# KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD–32

## PRACTICE PAPER 04 (2023-24)

## CHAPTER 04 DETERMINANTS

## (ANSWERS)

**SUBJECT: MATHEMATICS** **MAX. MARKS : 40**

**CLASS : XII** **DURATION : $1\frac{1}{2}$ hrs**

**General Instructions:**

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

## SECTION – A

**Questions 1 to 10 carry 1 mark each.**

**1.** For what value of k ∈ N, $\begin{vmatrix} k & 3 \\ 4 & k \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 0 & 1 \end{vmatrix}$ is .

(a) 4 (b) 1 (c) 3 (d) 0

Ans: (a) 4

Given, $\begin{vmatrix} k & 3 \\ 4 & k \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 0 & 1 \end{vmatrix}$

$\Rightarrow k^2 - 12 = 4 - 0 \Rightarrow k^2 = 16 \quad \Rightarrow k = \pm 4 \quad \Rightarrow k = 4 \in N$

**2.** Find the cofactor of $a_{12}$ in the following: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

(a) -46 (b) 46 (c) 0 (d) 1

Ans: (b) 46

Minor of $a_{12}$ is $M_{12} = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} = -42 - 4 = -46$

Cofactor $C_{12} = (-1)^{1+2} M_{12} = (-1)^3 (-46) = 46$

**3.** If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$, then the value of x is:

(a) 6 (b) 3 (c) 7 (d) 1

Ans: (a) 6

$\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix} \Rightarrow (4 - 2)^2 = (3x - 2) - (x + 6)$

$\Rightarrow 4 = 3x - 2 - x - 6 \Rightarrow 2x = 12 \Rightarrow x = 6$

**4.** If A and B are square matrices of order 3 such that |A| = 1 and |B| = 3, then the value of |3AB| is:

(a) 3 (b) 9 (c) 27 (d) 81

Ans: (d) 81

As AB is of order 3 and

$|3AB| = 3^3|AB|$

$= 27|A||B| = 27 \times 1 \times 3 = 81$

**5.** If one root of the equation $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 7$ is x = -9, then the other two roots are:

(a) 6, 3 (b) 6, -3 (c) -2, -7 (d) 2, 6

Ans: (c) -2, -7

$\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 7(7x - 6) - 6(14 - 2x) + x(6 - x^2)$

$= -x^3 + 67x - 126$

$= (x + 9)(-x^2 + 9x - 14)$

$= (x + 9)(-x - 2)(x + 7)$

Hence the other two roots are -2 and -7.

**6.** Let A be a non-singular matrix of order (3 × 3). Then |adj.A| is equal to

(a) $|A|$ (b) $|A|^2$ (c) $|A|^3$ (d) $3|A|$

Ans: (b) $|A|^2$

If A is a matrix of order n × n then $|\text{adj } A| = |A|^{n-1}$

**7.** The maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$, where θ is a real number is:

(a) 1 (b) $\frac{1}{2}$ (c) 3 (d) -1

Ans: (b) $\frac{1}{2}$

We have, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$

$= 1(1 + \sin\theta - 1) - 1(1 - 1 - \cos\theta) + (1 - (1 + \sin\theta)(1 + \cos\theta))$

$= \sin\theta + \cos\theta + [1 - 1 - \sin\theta - \cos\theta - \sin\theta\cos\theta]$

$= \sin\theta + \cos\theta - \sin\theta - \cos\theta - \sin\theta\cos\theta$

$= -\sin\theta\cos\theta = -\frac{1}{2}\sin 2\theta$

Max. $\Delta = -\frac{1}{2}(-1) = \frac{1}{2}$ [$\because 1 \le \sin\theta \le 1$]

**8.** $A$ and $B$ are invertible matrices of the same order such that $|(AB)^{-1}| = 8$, If $|A| = 2$, then $|B|$ is

(*a*) 16 (*b*) 4 (*c*) 6 (*d*) 1/16

Ans: (*d*) 1/16

$|(AB)^{-1}| = \frac{1}{|AB|} = \frac{1}{|A||B|} \Rightarrow 8 = \frac{1}{2|B|} \Rightarrow B = \frac{1}{16}$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a)Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b)Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c)Assertion (A) is true but reason (R) is false.

(d)Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The matrix A = $\begin{bmatrix} 2 & 3 & -1/2 \\ 7 & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ is singular.

**Reason (R):** The value of determinant of matrix A is zero.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. **Assertion (A):** The value of determinant of a matrix and the value of determinant of its transpose are equal.
**Reason (R):** The value of determinant remains unchanged if its rows and columns are interchanged.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

## SECTION – B

**Questions 11 to 14 carry 2 marks each.**

11. Find the value of $x$, such that the points (0, 2), (1, $x$) and (3, 1) are collinear.
Ans: If points are collinear then area of triangle = 0.

