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PRACTICE PAPER 13 (2023-24)
CHAPTER 12 LINEAR PROGRAMMING (ANSWERS)

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**
CLASS : XII — **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

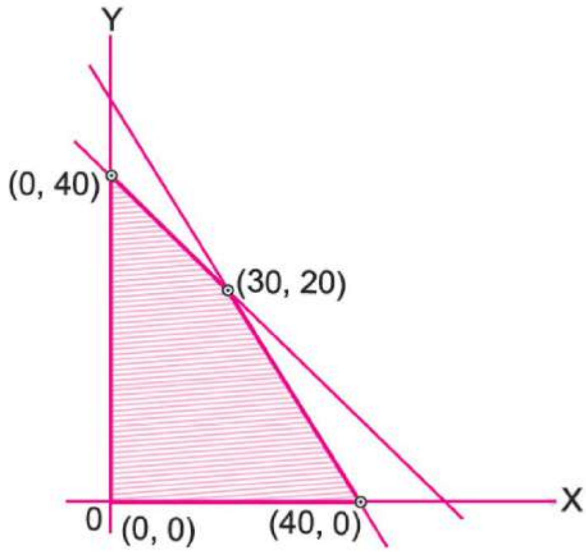
SECTION – A
Questions 1 to 10 carry 1 mark each.

1. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).
Let F =4x + 6y be the objective function. The minimum value of F occurs at
(a) Only (0, 2)
(b) Only (3, 0)
(c) the mid-point of the line segment joining the points (0, 2) and (3, 0)
(d) any point on the line segment joining the points (0, 2) and (3, 0)
Ans: (d) any point on the line segment joining the points (0, 2) and (3, 0)

Corner points	Corresponding value of $F = 4x + 6y$
(0, 2)	12← Minimum
(3, 0)	12← Minimum
(6, 0)	24
(6, 8)	72← Maximum
(0, 5)	30

Hence, minimum value of F occurs at any points on the line segment joining the points (0, 2) and (3, 0).

2. Feasible region (shaded) for a LPP is shown in the given figure.
The maximum value of the Z = 0.4x + y is



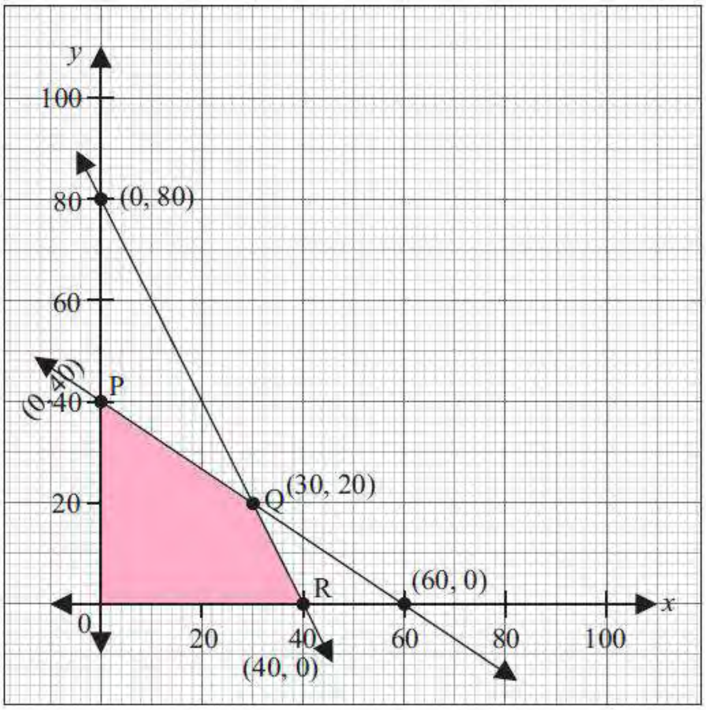
(a) 45 (b) 40 (c) 50 (d) 41

Ans: (d) 41

3. A set of values of decision variables that satisfies the linear constraints and non-negativity conditions of an L.P.P. is called its:
(a) Unbounded solution (b) Optimum solution
(c) Feasible solution (d) None of these
Ans: (c) Feasible solution

4. The corner points of the feasible region determined by the following system of linear inequalities: $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0,0), (5,0), (3,4), (0,5). Let $Z = px + qy$, where $p,q > 0$. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is
(a) $p = q$ (b) $p = 2q$ (c) $p = 3q$ (d) $q = 3p$
Ans: (d) $q = 3p$

