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PRACTICE PAPER 10 (2023-24)
CHAPTER 10 VECTOR ALGEBRA
(ANSWERS)

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**
CLASS : XII — **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The vector of the direction of the vector $\hat{i}-2\hat{j}+2\hat{k}$ that has magnitude 9 is

(a) $\hat{i}-2\hat{j}+2\hat{k}$ (b) $\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$ (c) $3(\hat{i}-2\hat{j}+2\hat{k})$ (d) $9(\hat{i}-2\hat{j}+2\hat{k})$

Ans: (c) $3(\hat{i}-2\hat{j}+2\hat{k})$

Any vector in the direction of a vector $\vec{a}$ is given by

$$\frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{1^2+2^2+2^2}} = \frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$$

∴ Vector in the direction of $\vec{a}$ with magnitude 9 is $9\frac{\vec{a}}{|\vec{a}|} = 9.\frac{\hat{i}-2\hat{j}+2\hat{k}}{3} = 3(\hat{i}-2\hat{j}+2\hat{k})$

2. The magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is 9/2, is

(a) 2 (b) 3 (c) 4 (d) 5

Ans: (b) 3

Given, $|\vec{a}| = |\vec{b}|$, $\theta = 60°$ and $\vec{a}\cdot\vec{b} = \frac{9}{2}$

Now, $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \cos 60° = \frac{9/2}{|\vec{a}|^2} \Rightarrow \frac{1}{2} = \frac{9/2}{|\vec{a}|^2}$

$\Rightarrow |\vec{a}|^2 = 9 \Rightarrow |\vec{a}| = 3 \therefore |\vec{a}| = |\vec{b}| = 3$

3. The projection of the vector $2\hat{i}+3\hat{j}+2\hat{k}$ *on the vector* $\hat{i}+2\hat{j}+\hat{k}$ is

(a) 10/√6 (b) 10/√3 (c) 5/√6 (d) 5/√3

Ans:

We have, $\vec{a} = 2\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$

$\therefore \vec{a}\cdot\vec{b} = (2\hat{i}+3\hat{j}+2\hat{k})\cdot(\hat{i}+2\hat{j}+\hat{k}) = 2+6+2 = 10$

and $|\vec{b}| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$

Hence, projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{10}{\sqrt{6}}$.

4. Find the angle between the vectors $\vec{a} = \hat{i}-\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+\hat{j}-\hat{k}$

(a) $\cos^{-1}\left(-\frac{1}{2}\right)$ (b) 60° (c) $\cos^{-1}\left(-\frac{1}{3}\right)$ (d) $\cos^{-1}\left(-\frac{2}{3}\right)$

Ans: (c) $\cos^{-1}\left(-\frac{1}{3}\right)$

$\vec{a} = \hat{i} - \hat{j} + \hat{k} \Rightarrow |\vec{a}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$

$\vec{b} = \hat{i} + \hat{j} - \hat{k} \Rightarrow |\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-1)^2} = \sqrt{3}$

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 1 - 1 - 1 = \sqrt{3}\sqrt{3}\cos\theta \Rightarrow -1 = 3\cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(-\frac{1}{3}\right)$

5. If $(\hat{i}+3\hat{j}+8\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k}) = 0$, then λ and μ are respectively:

(a) 27, -9 (b) 9, 9 (c) -9, 18 (d) -1, 1

Ans: (a) 27, -9

Given, $(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k}) = 0$

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix} = \vec{0}$$

$\hat{i}(3\mu+9\lambda) - \hat{j}(\mu-27) + \hat{k}(-\lambda-9) = 0\hat{i} + 0\hat{j} + 0\hat{k}$

On comparing the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$ we get,

3μ + 9λ = 0, -μ + 27 = 0 and -λ – 9 = 0

μ = 27 and -λ = 9

or μ = 27 and λ = -9

6. The value of λ such that the vector $\vec{a} = 2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$ are orthogonal is:

