad \frac{dy}{\text{cosec } y} = x^3.dx$

Integrating both sides, we get $\int \sin y \, dy = \int x^3 \, dx$

$\Rightarrow -\cos y = \frac{x^4}{4} + c \qquad \Rightarrow \quad -\cos y = \frac{0}{4} + c$

$\Rightarrow -1 = c$

Putting the value of c in (i), we get

$-\cos y = \frac{x^4}{4} - 1 \qquad \therefore \ \cos y = \left(1 - \frac{x^4}{4}\right)$

13. Solve : $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$

Ans: The given differential equation is $x^2(1-y)dy + y^2(1+x^2)dx = 0$

$\Rightarrow \quad x^2(1-y)dy = -y^2(1+x^2)dx$

$\Rightarrow \quad \frac{1-y}{y^2}dy = -\left(\frac{1+x^2}{x^2}\right)dx$, if $x, y \neq 0$

$\Rightarrow \quad \left(\frac{1}{y} - \frac{1}{y^2}\right)dy = \left(\frac{1}{x^2} + 1\right)dx \quad \Rightarrow \quad \int\left(\frac{1}{y} - \frac{1}{y^2}\right)dy = \int\left(\frac{1}{x^2} + 1\right)dx$

$\Rightarrow \quad \log|y| + \frac{1}{y} = -\frac{1}{x} + x + C$, which is the general solution of the differential equation.

14. Solve the differential equation: $x\frac{dy}{dx} - y = x^2$

Ans: $x\frac{dy}{dx} - y = x^2 \Rightarrow \frac{dy}{dx} - \frac{y}{x} = x$

The equation is of the form $\frac{dy}{dx} + Py = Q$, where P = $-\frac{1}{x}$ and Q = x

Integrating factor, IF = $e^{\int Pdx} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Solution is $y.\frac{1}{x} = \int x.\frac{1}{x} + C = \int 1.dx + C$

$\Rightarrow \frac{y}{x} = x + C \Rightarrow y = x^2 + Cx$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the general solution of the following differential equation; x dy – (y + 2x^2)dx = 0

Ans:

$x\,dy - (y + 2x^2)\,dx = 0 \Rightarrow x\,dy = (y + 2x^2)\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{y + 2x^2}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{2x^2}{x} \Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$

On comparing with $\frac{dy}{dx} + Py = Q$; Here 'P' $= \frac{-1}{x}$, $Q = 2x$

I.F. $= e^{\int P\,dx} = e^{\int \frac{-1}{x}dx} = e^{-\log|x|} = e^{\log|x^{-1}|} = x^{-1} = \frac{1}{x}$;

Hence the required solution is $y(\text{I.F.}) = \int Q(\text{I.F.})dx$

$y\left(\frac{1}{x}\right) = \int 2x.\frac{1}{x}dx = \int 2\,dx \Rightarrow \frac{y}{x} = 2x + C$

16. Solve : (x^2 + y^2) dx – 2xydy = 0

Ans:

We have, $(x^2 + y^2)\,dx - 2xydy = 0 \Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ ... (i)

Let $f(x, y) = \frac{x^2 + y^2}{2xy}$, so $f(\lambda x, \lambda y) = \frac{\lambda^2 x^2 + \lambda^2 y^2}{2\lambda x \cdot \lambda y} = \frac{\lambda^2}{\lambda^2} f(x, y) = \lambda^0 f(x, y)$

$\therefore$ This is homogeneous differential equation.

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

So, equation (i) becomes $v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x \cdot vx} = \frac{x^2(1 + v^2)}{x^2 \cdot 2v} = \frac{1 + v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v = \frac{1 + v^2 - 2v^2}{2v} = \frac{1 - v^2}{2v}$

Separating the variables, we get $\frac{2v}{1 - v^2}dv = \frac{dx}{x}$

$\Rightarrow \int \frac{2v}{1 - v^2}dv = \int \frac{dx}{x} \Rightarrow -\log|1 - v^2| = \log x + C \Rightarrow \log x + \log|1 - v^2| = -C$

$\Rightarrow \log x(1 - v^2) = -C \Rightarrow x\left(1 - \frac{y^2}{x^2}\right) = e^{-C} \Rightarrow x^2 - y^2 = Ax$, where $e^{-C} = A$

Hence, $x^2 - y^2 = Ax$ is the general solution of the given differential equation.