5. For an L.P.P. the objective function is $Z = 4x + 3y$, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



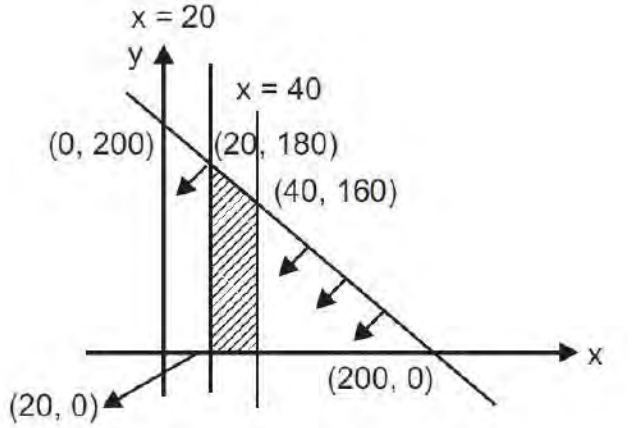
Which one of the following statements is true?
(a) Maximum value of Z is at R.
(b) Maximum value of Z is at Q.
(c) Value of Z at R is less than the value at P.
(d) Value of Z at Q is less than the value at R.
Ans: (b) Maximum value of Z is at Q.
$Z = 4x + 3y$
at P (0, 40), $Z = 4(0) + 3(40) = 120$
at Q (30, 20), $Z = 4(30) + 3(20) = 180$
at R (40, 0), $Z = 4(40) + 3(0) = 160$
$\therefore Z_{max} = 180$ at Q (30, 20)

6. Corner points of the feasible region for an LPP are (0, 3), (1,1) and (3,0). Let $Z = px + qy$, where $p, q > 0$, be the objective function. The condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is

(a) $p = q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p=q$

Ans: (b) $p = \frac{q}{2}$

7. For an L.P.P. the objective function is Z = 400x + 300y, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Find the coordinates at which the objective function is maximum.

(a) (20, 0) (b) (40, 0) (c) (40, 160) (d) (20, 180)

Ans: (c) (40, 160)

Value of z at each corner point

z at (20, 0), z = 400 × 20 + 300 × 0 = 8000

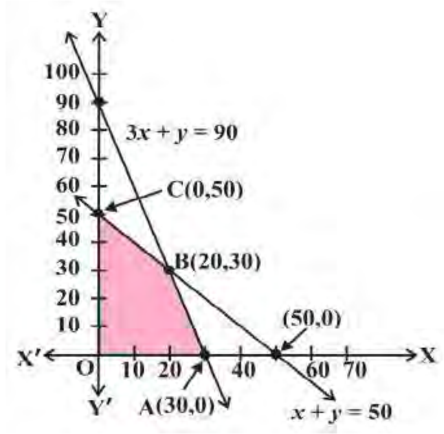
z at (40, 0) = 400 × 40 + 300 × 0 = 16000

z at (40, 160) = 400 × 40 + 300 × 160 = 16000 + 48000 = 64000

z at (20, 180) = 400 × 20 + 300 × 180 = 8000 + 54000 = 62000

max z = 64000 for x = 40, y = 160

8. The corner points of the shaded bounded feasible region of an LPP are (0,0),(30,0),(20,30) and (0,50) as shown in the figure .



The maximum value of the objective function Z = 4x+y is

(a) 120 (b) 130 (c) 140 (d) 150

Ans: (a) 120

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The maximum value of Z = 5x + 3y, satisfying the conditions x ≥ 20, y ≥ 0 and 5x + 2y ≤ 10, is 15.

Reason (R): A feasible region may be bounded or unbounded.

Ans: We have, corner points (0, 0), (2, 0), (0, 5).

∴ $Z_{max} = 5 \times 0 + 3 \times 5 = 15$ at (0, 5)

Clearly, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

∴ Option (b) is correct.

10. Assertion (A): The maximum value of Z = x + 3y. Such that $2x + y \le 20$, $x + 2y \le 20$, $x, y \ge 0$ is 30.
Reason (R): The variables that enter into the problem are called decision variables.

Ans: We have, corner points be (0, 0), (10, 0), (20/3, 20/3) (0, 10).

