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## PRACTICE PAPER 01 (2023-24) (ANSWERS)
## CHAPTER 01 RELATIONS AND FUNCTIONS

**SUBJECT: MATHEMATICS** | **MAX. MARKS : 40**
**CLASS : XII** | **DURATION : 1½ hrs**

**General Instructions:**
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

## SECTION – A
**Questions 1 to 10 carry 1 mark each.**

**1.** The relation $R$ in the set of real numbers defined as $R = \{(a, b) \in R \times R : 1 + ab > 0\}$ is
(*a*) reflexive and transitive (*b*) symmetric and transitive
(*c*) reflexive and symmetric (*d*) equivalence relation
Ans: (*c*) reflexive and symmetric

**2.** Let the function '$f$' be defined by $f(x) = 5x^2 + 2, \forall x \in R$. Then '$f$' is
(*a*) onto function (*b*) one-one, onto function
(*c*) one-one, into function (*d*) many-one, into function
Ans : (*d*) many-one, into function

**3.** Let set X = {1, 2, 3} and a relation R is defined in X as : R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are
(a) {(1, 1), (2, 3), (1, 2)} (b) {(3, 3), (3, 1), (1, 2)}
(c) {(1, 1), (3, 3), (3, 1), (2, 3)} (d) {(1, 1), (3, 3), (3, 1), (1, 2)}
Ans: (c), For reflexive $(a, a) \in R$ for $a \in X$
So it can be (c) or (d)
For symmetric $(1, 3) \in R$, then (3, 1) should belong to R. Also (2, 3) should belong to R from above observation.

**4.** Let Z be the set of integers and R be a relation defined in Z such that aRb if (a – b) is divisible by 5. Then number of equivalence classes are
(a) 2 (b) 3 (c) 4 (d) 5
Ans: (d) 5
as remainder can be 0, 1, 2, 3, 4.

**5.** Let R be a relation defined as R = {(x, x), (y, y), (z, z), (x, z)} in set A = {x, y, z} then relation R is
(a) reflexive (b) symmetric (c) transitive (d) equivalence
Ans: (a) reflexive, as for all $a \in A$, $(a, a) \in R$.

**6.** If R = {(x, y) : x + 2y = 8} is a relation on N, then range of R is
(a) {3} (b) {1, 2, 3}
(c) {1, 2, 3, .... 8} (d) {1, 2}
Ans: (b), as R = {(x, y) : x + 2y = 8} is a relation on N.

$\therefore$ R = {(2, 3), (4, 2), (6, 1)},
$\therefore$ range = {1, 2, 3}.

**7.** Let A = {a, b, c}, then the total number of distinct relations in set A are
(a) 64 (b) 32 (c) 256 (d) 512
Ans: (d), as given $A = \{a, b, c\}$.
A relation is a subset of $A \times A$.
$n(A \times A) = 9$
we know total subsets of a set containing $n$ elements is $2^n$.
Total relations = $2^9 = 512$

**8.** Let X = $\{x^2 : x \in N\}$ and the function f : N → X is defined by $f(x) = x^2, x \in N$. Then this function is
(a) injective only (b) not bijective (c) surjective only (d) bijective
Ans: (*d*) Function is injective as for $x_1, x_2 \in N$,
$f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$, as $x_1, x_2 > 0$.
Function is surjective as for $y \in x$
There exists $x \in N$ such that $y = f(x)$
$\Rightarrow y = x^2 \Rightarrow x = \sqrt{y} \in N$.
Function is bijective

**For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.**
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**9.** Assertion (A): In set A = {1, 2, 3} a relation R defined as R = {(1, 1), (2, 2)} is reflexive.
Reason (R): A relation R is reflexive in set A if (a, a) ∈ R for all a ∈ A.
Ans: (d) *A* is false but *R* is true.

**10.** Assertion (*A*): In set $A = \{a, b, c\}$ relation $R$ in set $A$, given as $R = \{(a, c)\}$ is transitive.
Reason (*R*): A singleton relation is transitive.
Ans: (a) Both A and R are true and R is the correct explanation of A.