17. Solve the following differential equation: $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0$

Ans: We have $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0$

$\Rightarrow \sin^2\left(\frac{y}{x}\right) - \frac{y}{x} + \frac{dy}{dx} = 0$ ...(*i*)

This is a linear homogeneous differential equation

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (i) becomes

$$\sin^2 v - v + v + x\frac{dv}{dx} = 0$$

$$\Rightarrow x\frac{dv}{dx} + \sin^2 v = 0 \Rightarrow \operatorname{cosec}^2 v dv + \frac{dx}{x} = 0$$

Integrating both sides, we get

$-\cot v + \log x = C \Rightarrow -\cot\left(\frac{y}{x}\right) + \log x = C$ is the required solution.

SECTION – D

Questions 18 carry 5 marks.

18. Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by (x + y + 1) = A(1 – x – y – 2xy), where A is a parameter.

Ans:

The given equation is $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$.

$$\therefore \frac{dy}{y^2+y+1} = \frac{-dx}{x^2+x+1} \Rightarrow \int\frac{dy}{y^2+y+1} = -\int\frac{dx}{x^2+x+1}$$

$$\Rightarrow \int\frac{dy}{y^2+y+\frac{1}{4}+\left(1-\frac{1}{4}\right)} = -\int\frac{dx}{x^2+x+\frac{1}{4}+\left(1-\frac{1}{4}\right)} \Rightarrow \int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = -\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\therefore \frac{1}{\sqrt{3}/2}\tan^{-1}\left(\frac{y+\frac{1}{2}}{\sqrt{3}/2}\right) = -\frac{1}{\sqrt{3}/2}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\sqrt{3}/2}\right) + C$$

$$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = C \Rightarrow \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right] = C$$

$$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1-\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}\right] = C \Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\sqrt{3}(2y+1+2x+1)}{3-(2y+1)(2x+1)}\right] = C$$

$$\Rightarrow \tan^{-1}\left[\frac{2\sqrt{3}(x+y+1)}{2(1-x-y-2xy)}\right] = \frac{\sqrt{3}}{2}C \Rightarrow \left[\frac{2\sqrt{3}(x+y+1)}{2(1-x-y-2xy)}\right] = \tan\left(\frac{\sqrt{3}}{2}C\right)$$

$$\Rightarrow \frac{x+y+1}{1-x-y-2xy} = \frac{1}{\sqrt{3}}\tan\left(\frac{\sqrt{3}}{2}C\right) = A \Rightarrow (x+y+1) = A(1-x-y-2xy)$$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F. The room in which the cat was put is always at 70°F. The normal temperature of the cat is taken as 98.6°F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by

the differential equation: $\frac{dT}{dt} \propto (T - 70)$, where 70°F is the room temperature and T is the temperature of the object at time t.
Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{dT}{dt} = k(T - 70)$ where k is a constant of proportion, time of death is calculated.

(a) State the degree of the above given differential equation. (1)
(b) Write method of solving a differential equation helped in calculation of the time of death? (1)
(c) Find the solution of the differential equation $\frac{dT}{dt} = k(T - 70)$. (1)
(d) If t = 0 when T is 72, then find the value of c (1)

Ans: (a) Degree is 1
(b) Variable separable method
(c) We have $\frac{dT}{dt} = k(T-70) \Rightarrow \int \frac{dT}{T-70} = \int kdt$
$\Rightarrow \log|T-70| = kt + C$
(d) Given t =0 when T =72
Now, log |T – 70| = kt + C
= log|72 – 70| = k. 0 + C
= log2 = C

20. Case-Study 2: Read the following passage and answer the questions given below.
Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50 - y)$ where x denotes the number of weeks and y the number of children who have been given the drops.

(a) Find the solution of the differential equation $\frac{dy}{dx} = k(50 - y)$ (1)

(b) Find the value of c in the particular solution given that y(0) = 0 and k = 0.049 (1)

(c) Find the solution that may be used to find the number of children who have been given the polio drops. (2)

Ans: (a) We have, $\frac{dy}{dx} = k(50 - y)$

$$\int \frac{dy}{50-y} = \int k dx \Rightarrow -\log|50-y| = kx + C$$

(b) Given y(0) = 0 and k = 0.049

$-\log|50 - y| = kx + C$

$\Rightarrow -\log|50 - 0| = 0.049 (0) + C$

$\Rightarrow -\log 50 = C \Rightarrow C = \log\frac{1}{50}$

(c) We have, $-\log|50-y| = kx + \log\frac{1}{50}$ [From (a) and (b)]

$$\Rightarrow -kx = \log|50-y| + \log\frac{1}{50} \Rightarrow -kx = \log\frac{50-y}{50} \Rightarrow e^{-kx} = \frac{50-y}{50} = 1 - \frac{y}{50}$$

$$\Rightarrow \frac{y}{50} = 1 - e^{-kx} \Rightarrow y = 50(1 - e^{-kx})$$

This is the required solution to find the number of children who have been given the polio drops.