PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD–32
PRACTICE PAPER 2 (2023-24)
STRAIGHT LINES AND CONIC SECTIONS
(ANSWERS)

SUBJECT: MATHEMATICS MAX. MARKS : 40

CLASS : XI DURATION : $1\frac{1}{2}$ hrs

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Slope of a line which cuts off intercepts of equal lengths on the axes is

(a) −1 (b) 0 (c) 2 (d) $\sqrt{3}$

Ans: (a) −1

Let equation of line be $\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow x + y = a \Rightarrow y = -x + a$

$\therefore$ Required slope = −1

2. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2), then the equation of the line will be

(a) 2x + 3y =12 (b) 3x + 2y =12 (c)4x − 3y = 6 (d) 5x −2y =10

Ans: (a) 2x + 3y =12

Since, the coordinates of the middle point are P(3, 2).

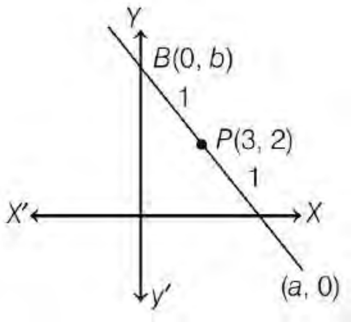
$\therefore \quad 3=\frac{1\cdot 0+1\cdot a}{1+1}$

$\Rightarrow \quad 3=\frac{a}{2} \Rightarrow a=6$

Similarly, $b=4$

$\therefore$ Equation of the line is $\frac{x}{6}+\frac{y}{4}=1$

$\Rightarrow 2x+3y=12$



3. The equation of the line through (− 2, 3) with slope − 4 is

(a) x + 4y −10 =0 (b) 4x + y + 5 =0 (c) x + y −1 =0 (d) 3x + 4y − 6 =0

Ans: Here, m = − 4 and given point (x0, y0) is (−2, 3).

By slope-point form, Equation of the given line is y − 3 = − 4 (x + 2)

or 4x + y + 5 = 0, which is the required equation.

4. The perpendicular distance from origin to the line 5x + 12y − 13 = 0 is

(a) 10 unit (b) 5 unit (c) 2 unit (d) 1 unit

Ans: (d) 1 unit

Given equation of line is 5x + 12y −13 = 0 …(i)

Length of perpendicular from origin to the line (i) is $p = \frac{|-13|}{\sqrt{5^2+12^2}}$

$\left[\because \text{ perpendicular length from origin to the line } ax+by+c=0 \text{ is } p = \frac{|c|}{\sqrt{a^2+b^2}}\right]$

$$= \frac{|-13|}{\sqrt{25+144}} = \frac{|-13|}{\sqrt{169}} = \frac{13}{13} = 1 \quad [\because |x| = -x,\ x<0]$$

5. The equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin is

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$ (b) $x^2 + y^2 - 2x - 2y - 1 = 0$

(c) $x^2 + y^2 - 2x - 2y = 0$ (d) $x^2 + y^2 - 2x + 2y - 1 = 0$

Ans: (a) $x^2 + y^2 - 2x - 2y + 1 = 0$

Since the equation $(x-1)^2 + (y-1)^2 = 1$ represents a circle touching both the axes with its centre (1, 1) and radius one unit, therefore required equation is

$x^2 + y^2 - 2x - 2y + 1 = 0$.

6. The equation of the ellipse whose centre is at the origin and the x-axis, the major axis, which passes through the points (–6, 1) and (4, –4) is

(a) $3x^2 - 4y^2 = 32$ (b) $3x^2 + 4y^2 = 112$ (c) $4x^2 - 3y^2 = 112$ (d) $4x^2 + 3y^2 = 112$

Ans: (b) $3x^2 + 4y^2 = 112$

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the equation of the ellipse, $a > b$.

Then according to the given conditions, we have

$\frac{36}{a^2} + \frac{1}{b^2} = 1 \quad \ldots$(i) and $\frac{16}{a^2} + \frac{16}{b^2} = 1 \quad \ldots$(ii)

Solving (i) and (ii), we get

$a^2 = \frac{112}{3}$ and $b^2 = \frac{112}{4}$.

Hence, required equation of ellipse is $3x^2 + 4y^2 = 112$.

