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PRACTICE PAPER 02 (2023-24)

CHAPTER 02 INVERSE TRIGONOMETRIC FUNCTIONS

(ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**

CLASS : XII | **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The value of $\tan^{-1}(\sqrt{3})+\cos^{-1}\left(-\frac{1}{2}\right)$ corresponding to principal branches is

(a) $-\frac{\pi}{12}$ (b) 0 (c) π (d) $\frac{\pi}{3}$

Ans: (c) π

$$\tan^{-1}(\sqrt{3})+\cos^{-1}\left(-\frac{1}{2}\right)$$
$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)+\cos^{-1}\left(-\cos\frac{\pi}{3}\right)$$
$$= \frac{\pi}{3}+\cos^{-1}\left[\cos\left(\pi-\frac{\pi}{3}\right)\right] = \frac{\pi}{3}+\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$$
$$= \frac{\pi}{3}+\frac{2\pi}{3}=\frac{3\pi}{3}=\pi$$

2. The value of $\sin^{-1}\left(\cos\frac{\pi}{9}\right)$ is

(a) $\frac{\pi}{9}$ (b) $\frac{5\pi}{9}$ (c) $\frac{-5\pi}{9}$ (d) $\frac{7\pi}{18}$

Ans: (d) $\frac{7\pi}{18}$

$$\sin^{-1}\left(\cos\frac{\pi}{9}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2}-\frac{\pi}{9}\right)\right)$$
$$= \sin^{-1}\left(\sin\frac{7\pi}{18}\right) = \frac{7\pi}{18}$$

3. The domain of the function defined by $\sin^{-1}\sqrt{x-1}$ is

(a) [1, 2] (b) [–1, 1] (c) [0, 1] (d) none of these

Ans: (a) [1, 2]

$\because\ f(x)=\sin^{-1}\sqrt{x-1}$

$\Rightarrow\ 0 \le x-1 \le 1 \quad [\because\ \sqrt{x-1}\ge 0 \text{ and } -1\le\sqrt{x-1}\le 1]$

$\Rightarrow\ 1 \le x \le 2$

$\therefore\ x\in[1,2]$

4. The value of $\tan^2(\sec^{-1}2) + \cot^2(\text{cosec}^{-1}3)$ is

(a) 5 (b) 11 (c) 13 (d) 15

Ans: (b) 11

$\tan^2\left(\sec^{-1}2\right) + \cot^2\left(\text{cosec}^{-1}3\right)$

$= \sec^2\left(\sec^{-1}2\right) - 1 + \text{cosec}^2\left(\text{cosec}^{-1}3\right) - 1$

$= 2^2 - 1 + 3^2 - 1 = 4 + 9 - 2 = 11$

5. The value of $\tan^{-1}\left(\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{4}\right)$ is

(a) $\frac{7}{24}$ (b) $\frac{24}{7}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

Ans: (b) $\frac{24}{7}$

Let $\sin^{-1}\frac{3}{5} = \theta \Rightarrow \sin\theta = \frac{3}{5} \Rightarrow \cos\theta = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\frac{3}{4}$

Now,

$\tan\left(\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{4}\right) = \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{4}\right)$

$= \tan\left(2\tan^{-1}\frac{3}{4}\right) = \tan\left(\tan^{-1}\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}\right) = \tan\left(\tan^{-1}\frac{24}{7}\right) = \frac{24}{7}$

6. If $\alpha \le 2\sin^{-1}x + \cos^{-1}x \le \beta$, then

(a) $\alpha = \frac{-\pi}{2}, \beta = \frac{\pi}{2}$ (b) $\alpha = 0, \beta = \pi$ (c) $\alpha = \frac{-\pi}{2}, \beta = \frac{3\pi}{2}$ (d) $\alpha = 0, \beta = 2\pi$

Ans: (b) $\alpha = 0, \beta = \pi$

We have $\frac{-\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$\Rightarrow \frac{-\pi}{2} + \frac{\pi}{2} \le \sin^{-1}x + \frac{\pi}{2} \le \frac{\pi}{2} + \frac{\pi}{2}$

