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PRACTICE PAPER 11 (2023-24)
CHAPTER 11 THREE DIMENSIONAL GEOMETRY (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**
CLASS : XII | **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. Two-line $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy plane has coordinates
(a) (2, 4, 7) (b) (-2, 4, 7) (c) (2, -4, -7) (d) (2, -4, 7)
Ans: (c) (2, -4, -7)

2. Direction ratios of the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$ are
(a) 2, 6, 3 (b) –2, 6, 3 (c) 2, – 6, 3 (d) none of these
Ans : (c) 2, – 6, 3

3. The vector equation of the line joining the points (3, -2, -5) and (3, -2, 6) is:
(a) $(4\hat{i}-4\hat{j}+5\hat{k})+\lambda(12\hat{k})$ (b) $(4\hat{i}-4\hat{j}+5\hat{k})+\lambda(12\hat{k})$
(c) $(6\hat{i}-2\hat{j}+2\hat{k})+\lambda(5\hat{k})$ (d) $(9\hat{i}-9\hat{j}-2\hat{k})+\lambda(2\hat{k})$
Ans: (a) $(4\hat{i}-4\hat{j}+5\hat{k})+\lambda(12\hat{k})$
The vector equation of a line joining the points (3, -2, -5) and (3, -2, 6) is
$\vec{r}=(3\hat{i}-2\hat{j}-5\hat{k})+\lambda[(3-3)\hat{i}+(-2+2)\hat{j}+(6+5)\hat{k}$
$\Rightarrow \vec{r}=(3\hat{i}-2\hat{j}-5\hat{k})+\lambda(11\hat{k})$

4. A point that lies on the line $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{1-z}{7}$ is:
(a) (1, -3, 1) (b) (-2, 4, 7) (c) (-1, 3, 1) (d) (2, -4, -7)
Ans: (a) (1, -3, 1)
The equation of the Line can be written as $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{z-1}{-7}$
So, it passes through (1, -3, 1).

5. The direction ratios of the line 6x – 2 = 3y + 1 = 2z – 2 are:
(a) 6, 3, 2 (b) 1, 1, 2 (c) 1, 2, 3 (d) 1, 3, 2
Ans: (c) 1, 2, 3
Given the equation of a line is
6x – 2 = 3y + 1 = 2z – 2

$$\Rightarrow 6\left(x-\frac{1}{3}\right)=6\left(y+\frac{1}{3}\right)=2(z-1)\Rightarrow \frac{x-\frac{1}{3}}{1}=\frac{y+\frac{1}{3}}{2}=\frac{z-1}{3}$$

This shows that the given line passes through (1/3, -1/3, 1), and has direction ratios 1, 2, and 3.

6. The straight line $\frac{x-3}{3}=\frac{y-2}{1}=\frac{z-1}{0}$ is:

(a) parallel to x-axis (b) parallel to y-axis

(c) parallel to z-axis (d) perpendicular to z-axis

Ans: (c) parallel to z-axis

7. If the equation of a line AB is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$, find the direction ratios of a line parallel to AB.

(a) 1, 2, 4 (b) 1, 2, –4 (c) 1, –2, –4 (d) 1, –2, 4

Ans: (d) The direction ratios of line parallel to AB is 1, –2 and 4.

8. If a line makes angles α, β, γ with the positive direction of co-ordinates axes, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans: (b) 2

We know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

or $(1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A) : The angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is 90°.

Reason (R) : Skew lines are lines in different planes which are parallel and intersecting.

Ans: (c) A is true but R is false.

Assertion is correct.

Give that $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$

Direction ratios of lines are

$$a_1 = 2, b_1 = 5, c_1 = 4 \text{ and } a_2 = 1, b_2 = 2, c_2 = -3$$

As we know, the angle between the lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right).\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$$

$$\Rightarrow \quad \cos\theta = \frac{2\times1+5\times2+4\times(-3)}{\left(\sqrt{2^2+5^2+4^2}\right).\left(\sqrt{1^2+2^2+(-3)^3}\right)} = 0$$

$$\therefore \quad \theta = 90°$$

Reason (R) is wrong.

10. Assertion: If the cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$, then its vector form is

$\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

Reason: The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x-3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is $\frac{x+3}{-2}=\frac{y-4}{4}=\frac{z+8}{-5}$.

Ans: (c) A is true but R is false.

In assertion the given cartesian equation is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$

$\Rightarrow \vec{a}=5\hat{i}-4\hat{j}+6\hat{k}, \vec{b}=3\hat{i}+7\hat{j}+2\hat{k}$

The vector equation of the line is given by $\vec{r}=\vec{a}+\lambda\vec{b}$

$\Rightarrow \vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$

Thus Assertion is correct.

In reason it is given that the line passes through the point (–2, 4, –5) and is parallel to

$\frac{x-3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Clearly, the direction ratios of line are (3, 5, 6).