(a) 3/2 (b) −5/2 (c) −1/2 (d) 1/2

Ans: (b) −5/2

Since, two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal

$\therefore \bar{a}.\bar{b} = 0$

$\Rightarrow (2\hat{i}+\lambda\hat{j}+\hat{k}).(\hat{i}+2\hat{j}+3\hat{k}) = 0$

$\Rightarrow 2 + 2\lambda + 3 = 0 \Rightarrow \lambda = \frac{-5}{2}$

7. For any vector $\vec{a}$, the value of $|\vec{a}\cdot\hat{i}|^2 + |\vec{a}\cdot\hat{j}|^2 + |\vec{a}\cdot\hat{k}|^2$ is:

(a) a (b) a^2 (c) 1 (d) 0

Ans: (b) a^2

Let $\bar{a} = x\hat{i} + y\hat{j} + z\hat{k}$. Then,

$\bar{a}.\hat{i} = x; \bar{a}.\hat{j} = y$ and $\bar{a}.\hat{k} = z \Rightarrow x^2 + y^2 + z^2 = a^2$

8. The area of a parallelogram whose one diagonal is $2\hat{i}+\hat{j}-2\hat{k}$ and one side is $3\hat{i}+\hat{j}-\hat{k}$ is

(a) $\hat{i}-4\hat{j}-\hat{k}$ (b) 3√2 sq units (c) 6√2 sq units (d) 6 sq units

Ans: (b) 3√2 sq units

$$\text{area of parallelogram} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & 1 & -1 \end{vmatrix}$$

$= |\hat{i} - 4\hat{j} - \hat{k}| = \sqrt{1+16+1} = 3\sqrt{2}$ sq units

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): The value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}+\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$ is 1.

Reason (R): Since, $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=0$

Ans: (c) A is true but R is false.

As $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})=\hat{i}\cdot\hat{i}+\hat{j}\cdot(-\hat{j})+\hat{k}\cdot\hat{k}$

= 1 – 1 + 1 = 1

10. Assertion (A): The direction of cosines of vector $\vec{a}=2\hat{i}+4\hat{j}-5\hat{k}$ are $\frac{2}{\sqrt{45}},\frac{4}{\sqrt{45}},-\frac{5}{\sqrt{45}}$

Reason (R): A vector having zero magnitude and arbitrary direction is called 'zero vector' or 'null vector'.

Ans: (b) Both A and R are true but R is not the correct explanation of A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If $|\vec{a}\times\vec{b}|^2+(\vec{a}.\vec{b})^2=144$ and $|\vec{a}|=4$, then find the value of $|\vec{b}|$.

We know that, $|\vec{a}\times\vec{b}|+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

$\Rightarrow\ 144=(4)^2|\vec{b}|^2$

$\Rightarrow\ |\vec{b}|^2=\frac{144}{16}=9$

$\therefore\ |\vec{b}|=3$

12. Find the angle between the vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

The angle θ between the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$i.e., \cos\theta=\frac{(\hat{i}+\hat{j}-\hat{k}).(\hat{i}-\hat{j}+\hat{k})}{\sqrt{(1)^2+(1)^2+(-1)^2}.\sqrt{(1)^2+(-1)^2+(1)^2}}$$

$$i.e., \cos\theta=\frac{1-1-1}{\sqrt{3}.\sqrt{3}}$$

$$i.e., \cos\theta=-\frac{1}{3}\ \Rightarrow\ \theta=\cos^{-1}\left(-\frac{1}{3}\right).$$

13. Given, $\vec{p}=3\hat{i}+2\hat{j}+4\hat{k}, \vec{a}=\hat{i}+\hat{j}, \vec{b}=\hat{j}+\hat{k}, \vec{c}=\hat{i}+\hat{k}$ and $\vec{p}=x\vec{a}+y\vec{b}+z\vec{c}$, then find the value of x, y, z.