∴ $Z_{max} = x + 3y = 0 + 3 \times 10 = 30$ at (0, 10)

Clearly, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

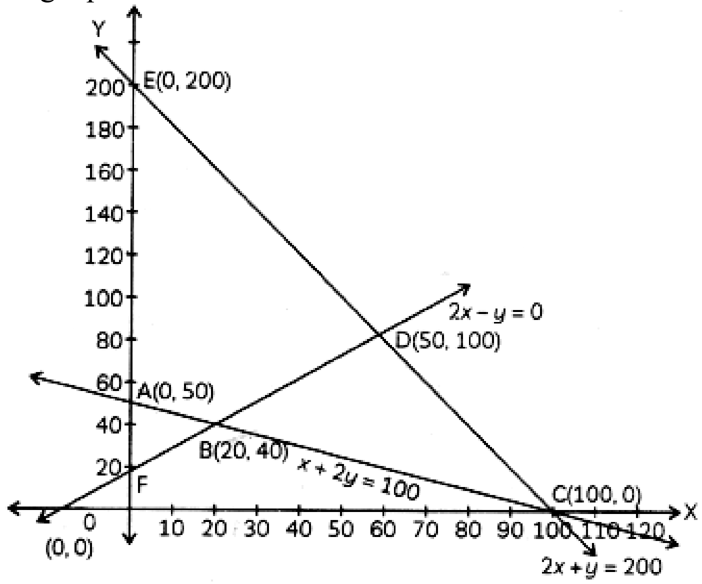
∴ Option (b) is correct.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In a linear programming problem, objective function, z = x + 2y. The subjective the constraints $x + 2y \ge 100$, $2x - y \le 0$, $2x + y \le 200$, $x \ge 0$, $y \ge 0$

The graph of the following equations is shown below.



Name the feasible region, and find the corner point at which the objective function is minimum.

Ans: Here the feasible region is ABCDEA

So, corner points are A(0, 50), B(20, 40), C(50, 100), E(0, 200)

Corner Points	Z = x + 2y	
A(0, 50)	100	Minimum
B(20, 40)	100	Minimum
C(50, 100)	250	
D(0, 200)	400	Maximum

The minimum value of z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

12. A manufacturer makes two types of furniture, chairs and tables. Both the products are processed on three machines A_1, A_2 and A_3. Machine A_1 requires 3 hours for a chair and 3 hours for a table, machine A_2 requires 5 hours for a chair and 2 hours for a table and machine A_3 requires 2 hours for a chair and 6 hours for a table. Maximum time available on machine A_1, A_2 and A_3 is 36 hours, 50 hours and 60 hours respectively. Profits are ₹ 20 per chair and ₹30 per table. Formulate the above as a linear programming problem to maximise the profit.

Ans:

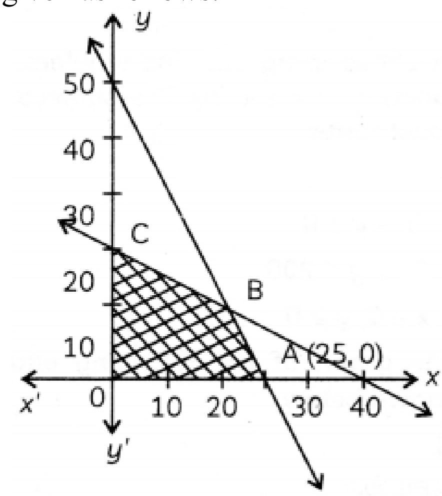
	Machine A_1	Machine A_2	Machine A_3	Profit
Chair	3 hrs	5 hrs	2 hrs	₹ 20
Table	3 hrs	2 hrs	6 hrs	₹ 30
	$\leq$ 36 hrs	$\leq$ 50 hrs	$\leq$ 60 hrs	

Let x chairs and y tables are manufactured.
Then *LPP* is
Maximise $P = 20x + 30y$
subject to the constraints, $x \geq 0$, $y \geq 0$, $3x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$.

OR

Two tailors *A* and *B* earn ₹150 and ₹200 per day respectively. *A* can stich 6 shirts and 4 pants per day while *B* can stich 10 shirts and 4 pants per day. Form a linear programming problem to minimise the labour cost to produce at least 60 shirts and 52 pants.
Ans: Let *A* works for x days and *B* works for y days
Then *LPP* is
To Minimise cost $Z = 150x + 200y$
subject to constraints,
$x \geq 0$, $y \geq 0$
$6x + 10y \geq 60$
$4x + 4y \geq 52$

13. The feasible region of a ∠PR is given as follows:



(i) Write the constraints with respect to the above in terms of x and y.
(ii) Find the coordinate of B and C and maximize, z = x + y.