$\Rightarrow 0 \le \sin^{-1}x + (\sin^{-1}x + \cos^{-1}x) \le \pi$

$\Rightarrow 0 \le 2\sin^{-1}x + \cos^{-1}x \le \pi$

7. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then the value of $\cos^{-1}x + \cos^{-1}y$ is

(a) $\frac{\pi}{2}$ (b) 0 (c) π (d) $\frac{2\pi}{3}$

Ans: (a) $\frac{\pi}{2}$

8. If $3\tan^{-1}x + \cot^{-1}x = \pi$, then x equals

(a) 0 (b) 1 (c) –1 (d) 1/2

Ans: (b) 1

Given, $3\tan^{-1}x + \cot^{-1}x = \pi$

$\Rightarrow\ 2\tan^{-1}x + \tan^{-1}x + \cot^{-1}x = \pi$

$\Rightarrow\ 2\tan^{-1}x = \pi - \frac{\pi}{2}$ $\quad\left[\because\ \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow\ 2\tan^{-1}x = \frac{\pi}{2}$

$\Rightarrow\ \tan^{-1}x = \frac{\pi}{4} \Rightarrow\ x = \tan\frac{\pi}{4} = 1 \Rightarrow\ x = 1$

Hence, only $x = 1$ satisfies the given equation.

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** Range of $\cot^{-1}$ x is $(0, \pi)$
Reason (R): Domain of $\tan^{-1}$ x is R.
Ans: (b) Both A and R are true but R is not the correct explanation of A.

10. **Assertion (A):** Principal value of $\tan^{-1}(-\sqrt{3})$ is $\frac{\pi}{3}$.

Reason (R): $\tan^{-1}$: IR$\rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so for any x ϵ IR, $\tan^{-1}(x)$ represents an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Ans: (a) Both A and R are true and R is the correct explanation of A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$

Ans:

$$\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right) = \sin^{-1}\cos\left(6\pi + \frac{3\pi}{5}\right) = \sin^{-1}\cos\left(\frac{3\pi}{5}\right)$$

$$= \frac{\pi}{2} - \cos^{-1}\cos\left(\frac{3\pi}{5}\right) = \frac{\pi}{2} - \frac{3\pi}{5} = -\frac{\pi}{10}.$$

12. Find the domain of $\sin^{-1}(x^2 - 4)$

Ans:

$$-1 \le \left(x^2 - 4\right) \le 1 \Rightarrow 3 \le x^2 \le 5 \Rightarrow \sqrt{3} \le |x| \le \sqrt{5}$$

$$\Rightarrow x \in \left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right].$$

So, required domain is $\left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$.

13. Find the value of $\sin^{-1}\left(\sin\left(\frac{13\pi}{7}\right)\right)$

Ans: $\sin^{-1}\left(\sin\left(\frac{13\pi}{7}\right)\right) = \sin^{-1}\left(\sin\left(2\pi - \frac{\pi}{7}\right)\right)$

$$= \sin^{-1}\left(\sin\left(-\frac{\pi}{7}\right)\right) = -\frac{\pi}{7}$$

14. Find the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

Ans: $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right) where, \frac{5\pi}{6} \in [0, \pi]$

$$\therefore \cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(\frac{5\pi}{6}\right)\right) = \frac{5\pi}{6} \quad (\because \cos(2\pi - \theta) = \cos\theta)$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the values of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

Ans: Let $\tan^{-1}(1) = x \Rightarrow \tan x = 1 = \tan\frac{\pi}{4} \Rightarrow x = \frac{\pi}{4}$ *where* $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \tan^{-1}(1) = \frac{\pi}{4}$$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y \Rightarrow \cos y = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3} \quad (\because \cos(\pi - \theta) = -\cos\theta)$

$$\Rightarrow y = \frac{2\pi}{3} \; where \; y \in [0, \pi]$$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z \Rightarrow \sin z = -\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right) \Rightarrow z = -\frac{\pi}{6}$ *where* $z \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) = x + y + z = \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

16. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$

Ans: *Let* $x = \cos y \Rightarrow y = \cos^{-1} x$

$$LHS = \tan^{-1}\left(\frac{\sqrt{1+\cos y} - \sqrt{1-\cos y}}{\sqrt{1+\cos y} + \sqrt{1-\cos y}}\right) = \tan^{-1}\left(\frac{2\cos\frac{y}{2} - 2\sin\frac{y}{2}}{2\cos\frac{y}{2} + 2\sin\frac{y}{2}}\right)$$

$$\left(\because 1 + \cos y = 2\cos^2\frac{y}{2} \; and \; 1 - \cos y = 2\sin^2\frac{y}{2}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{y}{2} - \sin\frac{y}{2}}{\cos\frac{y}{2} + \sin\frac{y}{2}}\right) = \tan^{-1}\left(\frac{1 - \tan\frac{y}{2}}{1 + \tan\frac{y}{2}}\right) = \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{y}{2}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

$$\left(\because \tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan x}{1 + \tan x}\right)$$

17. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

Ans: Given $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$

$$= \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\cos\frac{x}{2}\sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}\right)$$

$$\left(\because 1 - \cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}, \sin^2\frac{x}{2} + \cos^2\frac{x}{2} = 1 \ \ and \ \ \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right) = \tan^{-1}\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \frac{\pi}{4} + \frac{x}{2}$$

SECTION – D

Questions 18 carry 5 marks.

18. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

Ans:

Given $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

$$LHS = \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$$

$$= \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \times \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right)$$

(by rationalizing the denominator)

$$= \cot^{-1}\left(\frac{\left(\sqrt{1+\sin x} + \sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2 - \left(\sqrt{1-\sin x}\right)^2}\right) = \cot^{-1}\left(\frac{1 + \sin x + 1 - \sin x + 2\sqrt{1-\sin^2 x}}{1 + \sin x - 1 + \sin x}\right)$$

$$= \cot^{-1}\left(\frac{2 + 2\cos x}{\sin x}\right) = \cot^{-1}\left(\frac{2(1+\cos x)}{2\sin x}\right) = \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

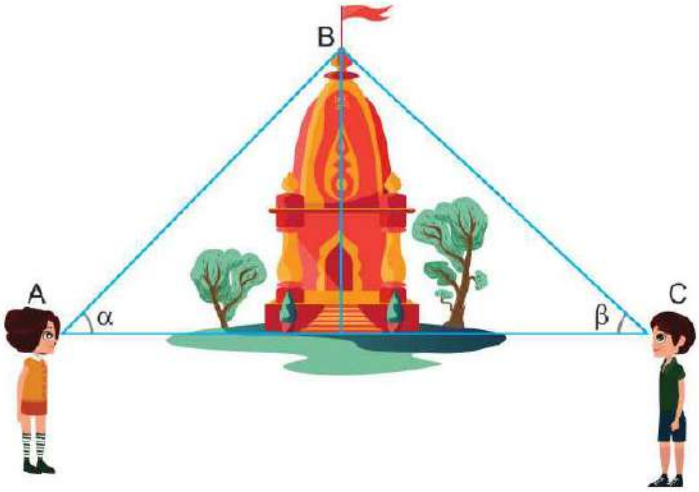
$$= \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) \qquad \left(\because 1 + \cos x = 2\cos^2\frac{x}{2} \ \ and \ \ \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\right)$$

$$= \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right) == \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2} = RHS$$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Two men on either side of a temple of 30 metres high from the level of eye observe its top at the angles of elevation α and β respectively. (as shown in the below figure). The distance between the two men is 40√3 metres and the distance between the first person A and the temple is 30√3 metres.

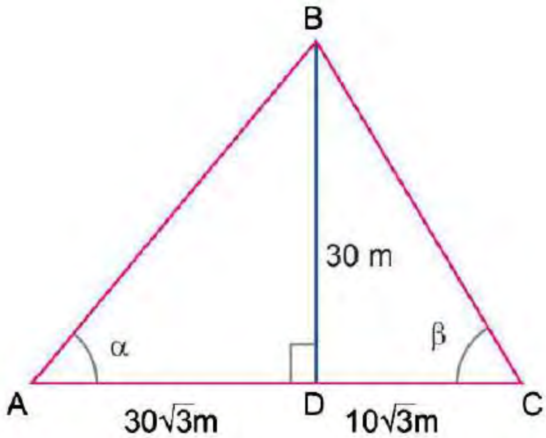


Based on the above information answer the following:

(i) Find the measure of ∠CAB in terms of $\sin^{-1}$ and $\cos^{-1}$.