Ans:

We have, $\vec{p}=x\vec{a}+y\vec{b}+z\vec{c}$

$\Rightarrow\ 3\hat{i}+2\hat{j}+4\hat{k}=x(\hat{i}+\hat{j})+y(\hat{j}+\hat{k})+z(\hat{i}+\hat{k})$

$\Rightarrow\ 3\hat{i}+2\hat{j}+4\hat{k}=(x+z)\hat{i}+(x+y)\hat{j}+(y+z)\hat{k}$

$\Rightarrow\ x+z=3$ ...(i), $x+y=2$

and $y+z=4$ ...(iii)

On solving (i), (ii) and (iii), we get $x=\frac{1}{2}, y=\frac{3}{2}, z=\frac{5}{2}$

14. Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

Ans:

Given, ΔABC with vertices $A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$

Now, $\overrightarrow{AB}(2-1)\hat{i}+(3-1)\hat{j}+(5-2)\hat{k} = \hat{i}+2\hat{j}+3\hat{k}$,

and $\overrightarrow{AC}(1-1)\hat{i}+(5-1)\hat{j}+(5-2)\hat{k} = 4\hat{j}+3\hat{k}$.

$$\therefore \quad (\overrightarrow{AB}\times\overrightarrow{AC}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{i}-3\hat{j}+4\hat{k}$$

Hence, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}| = \frac{1}{2}\sqrt{(-6)^2+(-3)^2+4^2} = \frac{1}{2}\sqrt{61}$ sq. units

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Show that the points A(1, 2, 7), B(2, 6, 3) and C(3, 10, –1) are collinear.

Ans: The given points are $A(1, 2, 7)$, B (2, 6, 3) and $C(3, 10, -1)$.

$\overrightarrow{AB} = (2\hat{i}+6\hat{j}+3\hat{k})-(\hat{i}+2\hat{j}+7\hat{k}) = \hat{i}+4\hat{j}-4\hat{k}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{1+16+16} = \sqrt{33}$

$\overrightarrow{BC} = (3\hat{i}+10\hat{j}-\hat{k})-(2\hat{i}+6\hat{j}+3\hat{k}) = \hat{i}+4\hat{j}-4\hat{k}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{1+16+16} = \sqrt{33}$

and $\overrightarrow{AC} = (3\hat{i}+10\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+7\hat{k}) = 2\hat{i}+8\hat{j}-8\hat{k}$

$\Rightarrow |\overrightarrow{AC}| = \sqrt{4+64+64} = \sqrt{132} = 2\sqrt{33}$

$\therefore |\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$

Hence, the given points A, B and C are collinear.

16. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = \hat{i}+2\hat{j}+3\hat{k}$.

Ans: We have $\vec{a}+\vec{b} = 2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{a}-\vec{b} = -\hat{j}-2\hat{k}$

A vector which is perpendicular to both $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is given by

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2\hat{i}+4\hat{j}-2\hat{k}(=\vec{c}, say)$$

Now, $|\vec{c}| = \sqrt{4+16+4} = \sqrt{24} = 2\sqrt{6}$

Therefore, the required unit vector is

$$\hat{c} = \frac{1}{|\vec{c}|}\vec{c} = \frac{1}{2\sqrt{6}}(-2\hat{i}+4\hat{j}-2\hat{k}) = \frac{-1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{2}{\sqrt{6}}\hat{k}$$

17. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

Ans: Two adjacent sides of a parallelogram are given by $\vec{a} = 2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}-3\hat{k}$

Then the diagonal of a parallelogram is given by $\vec{c} = \vec{a}+\vec{b}$

$\therefore \vec{c} = \vec{a}+\vec{b} = 2\hat{i}-4\hat{j}+5\hat{k}+\hat{i}-2\hat{j}-3\hat{k} = 3\hat{i}-6\hat{j}+2\hat{k}$

$\Rightarrow |\vec{c}| = \sqrt{9+36+4} = \sqrt{49} = 7$

Unit vector parallel to its diagonal = $\hat{c} = \frac{1}{|\vec{c}|}\vec{c} = \frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}) = \frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}=22\hat{i}+11\hat{j}+0\hat{k}$

Then the area of a parallelogram = $|\vec{a}\times\vec{b}|=\sqrt{484+121+0}=\sqrt{605}=11\sqrt{5}$ sq. units.