7. The length of the latus rectum of the ellipse $9x^2 + 16y^2 = 144$ is

(a) 4 (b) 11/4 (c) 7/2 (d) 9/2

Ans: (d) 9/2

Given equation of ellipse is $9x^2 + 16y^2 = 144$

or $\frac{9}{144}x^2 + \frac{16}{144}y^2 = 1$ or $\frac{x^2}{(4)^2} + \frac{y^2}{(3)^2} = 1$

$\therefore \quad a = 4, b = 3.$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \cdot 9}{4} = \frac{9}{2}$

8. The equation of hyperbola referred to its axes as axes of co-ordinate whose distance between the foci is 20 and eccentricity equals $\sqrt{2}$ is

(a) $x^2 - y^2 = 25$ (b) $x^2 - y^2 = 50$ (c) $x^2 - y^2 = 125$ (d) $x^2 + y^2 = 25$

Ans: (b) $x^2 - y^2 = 50$

Given, $2ae = 20$ and $e = \sqrt{2}$ $\therefore$ $a = \frac{20}{2e} = 5\sqrt{2}$

Again $b^2 = a^2(e^2 - 1) \Rightarrow b^2 = 25 \times 2(2-1) = 50$

$\therefore$ Equation of hyperbola is $x^2 - y^2 = 50$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** If $x\cos\theta + y\sin\theta = 2$ is perpendicular to the line $x - y = 3$, then one of the value of θ is $\pi/4$.
Reason (R): If two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular then $m_1 = m_2$.
Ans: (c) A is true but R is false.
Assertion: Since, slope of line $x \cos\theta + y \sin\theta = 2$ is $-\cot\theta$ and slope of line $x - y = 3$ is 1.
Also, these lines are perpendicular to each other.
$\therefore (-\cot\theta)(1) = -1$
$\Rightarrow \cot\theta = 1 = \cot \pi/4 \Rightarrow \theta = \pi/4$
Reason Condition of perpendicularity of two lines is $m_1 \times m_2 = -1$.
Hence, Assertion is true and Reason is false.

10. **Assertion (A):** The length of major and minor axes of the ellipse $5x^2 + 9y^2 - 54y + 36 = 0$ are 6 and 10, respectively.
Reason (R): The equation $5x^2 + 9y^2 - 54y + 36 = 0$ can be expressed as $5x^2 + 9(y - 3)^2 = 45$.
Ans: (d) A is false but R is true.
We have, $5x^2 + 9y^2 - 54y + 36 = 0$

$$\Rightarrow \quad 5x^2 + 9(y-3)^2 = 45 \Rightarrow \frac{x^2}{3^2} + \frac{(y-3)^2}{(\sqrt{5})^2} = 1$$

$\therefore$ Length of major axis $= 2 \times 3 = 6$
and length of minor axis $= 2\times\sqrt{5} = 2\sqrt{5}$

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the angle between the x-axis and the line joining the points $(3, -1)$ and $(4, -2)$.
Ans: Let A$(3, -1)$ and B$(4, -2)$ be the given points and let m be the slope of the line AB. Then,

$$m = \frac{-2-(-1)}{4-3} = \frac{-2+1}{1} = -1. \quad \left[\because\ m = \frac{(y_2 - y_1)}{(x_2 - x_1)}\right]$$

Let θ be the angle between the x-axis and the line AB. Then,

$$\tan\theta = m = -1 = -\tan 45^\circ = \tan(180^\circ - 45^\circ) = \tan 135^\circ.$$

$\therefore \quad \theta = 135^\circ.$

Hence, the required angle is 135°.

12. Find the equation of an ellipse whose vertices are at $(\pm5, 0)$ and foci at $(\pm 4, 0)$.
Ans: Since the vertices of the given ellipse are on the x-axis, so it is a horizontal ellipse.
Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 > b^2$.
Its vertices are $(\pm a, 0)$ and therefore, $a = 5$.
Its foci are $(\pm c, 0)$ and therefore, $c = 4$.
$\therefore c^2 = (a^2 - b^2) \Rightarrow b^2 = (a^2 - c^2) = (25 - 16) = 9$.
Thus, $a^2 = 5^2 = 25$ and $b^2 = 9$.
Hence, the required equation is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

13. If the line through the points (−2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24), find the value of x.

Ans: Let A(−2, 6), B(4, 8), C(8, 12) and D(x, 24) be the given points.

