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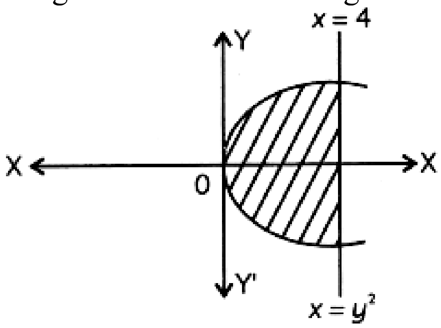
PRACTICE PAPER 08 (2023-24)
CHAPTER 08 APPLICATION OF INTEGRALS
(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XII **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The area (in sq. m) of the shaded region as shown in the figure is:



(a) 32/3 sq. units (b) 16/3 sq. units (c) 4 sq. units (d) 16 sq. units
Ans: (a) 32/3 sq. units
Given curves are x = y2 and x = 4.
So, their points of intersection are (4, 2) and (4, -2).

So required area, $A = \int_0^4 y.dx$

$$= 2\int_0^4 \sqrt{x}\,dx = \frac{2\left[x^{\frac{3}{2}}\right]_0^4}{\frac{3}{2}} = \left(4^{\frac{3}{2}} - 0\right)\times\frac{2}{3}\times 2 = \frac{32}{3} \text{ sq. units}$$

2. The area enclosed by the circle $x^2 + y^2 = 8$ is
(a) 16p sq units (b) $2\sqrt{2}\pi$ sq units (c) $8\pi^2$ sq units (d) 8π sq units
Ans: (*d*) 8π sq units
For circle $x^2 + y^2 = 8$, centre is (0, 0), radius = $\sqrt{8}$.

$\therefore$ Area = $4\int_0^{2\sqrt{2}} \sqrt{8 - x^2}\,dx = 8\pi$ sq units

3. The area bounded between the curves $y^2 = 6x$ and $x^2 = 6y$ is
(a) 6 sq units (b) 12 sq units (c) 36 sq units (d) 24 sq units
Ans: (*b*) 12 sq units
As both the curves cut at $x = 0$ and $x = 6$

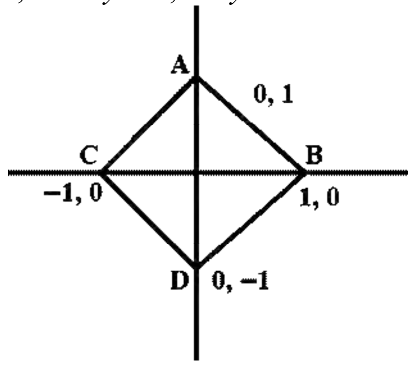
$\therefore$ Area = $\int_0^6 \left(\sqrt{6x} - \frac{x^2}{6}\right)dx = 12$ sq units

4. The area enclosed within the curve $|x| + |y| = 1$ is

(a) 21 (b) 1.5 (c) 2 (d) none of these

Ans: (*c*) 2

Curves are $x + y = 1, -x - y = 1, -x + y = 1, x - y = 1$



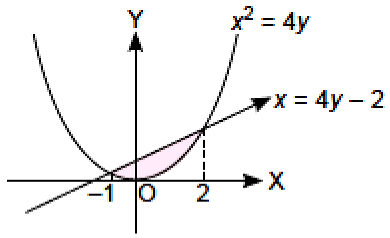
The diagram of function is shown this consist two triangle that is ABC and DBC both have base = 2 and height =1

Total area will be $\frac{1}{2}\times 2+\frac{1}{2}\times 2$ = 2 sq units

5. The area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$ (c) $\frac{7}{8}$ (d) $\frac{9}{8}$

Ans: (*d*) $\frac{9}{8}$



$\Rightarrow x = x^2 - 2 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = -1, 2$

as the two curves intersect at –1, 2

$$\therefore \text{Area } = \int_{-1}^{2}\left(\frac{x+2}{4}-\frac{x^2}{4}\right)dx = \int_{-1}^{2}\left(\frac{x+2}{4}\right)dx - \int_{-1}^{2}\frac{x^2}{4}dx$$

$$=\frac{1}{4}\left[\frac{x^2}{2}+2x\right]_{-1}^{2}-\frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}=\frac{1}{4}\left\{\frac{2^2}{2}+2\times 2-\left(\frac{1}{2}-2\right)\right\}-\frac{1}{12}[2^3-(-1)^3]$$

$$=\frac{1}{4}\left(6+\frac{3}{2}\right)-\frac{1}{12}\times 9=\frac{15}{8}-\frac{3}{4}=\frac{9}{8}\,sq\ unit$$

