PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD–32
PRACTICE PAPER 12 (2023-24)
CHAPTER 13 PROBABILITY (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XII **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. If P(A) = 1/4, P(B) = 1/3 and P(A ∩ B) = 1/5, then $P(\overline{B}/\overline{A})$ =?
(a) 11/15 (b) 11/45 (c) 37/45 (d) 37/60
Ans: (c) 37/45

2. If the following table represents a probability distribution for a random variable X:

X	1	2	3	4	5	6
P(X)	0.1	2k	K	0.2	3k	0.1

The value of k is:
(a) 0.01 (b) 0.1 (c) 1/1000 (d) 25
Ans: (b) 0.1
In the probability distribution of X, ΣP(X) = 1
⇒ (0.1) + 2k + k + (0.2) + 3k + (0.1) = 1
⇒ 6k = 0.6 ⇒ k = 0.1

3. A dice is tossed thrice. The probability of getting an odd number at least once is:
(a) 7/8 (b) 1/3 (c) 3/8 (d) 1/8
Ans: (a) 7/8
Required probability = 1 – Probability of getting no odd number
$= 1 - \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) = 1 - \frac{1}{8} = \frac{7}{8}$

4. Two numbers are selected at random from integers 1 through 9. If the sum is even, what is the probability that both numbers are odd?
(a) 5/8 (b) 1/6 (c) 4/9 (d) 2/3
Ans: (a) 5/8
Total outcome = $^5C_1$ x $^4C_1$ (Both the numbers are odd) + $^4C_1$ x $^3C_1$ (Both the numbers are even) = 32
Number of favourable outcomes = $^5C_1$ x $^4C_1$ =20
Thus, the probability that both numbers are odd will be = 20/32 = 5/8

5. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is
(a) 1/ 3 (b) 4/13 (c) 1/4 (d) 1/2
Ans: (c) 1/4

6. If A and B are two independent events with P(A) = 3/5 and P(B) = 4/9, then find $P(\overline{A}\cap\overline{B})$.
(a) 1/9 (b) 2/9 (c) 1/3 (d) 4/9
Ans: (b) 2/9

We know that, if A and B are independent events, then $\bar{A}$ and $\bar{B}$ are also independent.

$$\therefore P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B}) = (1 - P(A))(1 - P(B))$$
$$= \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right) = \frac{2}{5} \times \frac{5}{9} = \frac{2}{9}$$

7. If P(A) = $\frac{4}{5}$, and P(A ∩ B) = $\frac{7}{10}$, then P(B | A) is equal to

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$

Ans: (c) $\frac{7}{8}$

8. If P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6 then P(A U B) is equal to

(a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96

Ans: (d) 0.96

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \Rightarrow 0.6 = \frac{P(A \cap B)}{0.4}$$
$$\therefore P(A \cap B) = 0.6 \times 0.4 = 0.24$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.4 + 0.8 - 0.24 = 1.20 - 0.24 = 0.96$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, then P(E|F) = 2/3

Reason (R): Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, then P(E|F) = 1/3

Ans. (c) A is true but R is false.

Given that, P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

Hence, Assertion is true and Reason is false.

10. Assertion (A): Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$

Reason (R): Let A and B be two events with a random experiment then $P(A/B) = \frac{P(A \cap B)}{P(B)}$

Ans: A = Event of getting two heads and B = Event of getting at least one head

A = {HH}, B = {HT, TH, HH}

⇒ A ∩ B = {H H}

P(A ∩ B) = 1/4, P(B) = ¾

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = \frac{1}{3}$$

Both (A) and (R) are true and (R) is the correct explanation of (A).

∴ Option (a) is correct.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Find the probability that the problem is solved.

Ans: Let A, B, and C be the three students and P(A), P(B), P(C) be the probabilities of solving a problem respectively.

P(A) = 1/2, P(B) = 1/3, P(C) = 1/4

P[problem will be solved at least by 1] = $1 - P(\overline{A})P(\overline{B})P(\overline{C})$

$= 1 - [1 - P(A)]\,[1 - P(B)]\,[1 - P(C)] = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$

12. The random variable X can take only the values 0, 1, 2, 3. Given that:

P(X = 0) = P(X = 1) = P and P(X = 2) = P(X = 3) such that $\sum P_i x_i^2 = 2\sum P_i x_i$, find the value of P.