Ans: (i) Equation of line is: $\frac{x}{25} + \frac{y}{50} \leq 1 \Rightarrow 2x + y \leq 50$

Equation of second line is: $\frac{x}{40} + \frac{y}{20} \leq 1 \Rightarrow x + 2y \leq 40$

∴ Constraint are $2x + y \leq 50$, $x + 2y \leq 40$, $x \geq 0$, $y \geq 0$
(ii) Coordinates of B are (20, 10) and C(0, 20)
∴ For z = x + y

Corner points	z = x + y
(25, 0)	25
(20, 10)	30 Max
(0, 20)	20
(0, 0)	0

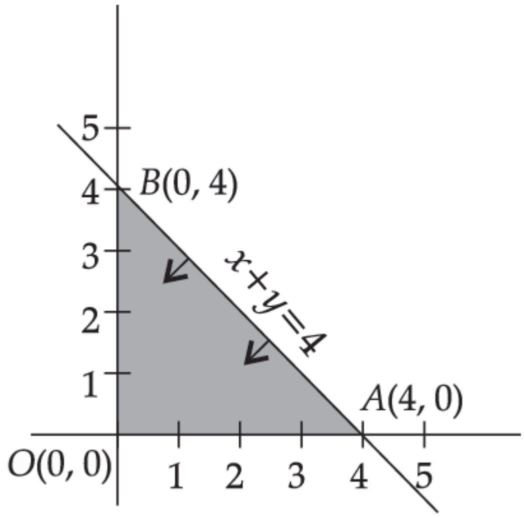
Hence, z is the maximum at the point (20, 10).

14. Solve the following LPP graphically:

Maximise Z = 3x + 4y
Subject to $x + y \leq 4$, $x \geq 0$ and $y \geq 0$.
Ans: Given, Max. Z = 3x + 4y
Subject to $x + y \leq 4$, $x, y \geq 0$
for x + y = 4

x	0	4
y	4	0

Also, x = 0 and y = 0

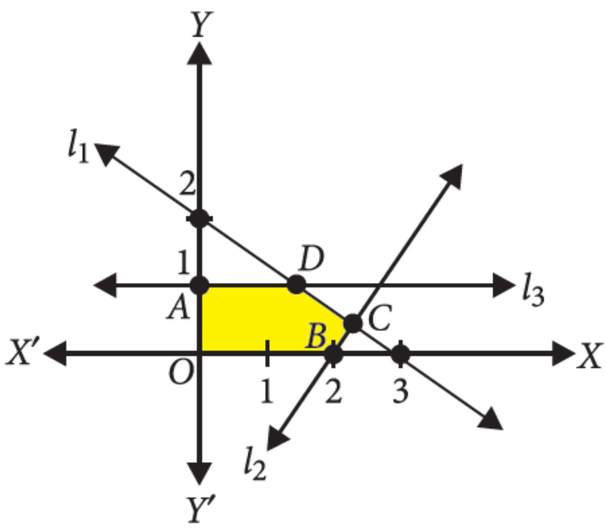


The feasible region is a triangle with vertices O(0, 0), A(4, 0) and B(0, 4)
ZO = 3 × 0 + 4 × 0 = 0
ZA = 3 × 4 + 4 × 0 = 12
ZB = 3 × 0 + 4 × 4 = 16
Thus, maximum of Z is at B(0, 4) and the maximum value is 16

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Solve the following Linear Programming Problem graphically:
Maximise z = 8x + 9y subject to the constraints: $2x + 3y \leq 6$, $3x - 2y \leq 6$, $y \leq 1$; $x, y \geq 0$
Ans:
Let $l_1 : 2x + 3y = 6$, $l_2 : 3x - 2y = 6$, $l_3 : y = 1$; x = 0, y = 0
Solving l_1 and l_3, we get D (1.5, 1)
Solving l_1 and l_2, we get $C\left(\frac{30}{13}, \frac{6}{13}\right)$



Shaded portion OADCB is the feasible region, where coordinates of the corner points are O(0, 0),