## SECTION – B

**Questions 11 to 14 carry 2 marks each.**

**11.** Show that the relation R in the set {1, 2, 3} given by R = {(1,1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.
Ans: Given R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} defined on R : {1, 2, 3} {1, 2, 3}
For reflexive: As (1, 1), (2,2), (3, 3) ∈ R. Hence, reflexive
For symmetric: (1, 2) ∈ R but (2, 1) ∉ R. Hence, not symmetric.
For transitive: (1, 2) ∈ R and (2, 3) ∈ R but (1, 3) ∉ R. Hence, not transitive.

**12.** Prove that the Greatest Integer Function f : R → R, given by f(x) = [x] is neither one-one nor onto. Where [x] denotes the greatest integer less than or equal to x.
Ans: $f$ : R → R given by $f(x) = [x]$
**Injectivity:** Let $x_1 = 2.5$ and $x_2 = 2$ be two elements of R.
$f(x_1) = f(2.5) = [2.5] = 2$
$f(x_2) = f(2) = [2] = 2$
$\therefore f(x_1) = f(x_2)$ for $x_1 \neq x_2$

$\Rightarrow f(x) = [x]$ is not one-one *i.e.*, not injective.
**Surjectivity:** Let $y = 2.5 \in R$ be any element.
$\therefore f(x) = 2.5 \Rightarrow [x] = 2.5$
Which is not possible as $[x]$ is always an integer.
$\Rightarrow f(x) = [x]$ is not onto *i.e.*, not surjective.

**13.** Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from $A$ to $B$. Show that $f$ is one-one.
Ans : Given $f: \{1, 2, 3\} \rightarrow \{4,5,6,7\}$
as $f = \{(1,4), (2,5), (3,6)\}$.
We have $f(1) = 4$, $f(2) = 5$, $f(3) = 6$.
We notice $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.
Hence, one-one.

**14.** Let the function $f : R \rightarrow R$ be defined by $f(x) = \cos x \ \forall \ x \in R$. Show that $f$ is neither one-one nor onto.
Ans: Given function $f(x) = \cos x$, $\forall x \in R$

$$\cos\frac{\pi}{3} = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$

So, f(x) is not one-one
Now, f(x) is also not onto as range is a subset of real numbers. $(-1 \leq \cos x \leq 1)$
e.g. for $y = 2 \in R$ (co-domain) there is no value of $x \in R$ (domain) such that
$y = f(x)$ i.e. $\cos x = 2$ $(\because -1 \leq \cos x \leq 1)$.

## SECTION – C

**Questions 15 to 17 carry 3 marks each.**

**15.** Let $T$ be the set of all triangles in a plane with $R$ a relation in $T$ given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$. Show that $R$ is an equivalence relation.
Ans: Given $R = \{(T_1, T_2) \in T \times T : T_1 \cong T_2\}$
For reflexive: $(T_1, T_1) \in R$ is true as $T_1 \cong T_1$ for all $T_1 \in T$ (i.e. triangle is congruent to itself).
Hence, $R$ is reflexive.
For symmetric: $(T_1, T_2) \in R \Rightarrow T_1 \cong T_2$ and $T_2 \cong T_1$ $(T_2, T_1) \in R$.
Hence, $R$ is symmetric.
For transitive: Let$(T_1, T_2) \in R$ and $(T_2, T_3) \in R \Rightarrow T_1 \cong T_2$ and $T_2 \cong T_3$
$\Rightarrow T_1 \cong T_3 \Rightarrow (T_1, T_3) \in R$.
Hence, $R$ is transitive.
Since $R$ is reflexive, symmetric and transitive, therefore $R$ is an equivalence relation.