Ans: Let P(X = 2) = P(X = 3) = a

$\sum p_i = 1 \Rightarrow a = \frac{1}{2} - p$

$\sum p_i x_i^2 = 2\sum p_i x_i$

$\Rightarrow 0(p) + 1(p) + 4(a) + 9(a) = 2(0(p) + 1(p) + 2(a) + 3(a))$

$\Rightarrow p + 13a = 2p + 10a$

$\Rightarrow p = 3\left(\frac{1}{2} - p\right) \Rightarrow p = \frac{3}{8}$

13. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then find the probability that both drawn balls are black.

Ans: Let E and F denote respectively the events that first and second ball drawn are black. We have to find P(E ∩ F).

Now, $P(E) = \frac{10}{15}$, $P(F \mid E) = \frac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E) \cdot P(F|E) = \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$

14. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random.

(a) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(b) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Ans: Let A be the event that a student reads Hindi newspaper and B be the event that a student reads English newspaper.

P(A) = 60/100 = 0.6, P(B)= 40/100 = 0.4 and P(A ∩ B) = 20/100 = 0.2

(a) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$

(b) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = \frac{1}{2}$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid.

Ans:

Let X be no. of selected scouts who are well trained in first aid. Here random variable X may have value 0, 1, 2.

Now, $P(X = 0) = \frac{^{20}C_2}{^{50}C_2} = \frac{20\times19}{50\times49} = \frac{38}{245}$

$P(X = 1) = \frac{^{20}C_1 \times {^{30}C_1}}{^{50}C_2} = \frac{20\times30\times2}{50\times49} = \frac{120}{245}$

$P(X = 2) = \frac{^{30}C_2}{^{50}C_2} = \frac{30\times29}{50\times49} = \frac{87}{245}$

Now probability distribution table is

X	0	1	2
P(X)	$\frac{38}{245}$	$\frac{120}{245}$	$\frac{87}{245}$

16. An urn contains 5 white and 8 white black balls. Two successive drawing of three balls at a time are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and second draw gives 3 black balls.

Ans:

Consider the following events

A = Drawing 3 white balls in first draw

B = Drawing 3 black balls in the second draw.

Required probability $= P(A \cap B) = P(A)\,P(B/A)$ ...(i)

Now, $P(A) = \frac{^5C_3}{^{13}C_3} = \frac{10}{286} = \frac{5}{143}$

After drawing 3 white balls in first draw 10 balls are left in the bag, out of which 8 are black balls.

$\therefore P(B/A) = \frac{^8C_3}{^{10}C_3} = \frac{56}{120} = \frac{7}{15}$

Substituting these values in Eq. (i), we get

Required probability $= P(A \cap B) = P(A)\,P(B/A)$

$= \frac{5}{143} \times \frac{7}{15} = \frac{7}{429}$

17. Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X.

Ans: First 7 natural numbers are 1, 2, 3, 4, 5, 6, 7.

$$S = \begin{Bmatrix} (1,2)\,(1,3)\,(1,4)\,(1,5)\,(1,6)\,(1,7) \\ (2,1)\,(2,3)\,(2,4)\,(2,5)\,(2,6)\,(2,7) \\ (3,1)\,(3,2)\,(3,4)\,(3,5)\,(3,6)\,(3,7) \\ (4,1)\,(4,2)\,(4,3)\,(4,5)\,(4,6)\,(4,7) \\ (5,1)\,(5,2)\,(5,3)\,(5,4)\,(5,6)\,(5,7) \\ (6,1)\,(6,2)\,(6,3)\,(6,4)\,(6,5)\,(6,7) \\ (7,1)\,(7,2)\,(7,3)\,(7,4)\,(7,5)\,(7,6) \end{Bmatrix} \text{i.e. 42 ways}$$

$P(X=1) = \frac{12}{42} = \frac{2}{7}, P(X=2) = \frac{10}{42} = \frac{5}{21}, P(X=3) = \frac{8}{42} = \frac{4}{21}$

$P(X=4) = \frac{6}{42} = \frac{1}{7}, P(X=5) = \frac{4}{42} = \frac{2}{21}, P(X=6) = \frac{2}{42} = \frac{1}{21}$

∴ Probability distribution is

X	1	2	3	4	5	6
P(X)	$\frac{2}{7}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{1}{7}$	$\frac{2}{21}$	$\frac{1}{21}$

SECTION – D

Questions 18 carry 5 marks.