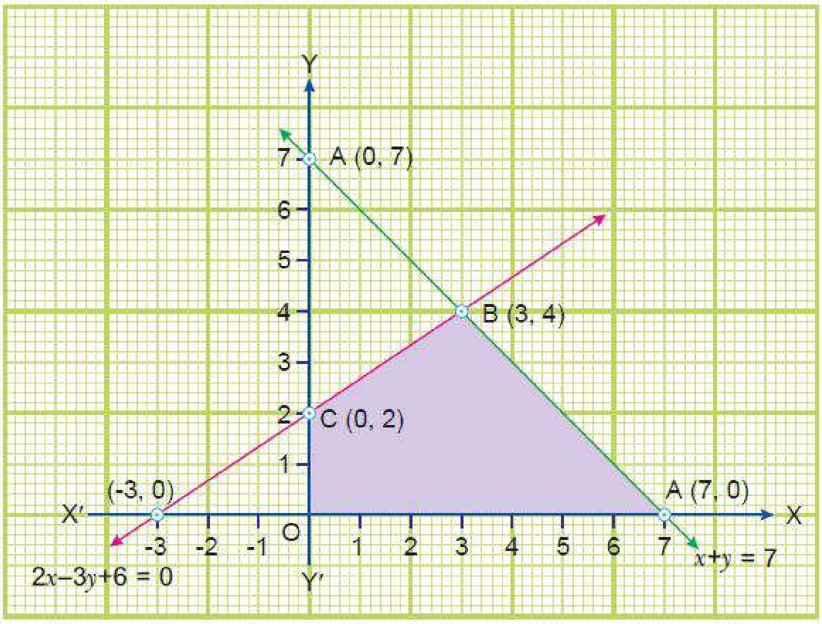
A(0, 1), D(1.5, 1), C$\left(\frac{30}{13}, \frac{6}{13}\right)$, B(2, 0).

The value of the objective function at these points are:

Corner points	**Value of the objective function z = 8x + 9y**
O (0, 0)	$8 \times 0 + 9 \times 0 = 0$
A (0, 1)	$8 \times 0 + 9 \times 1 = 9$
D (1.5, 1)	$8 \times 1.5 + 9 \times 1 = 21$
C$\left(\frac{30}{13}, \frac{6}{13}\right)$	$8 \times \frac{30}{13} + 9 \times \frac{6}{13} = 22.6$ (Maximum)
B (2, 0)	$8 \times 2 + 9 \times 0 = 16$

The maximum value of z is 22.6, which is at C$\left(\frac{30}{13}, \frac{6}{13}\right)$

16. Solve the following Linear Programming Problem graphically:
Minimise Z = 13x – 15y subject to the constraints x + y ≤ 7, 2x – 3y + 6 ≥ 0, x ≥ 0 and y ≥ 0.
Ans: Minimise Z = 13x – 15y ...(i)
Subject to the constraints
x + y ≤ 7 ...(ii)
2x – 3y + 6≥ 0 ...(iii)
x ≥ 0, y ≥ 0 ...(iv)
Shaded region shown as OABC is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively.

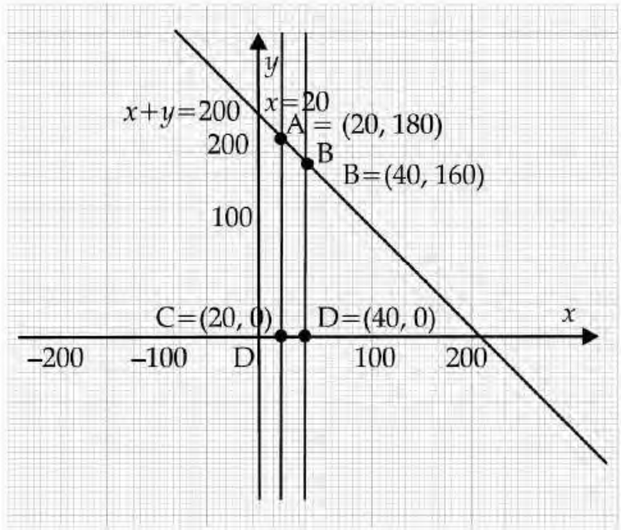


Corner Points	$Z = 13x - 15y$
O (0, 0)	0
A (7, 0)	91
B (3, 4)	–21
C (0, 2)	–30 ← Minimum

Hence, the minimum value of Z is –30 at (0, 2).