**16.** Show that the relation $S$ in the set $R$ of real numbers, defined as $S = \{(a, b) : a, b \in R$ and $a \leq b^3\}$ is neither reflexive, nor symmetric, nor transitive.
Ans: Given $S = \{(a, b) \in R \mid a \leq b^3\}$
We can consider counter example.
For reflexive: Let $(-2, -2) \in S \Rightarrow -2 \leq (-2)^3 \Rightarrow -2 \leq -8$, false, Hence, not reflexive.
For symmetric: Let $(-1, 2) \in S \Rightarrow -1 \leq (2)^3 \Rightarrow -1 \leq 8$ true,
If symmetric then $(2, -1) \in S$
$\Rightarrow 2 \leq (-1)^3 \Rightarrow 2 \leq -1$, false, Hence, not symmetric.
For transitive: Let $(25, 3) \in S$ and $(3, 2) \in S$
$\Rightarrow 25 \leq (3)^3$ and $3 \leq (2)^3 \Rightarrow 25 \leq 27$ and $3 \leq 8$, true in both cases.
If transitive then $(25, 2) \in S \Rightarrow 25 \leq (2)^3 \Rightarrow 25 \leq 8$,false
Hence, not transitive.

**17.** Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall\, x \in R$ is neither one-one nor onto.

Ans: For one-one: For $x_1, x_2 \in R$

$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$

$\Rightarrow x_1x_2^2 + x_1 = x_1^2x_2 + x_2 \Rightarrow x_1x_2(x_2 - x_1) + (x_1 - x_2) = 0$

$\Rightarrow (x_2 - x_1)(x_1x_2 - 1) = 0 \Rightarrow x_2 - x_1 = 0$ or $x_1x_2 = 1$

$\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$

Let $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we notice $f(x_1) = f(x_2)$ but $2 \neq \frac{1}{2}$. Hence, not one-one

Here we notice $f(x) \neq 1$ for any $x \in R$

Therefore, $1 \in R$ from co-domain does not have pre-image in domain. So, not onto.

## SECTION – D

**Questions 18 carry 5 marks.**

**18.** Let $N$ denote the set of all natural numbers and $R$ be the relation on $N \times N$ defined by $(a, b)\, R\, (c, d)$ if $ad(b + c) = bc(a + d)$. Show that $R$ is an equivalence relation.

Ans: Relation $R$ on $N \times N$ is given by

$(a, b)\, R(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$.

For reflexive:

For $(a, b) \in N \times N$

$(a, b)\, R(a, b) \Rightarrow ab(b + a) = ba(a + b)$,

true in $N$

Hence, reflexive

For symmetric:

For $(a, b), (c, d) \in N \times N$

$(a, b)\, R(c, d) \Rightarrow ad(b + c) = bc(a + d)$

$\Rightarrow cb(d + a) = da(c + b)$ ( × and + is commutative in $N$)

$\Rightarrow (c, d)\, R(a, b)\ \forall\, (a, b), (c, d) \in N \times N$.

Hence, symmetric

For transitive:

For $(a, b), (c, d), (e, f) \in N \times N$

Let $(a, b)\, R(c, d)$ and $(c, d)\, R(e, f)$

$\Rightarrow ad(b + c) = bc(a + d)$

$\Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$

and $cf(d + e) = de(c + f)$

$\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

$\Rightarrow \frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$

$af(e + b) = be(f + a)$

$\Rightarrow af(b + e) = be(a + f)$

$\Rightarrow (a, b)\, R(e, f)$

As $(a, b)\, R(c, d)$, $(c, d)\, R(e, f)$

$\Rightarrow (a, b)\, R(e, f)$ Hence, transitive.

As relation $R$ is reflexive, symmetric and transitive. Hence, $R$ is an equivalence relation.

## SECTION – E (Case Study Based Questions)

**Questions 19 to 20 carry 4 marks each.**

**19.** Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted that possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let $A$ be the set of players while $B$ be the set of all possible outcomes.