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 1 & x & 1 \\ 3 & 1 & 1 \end{vmatrix} = 0 \Rightarrow \frac{1}{2}[0 - 2(1 - 3) + 1(1 - 3x)] = 0$$

$$\Rightarrow 4 + 1 - 3x = 0 \Rightarrow x = \frac{5}{3}.$$

12. Area of a triangle with vertices ($k$, 0), (1, 1) and (0, 3) is 5 sq units. Find the value(s) of $k$.

Ans: $\Delta = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix} = \frac{1}{2}[k(1 - 3) + 1(3 - 0)] = \frac{1}{2}(-2k + 3)$

$$\frac{1}{2}(-2k + 3) = \pm 5 \Rightarrow -2k + 3 = \pm 10$$

$$\Rightarrow -2k = 7 \text{ or } -13 \Rightarrow k = -\frac{7}{2} \text{ or } \frac{13}{2}.$$

13. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of $x$.

Ans: We have, $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow (x + 1)(x + 2) - (x - 3)(x - 1) = 12 + 1$
$\Rightarrow x^2 + 3x + 2 - x^2 + 4x - 3 = 13$
$\Rightarrow 7x = 14 \Rightarrow x = 2.$

14. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of $k$ if $|2A| = k|A|$

Ans: Matrix $A$ is of order 2,
$|2A| = 2^2|A| = k|A| \Rightarrow k = 4.$

## SECTION – C

**Questions 15 to 17 carry 3 marks each.**

**15.** If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A'A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

Ans: $|A| = 1 + \tan^2 x \neq 0$, hence, $A^{-1}$ exists

$$A'A^{-1} = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$= \frac{1}{1+\tan^2 x}\begin{bmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1-\tan^2 x}{1+\tan^2 x} & \dfrac{-2\tan x}{1+\tan^2 x} \\ \dfrac{2\tan x}{1+\tan^2 x} & \dfrac{1-\tan^2 x}{1+\tan^2 x} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$$

**16.** Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = O$. Hence find $A^{-1}$.

Ans: Given $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ and the equation

$x^2 - 6x + 17 = O$

If $A$ satisfies the equation,

then $A^2 - 6A + 17I = O$

Consider $A^2 - 6A + 17I$

$$= \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} - 6\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} + 17\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-9 & -6-12 \\ 6+12 & -9+16 \end{bmatrix} - \begin{bmatrix} 12 & -18 \\ 18 & 24 \end{bmatrix} + \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix} - \begin{bmatrix} 12 & -18 \\ 18 & 24 \end{bmatrix} + \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} -5-12+17 & -18+18+0 \\ 18-18+0 & 7-24+17 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A^2 - 6A + 17I = O$

Multiplying both sides by $A^{-1}$

$A^{-1}(AA) - 6A^{-1}A + 17A^{-1}I = A^{-1}O$

$\Rightarrow (A^{-1}A)A - 6I + 17A^{-1} = O$

$\Rightarrow IA - 6I + 17A^{-1} = O$

$\Rightarrow A - 6I + 17A^{-1} = O$

$$\Rightarrow A^{-1} = \frac{1}{17}(6I - A) = \frac{1}{17}\left\{\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}\right\} = \frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}.$$

**17.** If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Ans:

$$AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 12+6 & 18+4 \\ 28+15 & 42+10 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 22 \\ 43 & 52 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 18 & 22 \\ 43 & 52 \end{vmatrix} = 18 \times 52 - 22 \times 43$$

$$= 936 - 946 = -10 \neq 0$$

$$\text{adj } (AB) = \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix}$$

$$\left[\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{then } adj\ A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right]$$

$$\therefore (AB)^{-1} = \frac{1}{|AB|} \text{adj } (AB)$$

$$= -\frac{1}{10}\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} \quad \ldots(i)$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}.$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \quad \ldots(ii)$$

$$|B| = \begin{vmatrix} 4 & 6 \\ 3 & 2 \end{vmatrix} = 8 - 18 = -10 \neq 0$$

$$\text{adj } B = \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}.$$

$$\therefore B^{-1} = \frac{1}{|B|} \text{adj } B = -\frac{1}{10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix} \ldots(iii)$$

$$B^{-1}A^{-1} = -\frac{1}{10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} 10+42 & -4-18 \\ -15-28 & 6+12 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} \quad \ldots(iv)$$

From (*i*) and (*iv*), we get

$(AB)^{-1} = B^{-1}A^{-1}$.

## SECTION – D

**Questions 18 carry 5 marks.**

**18.** If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find $A^{-1}$ and hence solve the system of linear equations: $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$; $x + y - 2z = -3$.