6. The area enclosed by the circle $x^2 + y^2 = 16$ is

(a) 20 (b) 20 π (c) 16 π (d) 256 π

Ans: (c) 16 π

7. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is

(a) 4 (b) 3 (c) 2 (d) 1

Ans: (a) 4

8. The area of the region bounded by the parabolas $y = x^2$ and $y^2 = x$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

Ans: (a) $\frac{1}{3}$

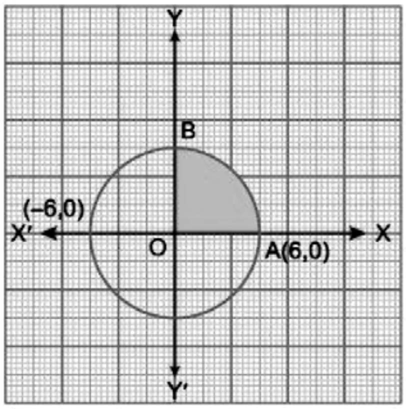
In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The area of the ellipse $2x^2 + 3y^2 = 6$ will be more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$.
Reason (R): The length of the semi-major axis of ellipse 2x2 + 3y2 = 6 is more than the radius of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$.
Ans: Clearly, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
∴ Option (b) is correct.

10. **Assertion (A):** Area enclosed by the circle $x^2 + y^2 = 36$ is equal to 36π sq. units.
Reason (R): Area enclosed by the circle $x^2 + y^2 = r^2$ is πr^2.
Ans:

$$\because \text{ Required area } = 4 \times ar(OABO) = 4 \times \int_0^6 \sqrt{6^2 - x^2}\,dx = 4 \times \left[\frac{x}{2}\sqrt{6^2 - x^2} + \frac{6^2}{2}\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$$

$$= 4 \times [(0 + 18\sin^{-1}(1)) - (0 + 0)] = 4 \times 18 \times \sin^{-1}(1) = 72 \times \frac{\pi}{2} = 36\pi \text{ sq. units}$$



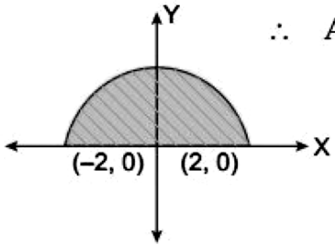
Clearly A is correct statement.
Also R is a correct statement and gives the correct explanation of statement A.
∴ Option (a) is correct.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Sketch the region $\{(x, 0): y = \sqrt{4 - x^2}\}$ and X-axis. Find the area of the region using integration.
Ans:
Given region is $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and X-axis.

We have, $y = \sqrt{4 - x^2} \Rightarrow y^2 = 4 - x^2 \Rightarrow x^2 + y^2 = 4$



$$\therefore \text{ Area of shaded region, } A = \int_{-2}^{2}\sqrt{4 - x^2}\,dx = \int_{-2}^{2}\sqrt{2^2 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\cdot\sin^{-1}\frac{x}{2}\right]_{-2}^{2}$$

$$= \frac{2}{2}\cdot 0 + 2\cdot\frac{\pi}{2} + \frac{2}{2}\cdot 0 - 2\sin^{-1}(-1) = 2\cdot\frac{\pi}{2} + 2\cdot\frac{\pi}{2} = 2\pi \text{ sq units.}$$

12. Find the area of the region bounded by the curve $y = \frac{1}{x}$, x-axis and between $x = 1$, $x = 4$.

Ans: Curve is $y = \frac{1}{x}$, x-axis and between $x = 1$, $x = 4$

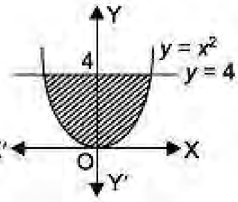
Area = $\int_1^4 \frac{1}{x} dx$ = $[\log |x|]_1^4$

= log 4 – log 1 = log 4 sq units.

13. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.
Ans:
Curve is symmetrical about y-axis only

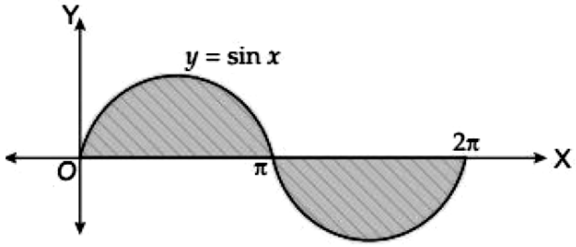
$$\text{Area} = 2\int_0^4 x\,dy = 2\int_0^4 \sqrt{y}\,dy$$

$$= \frac{4}{3}\left[y^{3/2}\right]_0^4 = \frac{4}{3} \times (8 - 0) = \frac{32}{3} \text{ sq units}$$



14. Find the area bounded by the curve y = sin x between x = 0 and x = 2π.
Ans:

$$\text{Required area} = \int_0^{2\pi} \sin x\,dx = \int_0^{\pi} \sin x\,dx + \left|\int_{\pi}^{2\pi} \sin x\,dx\right| = -[\cos x]_0^{\pi} + \left|[-\cos x]_{\pi}^{2\pi}\right|$$



$$= -[\cos\pi - \cos 0] + |-[\cos 2\pi - \cos\pi]|$$

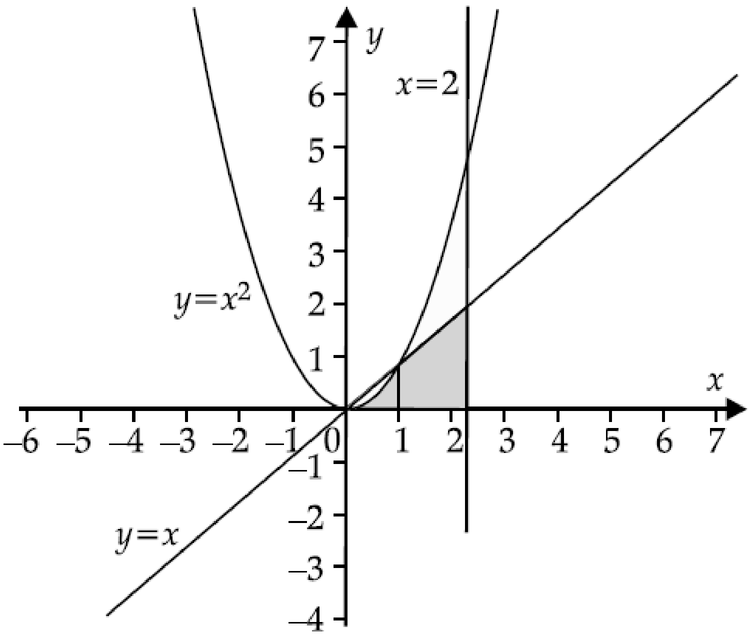
$$= -[-1 - 1] + |-(1 + 1)|$$

$$= 2 + 2 = 4 \text{ sq units.}$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2, 0 \le y \le x, 0 \le x \le 2\}$ and find the area of the region using integration.
Ans: The points of intersection of the parabola y = x2 and the line y = x are (0, 0) and (1, 1).



$$\text{Required Area} = \int_0^1 y_{\text{parabola}}\, dx + \int_1^2 y_{\text{line}}\, dx$$

$$\text{Required Area} = \int_0^1 x^2\, dx + \int_1^2 x\, dx$$

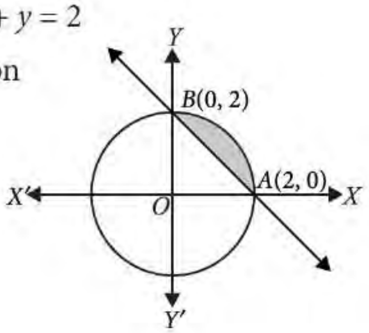
$$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$$

16. Using integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

Ans:

The given curves are $x^2 + y^2 = 4$ and $x + y = 2$

$\therefore$ Required area = area of shaded region



$$= \int_0^2 \left[\sqrt{4-x^2} - (2-x)\right] dx$$

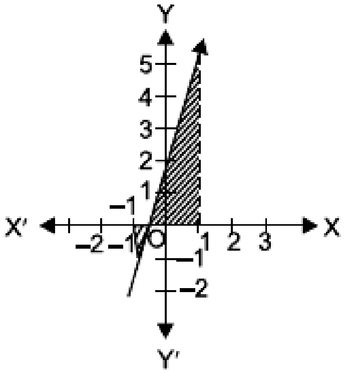
$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2} - 2x + \frac{x^2}{2}\right]_0^2$$

$$= 0 + 2\sin^{-1}(1) - 4 + 2 - 0$$

$$= 2\cdot\frac{\pi}{2} - 2 = (\pi - 2) \text{ sq.units.}$$

17. Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and ordinates at $x = -1$ and $x = 1$.