Now the equation of the line (in cartesian form) is

$\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6} \Rightarrow \frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

Hence, Reason is wrong.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

Ans: Vector equation of the line passing through (1, 2, 3) and (–3, 4, 3) is

$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$ where $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=-3\hat{i}+4\hat{j}+3\hat{k}$

$\Rightarrow \vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda[(-3-1)\hat{i}+(4-2)\hat{j}+(3-3)\hat{k}]$

$\Rightarrow \vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(-4\hat{i}+2\hat{j})$

Equation of z-axis is $\vec{r}=\mu\hat{k}$

Since $(-4\hat{i}+2\hat{j})\cdot\hat{k}=0$

$\therefore$ **Line (*i*) is $\perp$ to *z*-axis.**

12. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

Ans: Let A (1, –1, 2) and B (3, 4, – 2) be given points.

Direction ratios of AB are

(3 – 1), {(4 – (–1)}, (–2 –2) i.e., 2, 5, – 4.

Let C (0, 3, 2) and D (3, 5, 6) be given points.

Direction ratios of CD are

(3 – 0), (5 – 3), (6 – 2) i.e., 3, 2, 4.

We know that two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$\therefore 2 \times 3 + 5 \times 2 + (-4) \times 4 = 6 + 10 - 16 = 0$, which is true.

It will shows that lines AB and CD are perpendicular.

13. Find the angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{1-x}{-1}=\frac{y+2}{2}=\frac{3-z}{3}$.

Ans:

Given lines are $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{1-x}{-1}=\frac{y+2}{2}=\frac{3-z}{3}$

$\Rightarrow \frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$

$\therefore$ Angle θ between lines is

$$\cos\theta=\frac{2\times1+5\times2+4\times(-3)}{\sqrt{4+25+16}\sqrt{1+4+9}}=\frac{2+10-12}{\sqrt{45}\sqrt{14}}$$

$$\Rightarrow \cos\theta=0\Rightarrow\theta=90^\circ$$

14. Find the coordinates of the point where the line $\frac{x+3}{3}=\frac{y-1}{-1}=\frac{z-5}{-5}$ cuts the XY plane.

Ans:

Given. As line cuts the xy plane $z=0$

We have, $\frac{x+3}{3}=\frac{y-1}{-1}=\frac{z-5}{-5} \Rightarrow \frac{x+3}{3}=\frac{y-1}{-1}=\frac{z-5}{-5}=\lambda$

Now, Put $z=0$

$\frac{z-5}{-5}=\lambda \Rightarrow z=-5\lambda+5 \Rightarrow 0=-5\lambda+5 \Rightarrow \lambda=1$

Now, $\frac{x+3}{3}=1 \Rightarrow x=3-3=0$ | $\frac{y-1}{-1}=1 \Rightarrow y=-1+1=0$

$\therefore x=0, y=0, z=0$ $\therefore$ **Required Point (0, 0, 0)**

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3).

Ans: Given, the equation of a line is: $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}=\lambda$ (say)

$\Rightarrow x=3\lambda-2,\ y=2\lambda-1,\ z=2\lambda+3$

So, we have a point on the line is:

$Q(3\lambda-2, 2\lambda-1, 2\lambda+3)$ …….(i)

Now, given that distance between two points P(1, 3, 3) and $Q(3\lambda-2, 2\lambda-1, 2\lambda+3)$ is 5 units

i.e. PQ = 5

$\Rightarrow \sqrt{[(3\lambda-2-1)^2+(2\lambda-1-3)^2+(2\lambda+3-3)^2]}=5$

On Squaring both sides, we get

$(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2=25$

$\Rightarrow 9\lambda^2+9-18\lambda+4\lambda^2+16-16\lambda+4\lambda^2=25$

$\Rightarrow 17\lambda^2-34\lambda=0$

$\Rightarrow 17\lambda(\lambda-2)=0$

Either $17\lambda=0$ or $\lambda-2=0$

$\therefore \lambda=0$ or 2

On putting $\lambda=0$ and $\lambda=2$ in equation (i),

we get the required point as (-2, -1, 3) or (4, 3, 7)

16. Find the vector equation of the line through the point (1, 2, –4) and perpendicular to the two lines $\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and $\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

Ans:

The given lines are

$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Equation of any line through (1, 2, –4) with d.r's

l, m, n is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t(l\hat{i} + m\hat{j} + n\hat{k})$ ...(i)

Since, the required line is perpendicular to both the given lines.

$\therefore\ 3l - 16m + 7n = 0$ and $3l + 8m - 5n = 0$

$$\Rightarrow \frac{l}{80-56} = \frac{m}{21+15} = \frac{n}{24+48} \Rightarrow \frac{l}{2} = \frac{m}{3} = \frac{n}{6}$$

$\therefore$ From (i), the required line is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t(2\hat{i} + 3\hat{j} + 6\hat{k})$

Here, the position vector of passing point is $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$ and parallel vector is $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

17. Find the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

Ans:

We have, $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k},\ \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k},\ \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

$\therefore\ \vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 0\cdot\hat{j} + 3\hat{k} \Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 3^2} = 3\sqrt{2}$$

$$\Rightarrow (\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 1(-3) - 3(0) - 2(3) = -9.$$

$$\therefore\ d = \left|\frac{-9}{3\sqrt{2}}\right| = \frac{3}{\sqrt{2}} = \frac{3}{2}\sqrt{2} \text{ units}$$

SECTION – D

Questions 18 carry 5 marks.