18. There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head. What is probability that it was the two headed coin?

Ans: Let E_1 : Two headed coin is chosen

E_2 : Coin chosen is biased

E_3 : Coin chosen is unbiased

A : Coin shows head

$$P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{3}$$

$$P(A/E_1)=1, P(A/E_2)=\frac{75}{100}=\frac{3}{4}, P(A/E_3)=\frac{1}{2}$$

Using Baye's theorem,

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot(A|E_2)+P(E_3)\cdot(A|E_3)}$$

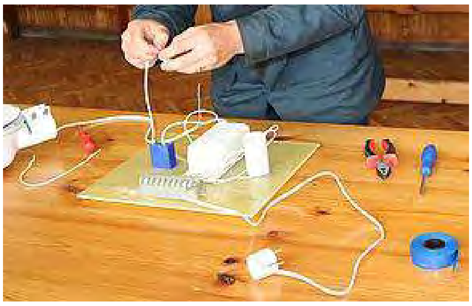
$$=\frac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right)\times\left(\frac{1}{3}\times\frac{3}{4}\right)\times\left(\frac{1}{3}\times\frac{1}{2}\right)}=\frac{\frac{1}{3}}{\frac{1}{3}\times\frac{3}{12}\times\frac{1}{6}}=\frac{\frac{1}{3}}{\frac{4+3+2}{12}}=\frac{1}{3}\times\frac{12}{9}=\frac{4}{9}$$

SECTION – E (Case Study Based Questions)

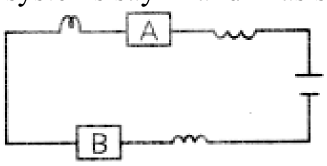
Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

An electric circuit includes a device that gives energy to the charged particles constituting the current, such as a battery or a generator; devices that use current, such as lamps, electric motors, or computers; and the connecting wires or transmission lines.



An electric circuit consists of two subsystems say A and B as shown below:



For previous testing procedures, the following probabilities are assumed to be known.

P(A fails) = 0.2, P(B fails alone) = 0.15, P(A and B fail) = 0.15

Based on the above information answer the following questions:

(a) What is the probability that B fails? [1]
(b) What is the probability that A fails alone? [1]
(c) Find the probability that the whole of the electric system fails? [2]

OR

Find the conditional probability that B fails when A has already failed. [2]
Ans:
(a) Consider the following events
E = A fails, F = B fails
Given P(E) = 0.2, P ($\vec{E}$ ∩ F) = 0.15, F(E ∩ F) = 0.15
Since, P($\vec{E}$ ∩ F) = 0.15
⇒ P(F) – P(E ∩ F) = 0.15
⇒ P(F) = 0.15 + P(E ∩ F)
⇒ P(F) = 0.15 + 0.15
⇒ P(F) = 0.30
(b) P(E ∩ $\vec{F}$) = P(E) – P(E ∩ F)
= 0.2 – 0.15
= 0.05
(c) If the electric system fails, we mean that A is also failed and B is also failed.
i.e., we have to find P(E ∪ F), where E is an event when A fails and B is an event when B fails.
∴ P(E ∪ F) = P(E) + P(F) – P(E ∩ F)
= 0.2 + 0.3 – 0.15
= 0.5 – 0.15
= 0.35

OR

Let E = A fail, F = B fail
∴ P(E) = 0.2, P(E ∩ F) = 0.15, P($\vec{F}$ ∩ F) = 0.15

$$P\left(\frac{B\ \ fails}{A\ \ fails}\right) = P\left(\frac{F}{E}\right) = \frac{P(F \cap E)}{P(E)} = \frac{0.15}{0.20} = \frac{3}{4}$$