Let m_1 and m_2 be the slopes of AB and CD respectively. Then,

$$m_1 = \text{slope of } AB = \frac{(8-6)}{4-(-2)} = \frac{2}{6} = \frac{1}{3}.$$

$$m_2 = \text{slope of } CD = \frac{(24-12)}{(x-8)} = \frac{12}{(x-8)}.$$

Now, $AB \perp CD \Leftrightarrow m_1 m_2 = -1$

$$\Leftrightarrow \frac{1}{3} \times \frac{12}{(x-8)} = -1$$

$$\Leftrightarrow -x + 8 = 4 \Leftrightarrow x = 4.$$

Hence, the required value of x is 4.

14. Show that the lines 27x − 18y + 25 = 0 and 2x + 3y + 7 = 0 are perpendicular to each other.

Ans: Let the slopes of the given lines be m_1 and m_2 respectively.

Then, $27x - 18y + 25 = 0 \Rightarrow 18y = 27x + 25$

$$\Rightarrow y = \frac{3}{2}x + \frac{25}{18}$$

And, $2x + 3y + 7 = 0 \Rightarrow 3y = -2x - 7$

$$\Rightarrow y = \frac{-2}{3}x - \frac{7}{3}.$$

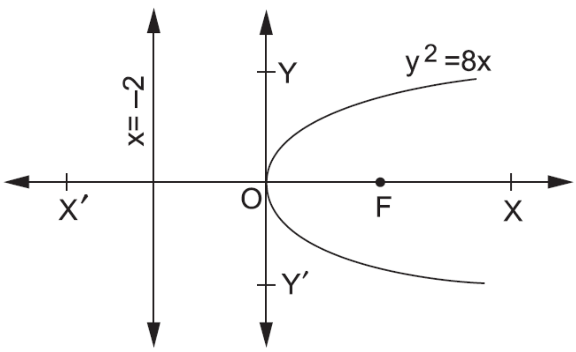
$$\therefore \quad m_1 = \frac{3}{2} \text{ and } m_2 = \frac{-2}{3} \Rightarrow m_1 m_2 = \left(\frac{3}{2}\right) \times \left(\frac{-2}{3}\right) = -1$$

Hence, the given lines are perpendicular to each other.

15. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola $y^2 = 8x$.

Ans: The given equation is of the form $y^2 = 4ax$, where 4a = 8, i.e., a = 2.

This is a right-handed parabola.



Its focus is F(a, 0), i.e., F(2, 0).

Its vertex isO(0, 0).

The equation of the directrix is x = − a, i.e., x = −2.

Its axis is x-axis, whose equation is y = 0.

Length of latus rectum = 4a = (4 × 2) units = 8 units.

SECTION – C

Questions 15 to 17 carry 3 marks each.

16. Find the equation of the perpendicular bisector of the line joining the points A(2, 3) and B(6, −5).

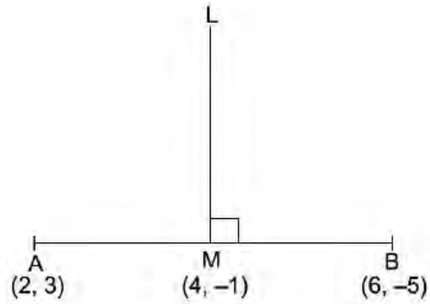
Ans: Here, A(2, 3) and B(6, −5) are the end points of the given line segment.

$M\left(\frac{2+6}{2}, \frac{3-5}{2}\right)$, i.e., $M(4, -1)$.

Slope of $AB = \frac{-5-3}{6-2} = -2$.

Let $LM \perp AB$ and let the slope of LM be m.

Then, $m \times (-2) = -1 \Rightarrow m = \frac{1}{2}$ [$\because$ $LM \perp AB$].



Clearly, LM is the perpendicular bisector of AB.

Now, LM is a line with slope $= \frac{1}{2}$ and passing through $M(4, -1)$.

So, the required equation is

$$\frac{y-(-1)}{x-4} = \frac{1}{2} \Rightarrow 2(y+1) = (x-4) \Rightarrow x - 2y - 6 = 0.$$

17. Find the equation of the line which passes through the point (3, 4) and the sum of whose intercepts on the axes is 14.

Ans: Let the intercepts made by the line on the x-axis and y-axis be a and $(14 - a)$ respectively.

Then, its equation is $\frac{x}{a} + \frac{y}{(14-a)} = 1$ [intercept form].

Since it passes through the point (3, 4), we have

$$\frac{3}{a} + \frac{4}{(14-a)} = 1 \Leftrightarrow 3(14-a) + 4a = a(14-a)$$
$$\Leftrightarrow a^2 - 13a + 42 = 0 \Leftrightarrow (a-6)(a-7) = 0$$
$$\Leftrightarrow a = 6 \quad \text{or} \quad a = 7.$$

Now, $a = b \Leftrightarrow b = 14 - 6 = 8$.