18. Find the coordinates of the foot of perpendicular drawn from the point A(–1, 8, 4) to the line joining the points B(0, –1, 3) and C(2, –3, –1). Hence find the image of the point A in the line BC.

Ans: Let P be the foot of the perpendicular drawn from point A on the line joining points B and C.

Equation of the line joining the points B(0,–1,3) and C(2,–3,–1) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \frac{x-0}{2-0} = \frac{y+1}{-3+1} = \frac{z-3}{-1-3} \Rightarrow \frac{x-0}{2} = \frac{y+1}{-2} = \frac{z-3}{-4}$$

Let $\dfrac{x-0}{2} = \dfrac{y+1}{-2} = \dfrac{z-3}{-4} = \lambda$

General coordinates of P is $(2\lambda, -2\lambda-1, -4\lambda+3)$

Direction ratios of AP$(2\lambda+1, -2\lambda-9, -4\lambda-1)$

$\because$ Both the lines AP and BC are perpendicular to each other.

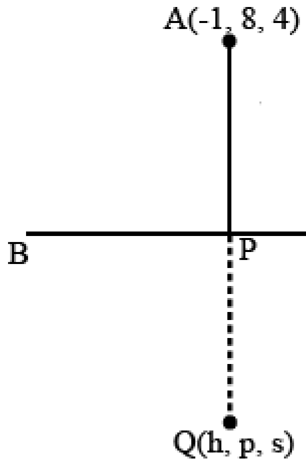
$\therefore 2(2\lambda+1) - 2(-2\lambda-9) - 4(-4\lambda-1) = 0$

$\Rightarrow 24\lambda + 24 = 0$

$\Rightarrow \lambda = -1$

$\therefore$ P(−2, 1, 7)

So, the coordinates of the foot of perpendicular drawn from the point A to BC is P(−2, 1, 7).



Let Q(h, p, s) be the image of A in the line BC,
So P must be the mid-point of AQ.

$$\therefore P\left(\frac{h-1}{2},\frac{p+8}{2},\frac{s+4}{2}\right)=P(-2,1,7)$$

On comparing the coordinates, we get h = -3, p=-6, s= 10,
Hence the image is Q(-3, -6, 10).

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.
The equation of motion of a missile are x = 3t, y = - 4t, z = t, where the time 't' is given in seconds, and the distance is measured in kilometers.

(a) Write the path of the missile.
(b) Find the distance of the rocket from the starting point (0, 0, 0) in 5 seconds.
(c) If the position of the rocket at a certain instant of the time is (5, -8, 10). Find the height of the rocket from the ground. (Ground considered as xy-plane)

OR

(c) Find the value of k for which the lines $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{k}$ and $\frac{x-2}{-2}=\frac{y-3}{-1}=\frac{z-5}{7}$; are perpendicular?

Ans:
(a) Given equation of motion of a missile be

$$x = 3t, y = -4t, z = t$$

$\Rightarrow \frac{x}{3}=\frac{y}{-4}=\frac{z}{1}$, which is a straight line.

Hence, the path of the missile is a straight line.
(b) After 5 seconds position of the rocket be

x = 3t = 3 x 5 = 15
y = -4t = -4 x 5 = -20
z = t = 5
∴ Point is (15, -20, 5).
Its distance from origin (0, 0, 0) is $\sqrt{(15-0)^2+(-20-0)^2+(5-0)^2} = \sqrt{225+400+25} = \sqrt{650}$ km.
(c) (a) Given position of the rocket at a time is (5, -8, 10).
Height of the rocket from the ground
= Distance between the points (5, -8, 10) and (5, -8, 0).
(Since ground is considered as the XY-Plane)
= $\sqrt{(5-5)^2+(-8+8)^2+(10-0)^2} = \sqrt{100}$ =10 km

OR

(c) Given lines are perpendicular if 2 x (-2) + 3 x (- 1) + 7 x k = 0
⇒ -7 + 7k = 0 ⇒ 7k = 7 ⇒ k = 1

20. Case-Study 2: Read the following passage and answer the questions given below.
Two non-parallel and non-intersecting straight lines are called skew lines. For skew lines, the line segment of the shortest distance will be perpendicular to both the lines. If the lines are $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$.

Then, shortest distance is given as $d = \left|\frac{(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$

Here, $\vec{a_1}, \vec{a_2}$ are position vectors of point through which the lines are passing and $\vec{b_1}, \vec{b_2}$ are the vectors in the direction of a line.
(a) If a line has the direction ratios -18, 12, -4 then what are its direction cosines? (1)
(b) Write the condition for which the given two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are not coplanar in vector form. (1)
(c) Write the distance of a point P(a, b, c) from the x-axis (1)
(d) If the cartesian form of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$ then write the vector equation of line. (1)

Ans: (a) $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$
(b) $(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1}) = 0$
(c) $\sqrt{b^2+c^2}$
(d) $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$