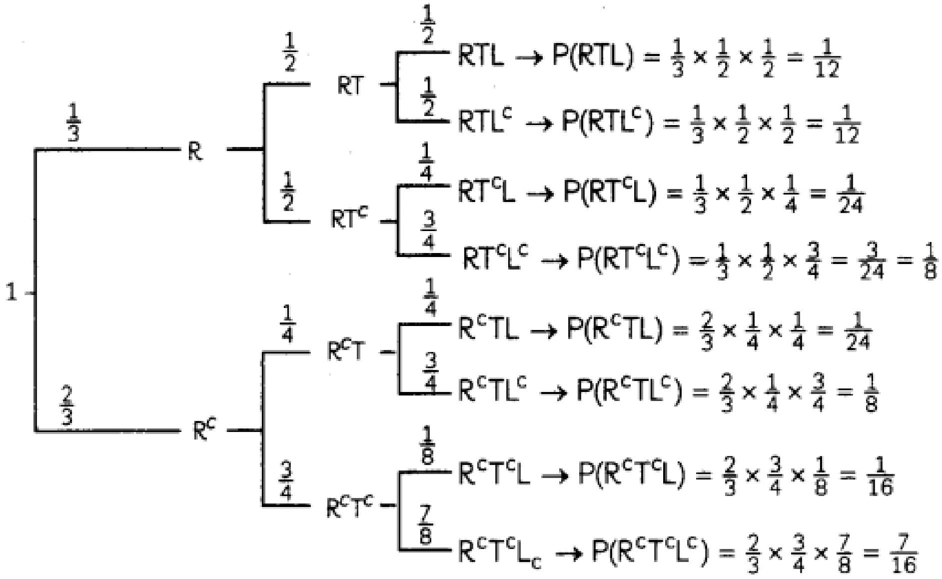
20. Case-Study 1: Read the following passage and answer the questions given below.
In a town, it's rainy one-third of the day. Given that it is rainy, there will be heavy traffic with probability 1/2. Given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, I arrive late for work with probability 1/2. On the other hand, the probability of being late is reduced to 1/8 if it, is not rainy and there is no heavy traffic. In other situations (rainy and no heavy traffic, net rainy and heavy traffic), the probability of being late is 1/4. You pick a random day.

Based on the above information, answer the following questions:
(i) What is the probability that it's not raining and there is heavy traffic and I am not late?

(ii) What is the probability that I am late?
(iii) Given that I arrived late at work, what is the probability that it rained that day?

OR

(iii) If P(not A) = 0.7, P(B) = 0.7 and P(B/A) = 0.5, then find the P(A/B)
Ans: From the given passage we can form a tree as below:



$\Rightarrow \quad P(A \cap B) = 0.15$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{3}{14}$$

(i) $P(R^C T L^C) = 1/8$

(ii) P(I am late) = Sum of probabilities corresponds to "I am late"

$= P(RTL) + (RT^C L) + (R^C TL) + (R^C T^C L)$

$$= \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \frac{11}{48}$$

(iii) $P\left(\frac{R}{L}\right) = \frac{P(R \cap L)}{P(L)}$

Now, P(R ∩ L) = Sum of probabilities in which R and L are common

$$= \frac{1}{12} + \frac{1}{24} = \frac{3}{24} = \frac{1}{8} \text{ and } P(L) = \frac{11}{48}$$

$$\therefore P\left(\frac{R}{L}\right) = \frac{P(R \cap L)}{P(L)} = \frac{\frac{1}{8}}{\frac{11}{48}} = \frac{1}{8} \times \frac{48}{11} = \frac{6}{11}$$

OR

Given, P(A') = 0.7 and P(B') = 0.7 and P(B/A) = 0.5

Clearly, P(A) = 1 – P(A') = 1 – 0.7 = 0.3

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \Rightarrow 0.5 = \frac{P(A \cap B)}{0.3} \Rightarrow P(A \cap B) = 0.15$

$$\therefore P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{3}{14}$$