And, $a = 7 \Leftrightarrow b = 14 - 7 = 7$.

So, the required equation is

$$\frac{x}{6} + \frac{y}{8} = 1 \quad \text{or} \quad \frac{x}{7} + \frac{y}{7} = 1$$

i.e., $4x + 3y - 24 = 0$ or $x + y - 7 = 0$.

18. Find the equation of the hyperbola whose vertices are (±7, 0) and the eccentricity is 4/3·

Ans: Since the vertices of the given hyperbola are of the form(±a, 0), it is a horizontal hyperbola.

Let the required equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Then, its vertices are $(\pm a, 0)$.

But, the vertices are $(\pm 7, 0)$.

$\therefore \quad a = 7 \Leftrightarrow a^2 = 49$.

Also, $e = \frac{c}{a} \Leftrightarrow c = ae = \left(7 \times \frac{4}{3}\right) = \frac{28}{3}$.

Now, $c^2 = (a^2 + b^2) \Leftrightarrow b^2 = (c^2 - a^2) = \left[\left(\frac{28}{3}\right)^2 - 49\right] = \frac{343}{9}$.

Thus, $a^2 = 49$ and $b^2 = \frac{343}{9}$.

$\therefore$ the required equation is

$$\frac{x^2}{49} - \frac{y^2}{(343/9)} = 1 \Leftrightarrow \frac{x^2}{49} - \frac{9y^2}{343} = 1 \Leftrightarrow 7x^2 - 9y^2 = 343.$$

SECTION – D

Questions 18 carry 5 marks.

19. Find the equation of the circle passing through the vertices of a triangle whose sides are represented by the equations $x + y = 2$, $3x - 4y = 6$ and $x - y = 0$.

Ans: Let the sides AB, BC and CA of CABC be represented by the equations $x + y = 2$, $3x - 4y = 6$ and $x - y = 0$ respectively.

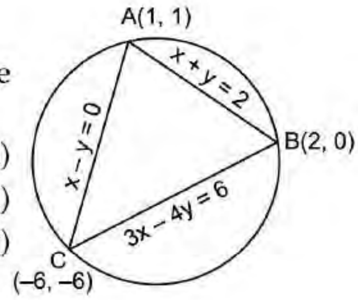
Solving $x + y = 2$ and $3x - 4y = 6$, we get B(2, 0).

Solving $3x - 4y = 6$ and $x - y = 0$, we get C(−6, − 6).

Solving $x + y = 2$ and $x - y = 0$, we get A(1, 1).

Let the required equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$. … (i)



Since it passes through $A(1, 1)$, $B(2, 0)$ and $C(-6, -6)$, each of these points must satisfy (i).

$\therefore \quad 1^2 + 1^2 + 2g + 2f + c = 0 \Rightarrow 2g + 2f + c + 2 = 0$ … (ii)

$2^2 + 0^2 + 4g + c = 0 \Rightarrow 4g + c + 4 = 0$ … (iii)

$(-6)^2 + (-6)^2 - 12g - 12f + c = 0 \Rightarrow -12g - 12f + c + 72 = 0$ … (iv)

Subtracting (ii) from (iii), we get

$2g - 2f + 2 = 0 \Leftrightarrow g - f = -1.$ … (v)

Subtracting (iv) from (iii), we get

$16g + 12f - 68 = 0 \Leftrightarrow 4g + 3f = 17.$ … (vi)

Solving (v) and (vi), we get $g = 2$ and $f = 3$.

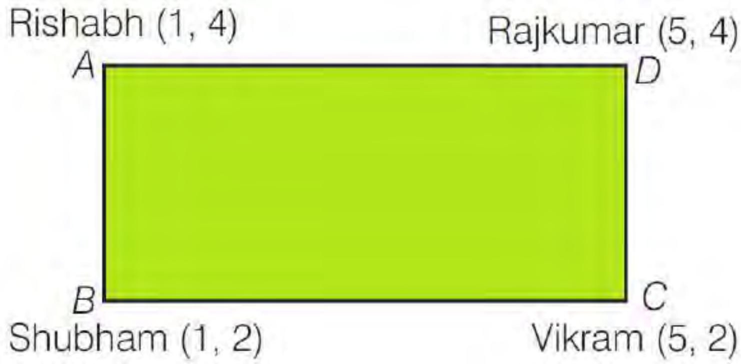
Putting $g = 2$ in (iii), we get $c = -12$.