(ii) Find the measure of ∠ABC.

Ans:



(*i*) Now in Δ*ABD* (right angled)

$$\tan\alpha = \frac{BD}{AD} = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \tan\alpha = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \quad \alpha = 30^\circ \quad \Rightarrow \quad \sin\alpha = \sin 30^\circ = \frac{1}{2} \quad \Rightarrow \quad \alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \cos\alpha = \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

(*ii*) In right Δ*BCD*, we have

$$\tan\beta = \frac{BD}{DC} \quad \Rightarrow \quad \tan\beta = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \quad \beta = \tan^{-1}(\sqrt{3}) = 60^\circ$$

In Δ*ABC*, we have,

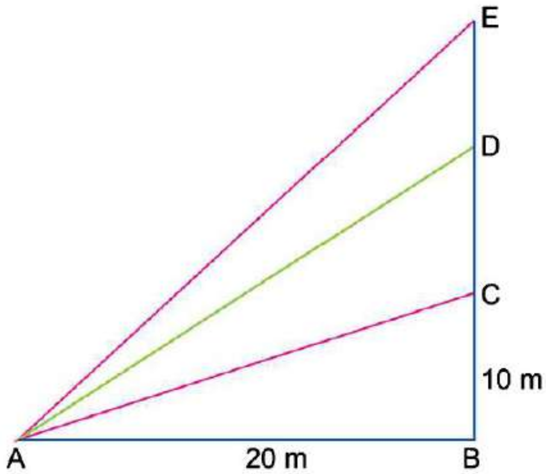
$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ABC + \alpha + \beta = 180° \Rightarrow \angle ABC + 30° + 60° = 180°$$

$$\Rightarrow \angle ABC = 90°$$

$$\Rightarrow \angle ABC = \frac{\pi}{2}$$

20. The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. “A” is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. “C” is at the height of 10 metres from the ground level. For the viewer A, the angle of elevation of “D” is double the angle of elevation of “C” The angle of elevation of “E” is triple the angle of elevation of “C” for the same viewer. Look at the figure given and based on the above information answer the following:

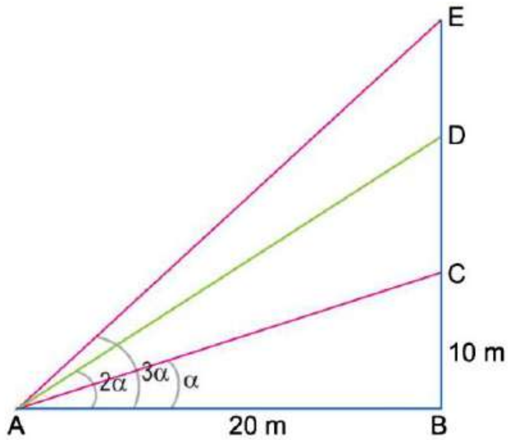


Based on the above information, answer the following questions:

(i) Find the measure of ∠DAB

(ii) Find the measure of ∠EAB

Ans:



(*i*) Let $\angle DAB = 2\alpha$

In ΔABC $\tan\alpha = \frac{BC}{AB} = \frac{10}{20} = \frac{1}{2}$

$$\therefore \quad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\times\frac{1}{2}}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\Rightarrow 2\alpha = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \angle DAB = \tan^{-1}\left(\frac{4}{3}\right)$$

(*ii*) Let $\angle EAB = 3\alpha$

We have, $\tan\alpha = \frac{1}{2}$ [From (*i*)]

$$\therefore \quad \tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha} = \frac{3\times\frac{1}{2} - \left(\frac{1}{2}\right)^3}{1 - 3\times\left(\frac{1}{2}\right)^2} = \frac{\frac{3}{2} - \frac{1}{8}}{1 - \frac{3}{4}} = \frac{\frac{11}{8}}{\frac{1}{4}} = \frac{11}{8}\times 4 = \frac{11}{2}$$

$$\Rightarrow \quad 3\alpha = \tan^{-1}\left(\frac{11}{2}\right) \Rightarrow \quad \angle EAB = \tan^{-1}\left(\frac{11}{2}\right).$$