17. Solve the following Linear Programming Problem graphically:
Maximize Z = 400x + 300y subject to $x + y \leq 200$, $x \leq 40$, $x \geq 20$, $y \geq 0$

Ans: We have Z = 400x + 300y subject to
$x + y < 200, x < 40, x > 20, y > 0$
The corner points of the feasible region are
C(20, 0) D(40, 0), B(40, 160), A(20, 80)



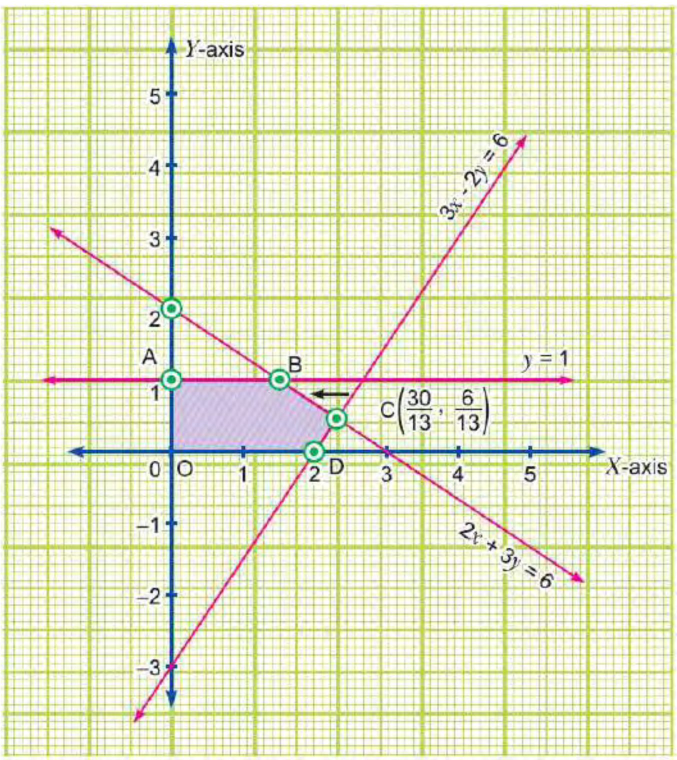
Corner Point	$Z = 400x + 300y$
C(20, 0)	8000
D(40, 0)	16000
B(40, 160)	64000
A(20, 180)	62000

Maximum profit occurs at x = 40, y = 160
and the maximum profit = Rs. 64, 000

SECTION – D

Questions 18 carry 5 marks.

18. Maximise Z = 8x + 9y subject to the constraints given below :
$2x + 3y \leq 6;\ 3x - 2y \leq 6;\ y \leq 1;\ x, y \geq 0$
Ans: For graph of $2x + 3y \leq 6$
We draw the graph of $2x + 3y = 6$
$2 \times 0 + 3 \times 0 \leq 6 \Rightarrow (0,0)$ satisfy the constraints.
Hence, feasible region lie towards origin side of line.
For graph of $3x - 2y \leq 6$
We draw the graph of line $3x - 2y = 6$.
$3 \times 0 - 2 \times 0 \leq 6 \Rightarrow$ Origin (0, 0) satisfy $3x - 2y = 6$.
Hence, feasible region lie towards origin side of line.



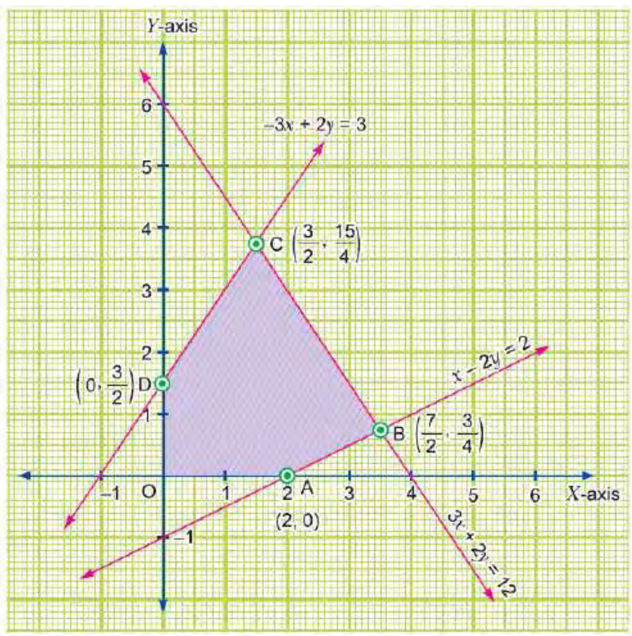
For graph of $y \le 1$
We draw the graph of line $y = 1$, which is parallel to x-axis and meet y-axis at 1.
$0 \le 1 \Rightarrow$ feasible region lie towards origin side of $y = 1$.
Also, $x \ge 0$, $y \ge 0$ says feasible region is in Ist quadrant.
Therefore, OABCDO is the required feasible region, having corner point O(0, 0), A(0, 1)
Here, feasible region is bounded. Now the value of objective function $Z = 8x + 9y$ is obtained as.