Ans: $y = 3x + 2$, the x-axis and ordinates at $x = -1$ and $x = 1$.



$$\text{Required Area} = \left|\int_{-1}^{-\frac{2}{3}} (3x+2)dx\right| + \int_{-\frac{2}{3}}^{1} (3x+2)dx$$

$$= \left|\left[\frac{3x^2}{2} + 2x\right]_{-1}^{-\frac{2}{3}}\right| + \left[\frac{3x^2}{2} + 2x\right]_{-\frac{2}{3}}^{1} = \left|\frac{2}{3} - \frac{4}{8} - \left(\frac{3}{2} - 2\right)\right| + \frac{3}{2} + 2 - \left(\frac{2}{3} - \frac{4}{3}\right)$$

$$= \left|-\frac{2}{3} + \frac{1}{2}\right| + \frac{7}{2} + \frac{2}{3} = \left|-\frac{1}{6}\right| + \frac{25}{6} = \frac{1}{6} + \frac{25}{6} = \frac{26}{6} = \frac{13}{3} \text{ sq. units}$$

SECTION – D

Questions 18 carry 5 marks.

18. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

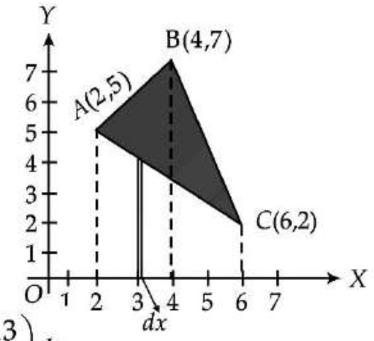
Ans:

From the given points,

Equation of AB : $y = x+3$

Equation of BC : $y = \frac{-5x}{2}+17$

Equation of AC : $y = \frac{-3x}{4}+\frac{13}{2}$



Required Area of ΔABC

$$= \int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2}+17\right)dx - \int_2^6 \left(\frac{-3x}{4}+\frac{13}{2}\right)dx$$

$$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{4}+17x\right]_4^6 - \left[\frac{-3x^2}{8}+\frac{13x}{2}\right]_2^6$$

$$= \left(\frac{49}{2}-\frac{25}{2}\right)+(57-48)-\left(\frac{51}{2}-\frac{23}{2}\right) = 12+9-14 = 21-14 = 7 \text{ sq. units}$$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

An architect designs a building whose lift (elevator) is from outside of the building attached to the walls. The floor (base) of the lift (elevator)) is in circular shape.

The floor of the elevator (lift) whose circular edge is given by the equation $x^2 + y^2 = 4$ and the straight edge (line) is given by the equation $y = 0$.

(i) Find the point of intersection of the circular edge and straight line edge.

(ii) Find the length of each vertical strip of the region bounded by the given curves.

(iii) (a) Find the area of a vertical strip between given circular edge and straight edge.

(b) Find the area of a horizontal strip between given circular strip and straight edge.

OR

(iii) Find the area of the region of the floor of the lift of the building (in square units).

Ans: (i) Given curve for circle and straight line are

$x^2 + y^2 = 4$ ....(1)

$y = 0$ ....(2)

From (1) and (2), we have

$x^2 = 4 \Rightarrow x = \pm 2$

∴ Points of intersection are (2, 0) and (-2, 0).

(ii) Given curve, is circle whose equation is $x^2 + y^2 = 4$

$\Rightarrow y^2 = 4 - x^2 \Rightarrow y = \sqrt{4-x^2}$

and $y = 0$

It represents x-axis.