$A = \{S, D\}$, $B = \{1, 2, 3, 4, 5, 6\}$

($i$) Let $R : B \to B$ be defined by $R = \{(x, y) : y \text{ is divisible by } x\}$. Show that relation $R$ is reflexive and transitive but not symmetric.

($ii$) Let $R$ be a relation on $B$ defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then check whether $R$ is an equivalence relation.

($iii$) Raji wants to know the number of functions from $A$ to $B$. How many number of functions are possible?

**OR**

*(iii)* Raji wants to know the number of relations possible from $A$ to $B$. How many numbers of relations are possible?

Ans: ($i$) Since every number is divisible by itself, So

$(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \in R$.

So, $R$ is reflexive relation on $B$. Also $(1, 2) \in R$ but $(2, 1)$ does not belong here non-symmetric.

($ii$) $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5),\}$

Since $(1, 1) \in R$, so $R$ is not reflexive.

Hence '$R$' is not an equivalence relation.

($iii$) As number of functions possible from set $A$ to set $B$, if set $A$ contains $m$ elements and set $B$ contains $n$ elements is given by $n^m$.

Now, $n(A) = 2$ ; $n(B) = 6$

Number of possible functions = $6^2$

**OR**

As, number of relations from a set with '$m$' elements to a set with $n$ elements is $2^{mn}$.

Now $n(A) = 2$ ; $n(B) = 6$

Required number of relations = $2^{12}$

**20.** A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

R = $\{(V_1,V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019$\}$

(*i*) Two neighbours $X$ and $Y \in I$. $X$ exercised his voting right while $Y$ did not cast her vote in general election-2019. Check whether $X$ is related to $Y$ or not.

(*ii*) Mr. '$X$' and his wife '$W$' both exercised their voting right in general election-2019. Show that $(X, W) \in R$ and $(W, X) \in R$.

(*iii*) Three friends $F_1$, $F_2$ and $F_3$ exercised their voting right in general election-2019. Show that $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$.

OR

Show that the relation $R$ defined on set $I$ is an equivalence relation.

Ans: R = $\{(V_1, V_2)$: $V_1, V_2 \in I$ and both use their voting rights$\}$

It is given that $X$ exercised his voting right and $Y$ didn't cast her vote.

So, $X$ is not related to $Y$, i.e. $(X, Y) \in R$.

(*ii*) $R = \{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting rights$\}$

It is given that Mr $X$ and his wife $W$ both exercised their voting rights in election.

So, $X$ is related to $W$ and $W$ is related to $X$, i.e.

$(X, W) \in R$ and $(W, X) \in R$

(*iii*) Since all the three friends $F_1$, $F_2$ and $F_3$ exercised their voting rights in election, so

$(F_1, F_2) \in R$,

$(F_2, F_3) \in R$ and $(F_1, F_3) \in$ R.

OR

Let $V$ be any person in $I$. Then $V$ and $V$ use their voting rights in election

Þ $(V, V) \in R$

Thus $(V, V) \in R$ for all $V \in I$.

So, $R$ is reflexive relation on $I$.

Let $V_1$ and $V_2$ be two persons in $A$ such that $(V_1, V_2) \in R$.

Then, $(V_1, V_2) \in R \Rightarrow$ $V_1$ and $V_2$ both use their voting rights

$\Rightarrow$ $V_2$ and $V_1$ both use their voting rights.

$\Rightarrow$ $(V_2, V_1) \in R$

$R$ is symmetric on $I$.

Let $V_1$, $V_2$, $V_3$ be three person in $I$ such that $(V_1, V_2) \in R$ and $(V_2, V_3) \in R$.

Then $(V_1, V_2) \in R \Rightarrow$ $V_1$ and $V_2$ both use their voting rights.

and $(V_2, V_3) \in R \Rightarrow V_2$ and $V_3$ both use their voting rights.

So, $V_1$ and $V_3$ both use their voting rights.

$\Rightarrow$ $(V_1, V_3) \in R$

So, $R$ is transitive on $I$.

Hence, $R$ is an equivalence relation.