Corner Points	**$Z = 8x + 9y$**
O (0, 0)	0
A (0, 1)	9
B (3/2, 1)	21
C (30/13, 6/13)	22.6
D(2, 0)	16

Z is maximum when $x = 30/13$ and $y = 6/13$.

OR

Minimize and maximize $Z = 5x + 2y$ subject to the following constraints:
$x - 2y \le 2$, $3x + 2y \le 12$, $-3x + 2y \le 3$, $x \ge 0$, $y \ge 0$
Ans: Here, objective function is $Z = 5x + 2y$ ...(i)
Subject to the constraints:
$x - 2y \le 2$ ...(ii)
$3x + 2y \le 12$ ...(iii)
$-3x + 2y \le 3$ ...(iv)
$x \ge 0$, $y \ge 0$ ...(v)
For Graph for $x - 2y \le 2$, We draw graph of $x - 2y = 2$
[By putting $x = y = 0$ in the equation]
i.e., (0, 0) satisfy (ii) $\Rightarrow$ feasible region lie origin side of line $x - 2y = 2$.
For Graph for $3x + 2y \le 12$, We draw the graph of $3x + 2y = 12$.
$3 \times 0 + 2 \times 0 \le 12$ [By putting $x = y = 0$ in the given equation]
i.e., (0, 0) satisfy (iii) $\Rightarrow$ feasible region lie origin side of line $3x + 2y = 12$.
For Graph for $-3x + 2y \le 3$, We draw the graph of $-3x + 2y = 3$
$-3 \times 0 + 2 \times 0 \le 3$ [By putting $x = y = 0$]
i.e., (0, 0) satisfy (iv) $\Rightarrow$ feasible region lie origin side of line $-3x + 2y = 3$.
$x \ge 0$, $y \ge 0 \Rightarrow$ feasible region is in Ist quadrant.
Now, we get shaded region having corner points O, A, B, C and D as feasible region.



The co-ordinates of O, A, B, C and D are O(0, 0), A(2, 0), B(7/2, 3/4), C(3/2, 15/4) and D(0, 3/2) respectively. Now, we evaluate Z at the corner points.

Corner Points	Z = 5x + 2y
O (0, 0)	0
A (2, 0)	10
B (7/2, 3/4)	19
C (3/2, 15/4)	15
D (0, 3/2)	3

Hence, Z is minimum at x = 0, y = 0 and minimum value = 0
also Z is maximum at x = 7/2, y = 3/4 and maximum value = 19.

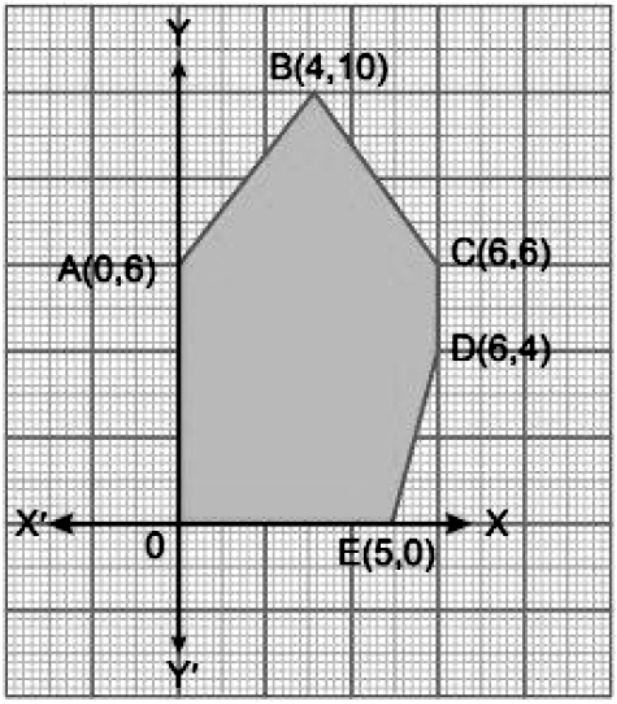
SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

Linear Programming Problem is a method of or finding the optimal values (maximum or minimum) of quantities subject to the constraints when relationship is expressed as a linear equations or linear inequations.