Hence, the required equation is $x^2 + y^2 + 4x + 6y - 12 = 0$.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

20. In a rectangle park, four friends Rishabh, Shubham, Vikram and Rajkumar are sitting at the corners and chatting through their phones. Their positions in the form of coordinates are given as Rishabh (1, 4), Rajkumar (5, 4) Shubham (1, 2) and Vikram (5, 2)



On the bases of the information answer the following.

(i) Find the equation formed by Shubham and Rajkumar. (2)

OR

(ii) Find the equation formed by Rishabh and Vikram. (2)

(iii) Find the Slope of equation of line formed by Rishabh and Rajkumar. (1)

(iv) Pair for the same slope is

(a) Rishabh-Rajkumar and Shubham-Vikram

(b) Rishabh-Rajkumar and Rajkumar-Vikram

(c)Rishabh-Rajkumar and Rishabh-Shubham

(d) None of the above

Ans: (i) We have the positions, Shubham B (1, 2) and Rajkumar D (5, 4)

$$\text{Slope, } m_1 = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$$

Taking point $(1, 2) = (x_1, y_1)$ and $m_1 = \frac{1}{2}$

Equation of line joining points B and D is $(y - y_1) = m_1(x - x_1)$

$\Rightarrow (y-2) = \frac{1}{2}(x-1) \Rightarrow 2y - 4 = x - 1 \Rightarrow x - 2y + 3 = 0$

(ii) We have the positions, Rishabh A (1, 4) and Vikram C (5, 2)

$$\text{Slope, } m_2 = \frac{2-4}{5-1} = \frac{-2}{4} = -\frac{1}{2}$$

Taking point $(x_1, y_1) = (1, 4)$ and $m_2 = -\frac{1}{2}$

Equation of line is $(y-4) = -\frac{1}{2}(x-1)$

$\Rightarrow 2y - 8 = -x + 1 \Rightarrow x + 2y - 9 = 0$

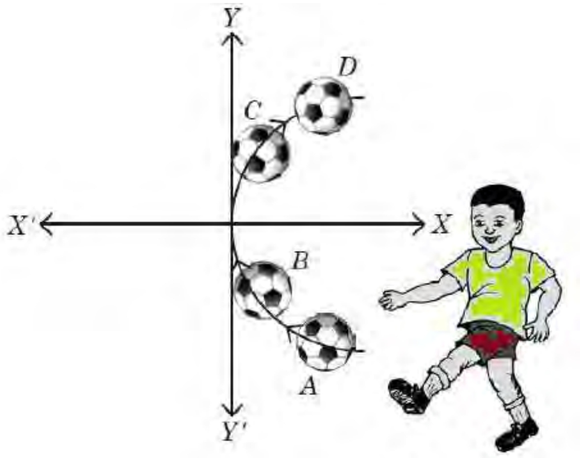
(iii) We have positions, Rishabh A (1, 4) and Rajkumar D (5, 4)

$$\text{Slope } AD = \frac{4-4}{5-1} = 0$$

Hence, slope is zero.

(iv) The line formed by Rishabh-Rajkumar is opposite and parallel to the line formed by Shubham and Vikram. Hence, first pair have same slope.

21. Aditya was playing a football match. When he kicked the football, the path formed by the football from ground level is parabolic, which is shown in the following graph. Consider the coordinates of point A as (3, –2).



On the bases of the information answer the following.

(i) Find the equation of path formed by the football. (1)

(ii) Find the equation of directrix of path formed by football. (1)

(iii) Find the extremities of latus rectum of given curve. (1)

(iv) Find the length of latus rectum of given curve. (1)

Ans: (i) The path formed by football is in the shape of parabola. We know that general equation of parabola is $y^2 = 4ax$.

Since, it passes through (3, –2)

$\therefore (-2)^2 = 4 \times a \times 3 \Rightarrow a = 1/3$

$\Rightarrow$ Hence, required equation of path formed by football is $y^2 = \frac{4x}{3} \Rightarrow 3y^2 = 4x$

(ii) Since, a = 1/3. Therefore, the equation of its directrix is $x + \frac{1}{3} = 0$

(iii) The extremities of latus rectum are (a, ±2a) = $\left(\frac{1}{3}, \pm\frac{2}{3}\right)$

(iv) The length of latus rectum = 4a = $4 \times \frac{1}{3} = \frac{4}{3}$