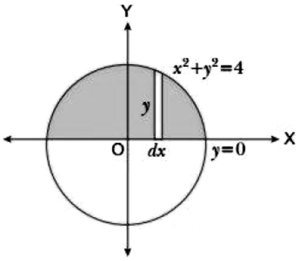
$\therefore$ Length of the vertical strip is $y = \sqrt{4 - x^2}$ i.e. $\sqrt{4 - x^2}$

(iii)

(*a*) We have,

Area of one vertical strip

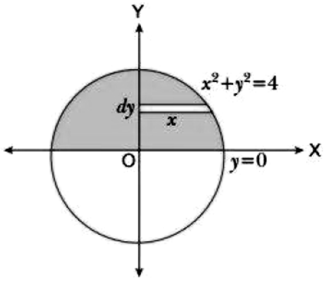
$= y.\,dx = \sqrt{4 - x^2}.\,dx$



(*b*) We have,

Area of one horizontal strip

$= x.\,dy = \sqrt{4 - y^2}.\,dy$



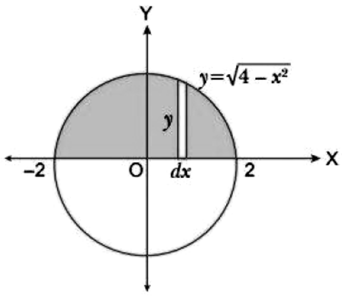
OR

(*iii*) We have,

$$\text{Area of the floor} = \int_{-2}^{2} y.dx = 2\int_{0}^{2} \sqrt{4 - x^2}.dx$$

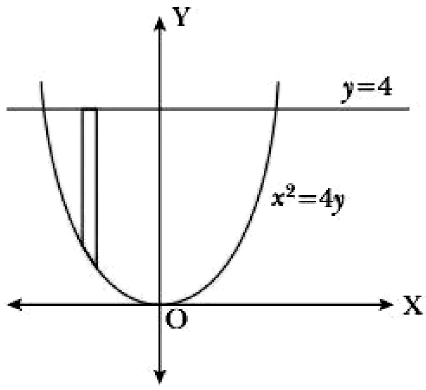
$$= 2\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= 4\sin^{-1} 1 - 0 = 4 \times \frac{\pi}{2} = 2\pi \text{ sq. units}$$



20. Case-Study 2: Read the following passage and answer the questions given below.

A student designs an open air Honeybee nest on the branch of a tree, whose plane figure is parabolic and the branch of tree is given by a straight line.



(i) Find point of intersection of the parabola and straight line.

(i) Find the area of each vertical strip.

(iii) (a) Find the length of each horizontal strip of the bounded region.

(b) Find the length of each vertical strip.

OR

(iii) Find the area of region bounded by parabola $x^2 = 4y$ and line $y = 4$ (in square units).

Ans: (i) Given equation of parabola is $x^2 = 4y$ and equation of straight line $y = 4$. .....(i)

From (i), we get $x^2 = 4 \times 4 = 16 \Rightarrow x = 4$

$\therefore$ Point of intersection are (4, 4) and (-4, 4).

(ii) Area of each (one) vertical strip

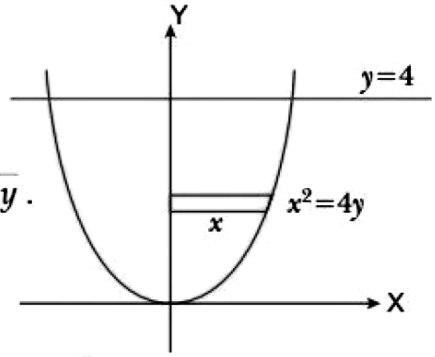
$= y.dx = 4dx - \frac{x^2}{4}dx = \left(4 - \frac{x^2}{4}\right).\ dx$

(*iii*) (*a*) We have,

$x^2 = 4y \qquad ...(i)$

$\Rightarrow \quad x = 2\sqrt{y}$

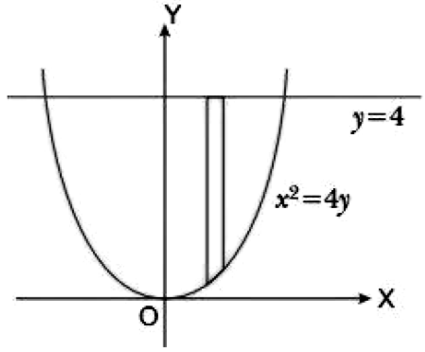
$\therefore$ Length of the horizontal strip be $2 \times 2\sqrt{y} = 4\sqrt{y}$.



(*b*) We have

Length of the vertical strip $= 4 - \frac{x^2}{4}$

$= \frac{1}{4}(16 - x^2)$



OR

(*iii*) We have

Area of required bounded region

$= 2\int_0^4 x\, dy$

$= 2\int_0^4 2\sqrt{y}\, dy = 4\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4$

$= \frac{8}{3}\left[(4)^{\frac{3}{2}} - 0\right] = \frac{64}{3}$ sq units