SECTION – D

Questions 18 carry 5 marks.

18. The magnitude of the vector product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

Ans: Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$

Now, $\vec{b}+\vec{c}=2\hat{i}+4\hat{j}-5\hat{k}+\lambda\hat{i}+2\hat{j}+3\hat{k}=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\Rightarrow|\vec{b}+\vec{c}|=\sqrt{(2+\lambda)^2+36+4}=\sqrt{4+\lambda^2+4\lambda+40}=\sqrt{\lambda^2+4\lambda+44}$

The vector product of $\hat{i}+\hat{j}+\hat{k}$ with this unit vector is $\sqrt{2}$.

$\therefore\left|\vec{a}\times\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}\right|=\sqrt{2}\Rightarrow\left|\frac{\vec{a}\times(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|}\right|=\sqrt{2}$

Now, $\vec{a}\times(\vec{b}+\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2+\lambda & 6 & -2\end{vmatrix}=(-2-6)\hat{i}-(-2-2-\lambda)\hat{j}-(6-2-\lambda)\hat{k}$

$=-8\hat{i}+(4+\lambda)\hat{j}+(4-\lambda)\hat{k}$

$\therefore\left|\frac{\vec{a}\times(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|}\right|=\sqrt{2}\Rightarrow\left|\frac{-8\hat{i}+(4+\lambda)\hat{j}+(4-\lambda)\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}\right|=\sqrt{2}$

$\Rightarrow\frac{\sqrt{64+(4+\lambda)^2+(4-\lambda)^2}}{\sqrt{\lambda^2+4\lambda+44}}=\sqrt{2}$

$\frac{64+(4+\lambda)^2+(4-\lambda)^2}{\lambda^2+4\lambda+44}=2\Rightarrow\frac{64+16+\lambda^2+8\lambda+16+\lambda^2-8\lambda}{\lambda^2+4\lambda+44}=2\Rightarrow\frac{96+2\lambda^2}{\lambda^2+4\lambda+44}=2$

$\Rightarrow 96+2\lambda^2=2(\lambda^2+4\lambda+44)\Rightarrow 96+2\lambda^2=2\lambda^2+8\lambda+88$

$\Rightarrow 8\lambda=8\Rightarrow\lambda=1$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

Solar panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture, suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1 (6, 8, 4), P_2 (21, 8, 4), P_3 (21, 16, 10) and P_4 (6,16,10).

(i) Find the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 – PV of P_1 and $\vec{B}$ = PV of P_4 – PV of P_1 where PV stands for position vector.

(ii) (a) Find the magnitudes of the vectors $\vec{A}$ and $\vec{B}$.

(b) Find the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof.

Ans:

Given points are P_1 (6, 8, 4), P_2 (21, 8, 4), P_3 (21, 16, 10) and P_4 (6, 16, 10).

(i) We have, $\vec{A} = PV \text{ of } P_2 - PV \text{ of } P_1 = (21\hat{i} + 8\hat{j} + 4\hat{k}) - (6\hat{i} + 8\hat{j} + 4\hat{k})$

$\vec{A} = 15\hat{i} + 0\hat{j} + 0\hat{k}$

∴ Components of $\vec{A}$ are 15, 0, 0.

and $\vec{B} = PV \text{ of } P_4 - PV \text{ of } P_1 = (6\hat{i} + 16\hat{j} + 10\hat{k}) - (6\hat{i} + 8\hat{j} + 4\hat{k}) = 0\hat{i} + 8\hat{j} + 6\hat{k}$

∴ Components of $\vec{B}$ are 0, 8, 6.