Ans:

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2(-4+4) + 3(-6+4) + 5(3-2) = 0 - 6 + 5 = -1 \neq 0.$$

$\therefore$ A is non-singular matrix and $A^{-1}$ exists.

$C_{11} = 0$; $C_{12} = 2$; $C_{13} = 1$; $C_{21} = -1$; $C_{22} = -9$; $C_{23} = -5$; $C_{31} = 2$; $C_{32} = 23$; $C_{33} = 13$.

$$\text{So, } adj A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations can be expressed as AX = B, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Now $AX = B \Rightarrow X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2, z = 3$

## SECTION – E (Case Study Based Questions)

**Questions 19 to 20 carry 4 marks each.**

**19. Case-Study 3:**

Two schools A and B want to award their selected students on the values of Honesty, Hard work and Punctuality. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of ₹ 2200. School B wants to spend ₹ 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school A). The total amount of award for one prize on each value is ₹ 1200.


Using the concept of matrices and determinants, answer the following questions.

(i) What is the award money for Honesty? [1]

(ii) What is the award money for Punctuality? [1]

(iii) What is the award money for Hard work? [1]
(iv) If a matrix P is both symmetric and skew-symmetric, then find |P|. [1]
Ans: Three equations are formed from the given statements:

3x + 2y + z = 2200
4x + y + 3z = 3100
and x + y + z = 1200

Converting the system of equations in matrix form, we get

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

i.e. PX = Q

$$\text{where } P=\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } Q=\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

|P| = 3(1 – 3) –2(4 – 3) + 1(4 – 1) = –6 – 2 + 3 = –5 ≠ 0
⇒ X = P–1 Q, provided P–1 exists.

$$\therefore \quad \text{adj } P=\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

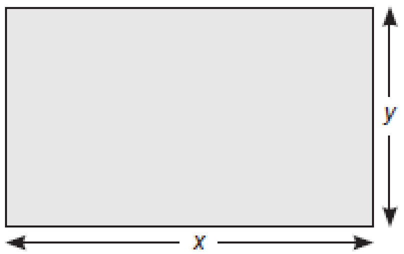
$$\therefore\ P^{-1}=\frac{1}{|P|}(\text{adj } P)=\frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}=\frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

$$\therefore X=\frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}=\frac{1}{5}\begin{bmatrix} 4400+3100-6000 \\ 2200-6200+6000 \\ -6600+3100+6000 \end{bmatrix}=\begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

$$\Rightarrow \quad x=300,\ y=400 \text{ and } z=500$$

(i) ₹ 300 (ii) ₹ 500 (iii) ₹ 400
(iv) If a matrix P is both symmetric and skew symmetric matrix then it will be a zero matrix. So, |P| = 0.

**20.** Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 $m^2$.




Based on the information given above, answer the following questions:
(a) Find the equations in terms of x and y (1)
(b) Find the value of x (length of rectangular field). (1)
(c) Find the value of y (breadth of rectangular field). (1)
(d) How much is the area of rectangular field? (1)
Ans: (a) Let Length of plot = x m
Breadth of plot = y m

Now, Area of plot = xy m2
Given that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same
(Length - 50) x (Breadth +50) = Area
(x – 50) x (y + 50) = xy
x (y + 50) – 50 (y + 50) = xy
xy + 50x – 50y – 2500 = xy
50x – 50y – 2500 = 0
50x – 50y = 2500
Dividing both sides by 50, we get
x - y = 50 ....(1)
Also, if length is decreased by 10m & breadth is decreased by 20m, then area will decrease by 5300 m2
(Length - 10) x (Breadth - 20) = Area - 5300
(x - 10) ×(y-20) = xy - 5300
x (y-20) 10 (y - 20) = xy - 5300
xy-20x10y + 200 = xy - 5300
-20x - 10y + 200 = -5300
-20x - 10y = -5300 - 200
-20x - 10y = -5500
20x+10y = 5500
Dividing both sides by 10, we get
2x + y = 550 ...(2)
Thus, our equation are x - y = 50 2x + y = 550
(b) Since our equations are x - y = 50 ...(1)
2x + y = 550 ...(2)
Adding (1) and (2)
(x - y) + (2x + y) = 50 + 550
⇒ 3x = 600
⇒ x = 600/2
⇒ x = 200 m
(c) Putting x = 200 in (1) , we get x – y = 50
⇒ 200 – y = 50
⇒ 200-50 = y
⇒ 150 = y
⇒ y = 150 m
(d) Area of rectangular field = Length x Breadth = 200 x 150 = 30,000 m2
= 30,000 sq. m