The corner points of a feasible region determined by the system of linear constraints are as shown below.



(i) Is this feasible region is bounded?

(i) Write the number of corner points in the feasible region.

(iii) (a) If Z = ax + by has maximum value at C (6, 6) and B (4, 10). Find the relationship between a & b.

OR

(iii) (b) If Z = 2x – 5y then find the minimum value of this objective function.

Ans:

(i) Yes the above feasible region is bounded.

(ii) Number of corner points = 6

(iii) (a) Z = ax + by

Z(6,6) = 6a + 6b

Also Z(4, 10) = 4a + 10b

From question, 6a + 6b = 4a + 10b ⇒ 2a = 4b ⇒ a = 2b

OR

(iii) (b)

Corner Points	Z = 2x – 3y
O (0, 0)	0
A (0, 6)	-30
B (4, 10)	-42
C (6, 6)	-18
D (6, 4)	-8
E(5, 0)	10

Minimum value of Z is -42 at the point B(4, 10).

20. Case-Study 2: Read the following passage and answer the questions given below.

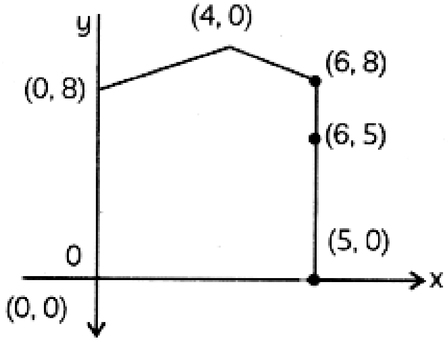
Let R be the feasible region of a linear programming problem and let Z = ax + by be the objective function. When Z has an optimal value (max. or min.), when the variable x and y are subject to constraints described by linear inequalities, this optimal value occurs at the corner point (vertex) of the feasible region.

Based on the above information, answer the following questions:

(i) What is an objective function of LPP? [1]

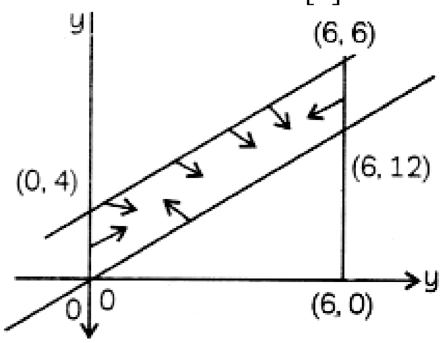
(ii) In solving an LPP "minimize f = 6x + 10y subject to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" which among is redundant constraint? [1]

(iii) The feasible region for an LPP is shown in the figure. Let Z = 3x – 4y, be the objective function. Then, at which point minimum of Z occurs? [2]



OR

The feasible region for an LPP is shown shaded in the figure. Let F = 3x – 4y be the objective function. Then, what is the maximum value of F. [2]



Ans: (i) Objective function is a linear function whose maximum or minimum values is to be found.

(ii) When $x \geq 6$ and $y \geq 2$, then

$2x + y \geq 2 \times 6 + 2 \Rightarrow, 2x + y \geq 14$

Hence, $x \geq 0$, $y \geq 0$, and $2x + y \geq 10$ are automatically satisfied by every point of the region.

Hence, answer is $2x + y \geq 10$, $x \geq 0$, $y \geq 0$.

(iii) Minimum of z = -32 at (0, 8)

Corner Point	z = 3x – 4y
(0, 0)	0
(5, 0)	3 x 5 – 4 x 0 = 15
(6, 5)	3 x 6 – 4 x 5 = -2
(6, 8)	3 x 6 – 4 x 8 = -14
(4, 10)	3 x 4 – 4 x 10 = -28
(0, 8)	3 x 0 – 4 x 8 = -32

Maximum of z = 0 at (0, 0)

OR

Corner Point	F = 3x – 4y
(0, 0)	0
(6,12)	3 x 6 – 4 x 12 = -30
(6, 16)	3 x 6 – 4 x 16 = -46
(0, 4)	3 x 0 – 4 x 4 = -16