(ii) (a) We have, $|\vec{A}| = \sqrt{(15)^2 + (0)^2 + (0)^2} = 15$ units

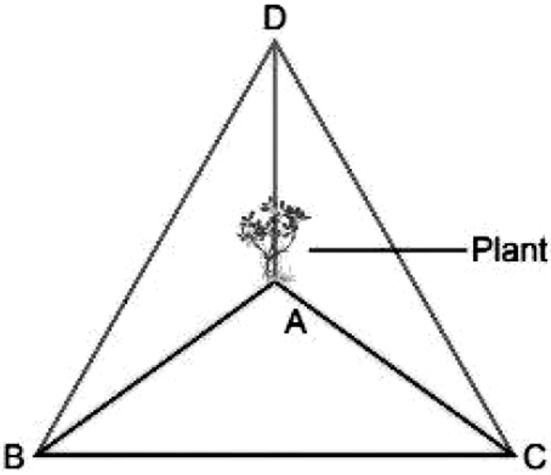
$|\vec{B}| = \sqrt{(0)^2 + (8)^2 + (6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$ units

(b) We have, $\vec{N} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(90-0) + \hat{k}(120-0) = 0\hat{i} - 90\hat{j} + 120\hat{k}$

Its components are 0, –90, 120.

20. Case-Study 2: Read the following passage and answer the questions given below.

Raghav purchased an air plant holder which is in shape of tetrahedron. Let A, B, C, D be the co-ordinates of the air plant holder where A = (1, 2, 3), B= (3, 2, 1), C = (2, 1, 2), D = (3, 4, 3).



(i) Find the vector $\overrightarrow{AB}$. (1)

(ii) Find the vector $\overrightarrow{CD}$. (1)

(iii) Find the unit vector along $\overrightarrow{BC}$ vector. (2)

OR

(iii) Find the area (ABCD). (2)

Ans:

$\because A \equiv (1, 2, 3), B \equiv (3, 2, 1), C \equiv (2, 1, 2), D \equiv (3, 4, 3)$

(i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (3-1)\hat{i} + (2-2)\hat{j} + (1-3)\hat{k} = 2\hat{i} - 2\hat{k}$

(ii) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = (3-2)\hat{i} + (4-1)\hat{j} + (3-2)\hat{k} = \hat{i} + 3\hat{j} + \hat{k}$

(iii) $\because \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (2-3)\hat{i} + (1-2)\hat{j} + (2-1)\hat{k} = -\hat{i} - \hat{j} + \hat{k}$

$$\because \widehat{BC} = \frac{\overrightarrow{BC}}{|\overrightarrow{BC}|} = \frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{(-1)^2 + (-1)^2 + 1^2}} = \frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}} = -\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

OR

(iii) $\because \overrightarrow{BC} = -\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB} = (3-3)\hat{i} + (4-2)\hat{j} + (3-1)\hat{k} = 2\hat{j} + 2\hat{k}$

$\because \overrightarrow{BD}$ and $\overrightarrow{BD}$ are adjacent sides of ΔBCD.

$$= \overrightarrow{BC} \times \overrightarrow{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ 0 & 2 & 2 \end{vmatrix} = \hat{i}(-2-2) - \hat{j}(-2-0) + k(-2+0) = -4\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\therefore \text{Area of } \Delta BCD = \frac{1}{2}|\overrightarrow{BC} \times \overrightarrow{BD}| = \frac{1}{2}\sqrt{(-4)^2 + 2^2 + (-2)^2} = \frac{1}{2} \times \sqrt{16+4+4}$$

$$= \frac{1}{2}\sqrt{24} = \frac{1}{2} \times 2\sqrt{6} = \sqrt{6} \text{ sq. units}$$
