PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD–32

PRACTICE PAPER 1 (2023-24)

STRAIGHT LINES AND CONIC SECTIONS

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : XI **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The value of y will be, so that the line through (3, y) and (2, 7) is parallel to the line through (−1, 4) and (0, 6).

(a) 7 (b) 8 (c) 9 (d) 10

Ans: (c) 9

Let A(3, y), B(2, 7), C(−1, 4) and D(0, 6) be the given points.

Then, m_1 = Slope of the line

AB = $\frac{7-y}{2-3} = y-7$

and m_2 = Slope of the line

CD = $\frac{6-4}{0-(-1)} = 2$

Since, AB and CD are parallel.

$\therefore m_1 = m_2 \Rightarrow y - 7 = 2 \Rightarrow y = 9$

2. The equations of the line which have slope 1/2 and cuts-off an intercept 4 on X -axis is

(a) x − 2y −4 = 0 (b) x + 2y −4 =0

(c) x + 2y + 4 =0 (d) x −2y + 4 =0

Ans: (a) x − 2y −4 = 0

Given, m = Slope of the line = 1/2

Here, d = Intercept of the line on X -axis = 4.

Hence, required equation of the line is

$y = (1/2)(x - 4)$

$\Rightarrow x - 2y - 4 = 0$ [$\because y = m(x - d)$]

3. Lines through the points (−2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). Then, the value of x is

(a) 2 (b) 6 (c) 8 (d) 4

Ans: (d) 4

Slope of the line through the points (– 2, 6) and (4, 8) is

$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

Slope of the line through the points (8, 12) and (x, 24) is

$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$

Since, two lines are perpendicular.

So, $m_1m_2 = -1 \Rightarrow \frac{1}{3}\times\frac{12}{x-8} = -1 \Rightarrow 8 - x = 4$ or $x = 4$

4. The distance between the lines 3x + 4y = 9 and 6x + 8y = 15 is

(a) 3/10 (b) 2/25 (c) 7/10 (d) 3/25

Ans: (a) 3/10

The equations of lines 3x + 4y = 9 and 6x + 8y =15 may be rewritten as 3x + 4y − 9 = 0 and 3x + 4y − 15/2 = 0

Since, the slope of these lines are same and hence they are parallel to each other.

Therefore, the distance between them is given by

$$\left|\frac{9-\frac{15}{2}}{\sqrt{3^2+4^2}}\right| = \frac{3}{10}$$

5. If parabola is passing through (5, 2), vertex (0, 0) and symmetric with respect to Y-axis, then the equation of parabola is

(a) $y^2 = \frac{25}{2}x$ (b) $y^2 = \frac{5}{2}x$ (c) $x^2 = \frac{25}{2}y$ (d) $x^2 = \frac{5}{2}y$

Ans: (c) $x^2 = \frac{25}{2}y$

Given, vertex = (0, 0), Point = (5, 2)

Since, point (5, 2) lies in first quadrant and axis is Y-axis.

Therefore, parabola is symmetric with respect to Y-axis.

Hence, equation of parabola will be of the form $x^2 = 4ay$, which passes through (5, 2) i.e.

Put x = 5, y = 2 in $x^2 = 4ay$

$\therefore$ (5)2 = 4a(2) $\Rightarrow$ a = 25/8

Hence, required equation of parabola is $x^2 = \frac{25}{2}y$

6. If the foci and vertices of an ellipse be (±1, 0) and (±2, 0) respectively, then the minor axis of the ellipse is

(a) 2√5 units (b) 2 units (c) 4 units (d) 2√3 units

Ans: (d) 2√3 units

Given that, ae = 1, a = 2 and e = 1/2

$$\Rightarrow \quad b = \sqrt{4\left(1-\frac{1}{4}\right)} = \sqrt{3} \qquad \left[\because e = \sqrt{1-\frac{b^2}{a^2}}\right]$$

Hence, minor axis 2√3 units.

7. The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is

(a) 4 units (b) 3 units (c) 8 units (d) 4/√3 units

Ans: (d) 4/√3 units

Given, equation of ellipse is $3x^2 + y^2 = 12 \Rightarrow \frac{x^2}{4} + \frac{y^2}{12} = 1$

$\therefore\ b^2 = 4 \Rightarrow b = 2$ and $a^2 = 12 \Rightarrow a = 2\sqrt{3}$

$\because\ a > b$

$\therefore$ Length of latusrectum $= \frac{2\times 4}{2\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{2b^2}{a}$ units.

8. The centre of the circle $x^2 + y^2 + 6x - 4y - 12 = 0$ is
(a) (–3, 2) (b) (3, 2) (c) (3, –2) (d) (–3, –2)
Ans: (a) (–3, 2)

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** The distance between the lines 4x + 3y = 11and 8x + 6y = 15 is 7/10.

Reason (R): The distance between the lines ax + by = c_1 and ax + by = c_2 is given by $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

Ans: (a) Both A and R are true and R is the correct explanation of A.
Assertion - Given lines are 4x + 3y =11 and 4x + 3y =15/2.
Distance between them

$$= \left|\frac{11 - \frac{15}{2}}{\sqrt{16+9}}\right| = \left|\frac{7}{2 \times 5}\right| = \frac{7}{10}$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

10. **Assertion (A):** A line through the focus and perpendicular to the directrix is called the x-axis of the parabola.
Reason (R): The point of intersection of parabola with the axis is called the vertex of the parabola.
Ans: (d) A is false but R is true.
A line through the focus and perpendicular to the directrix is called the axis of the parabola. The point of intersection of parabola with the axis is called the vertex of the parabola.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the value of k, if the straight line $2x + 3y + 4 + k(6x - y + 12) = 0$ is perpendicular to the line $7x + 5y - 4 = 0$.

Ans: Slope of line $2x + 3y + 4 + k(6x - y + 12) = 0$ is $-\frac{(2+6k)}{3-k}$ and slope of line $7x + 5y - 4 = 0$ is $-\frac{7}{5}$

If lines are perpendicular, then $\frac{(2+6k)}{3-k} \times -\frac{7}{5} = -1$

$\Rightarrow 14 + 42k = -15 + 5k \Rightarrow k = -\frac{29}{37}$

12. Find the focus, vertex and directrix for each of the following parabolas : $y^2 = -8x$
Ans: $y^2 = -8x$ is of the form $y^2 = 4ax$ with $a = -2$.
Focus is (– 2, 0), vertex is (0, 0) and directrix is $x = 2$.

13. If p is the length of perpendicular from origin to the line which makes intercepts a, b on the axes, prove that: $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Ans: As intercepts on axes are a and b, therefore, equation of line is

$\frac{x}{a}+\frac{y}{b}=1$ ...(*i*)

Its distance from (0, 0) is $p =$

$$\left|\frac{\frac{1}{a}.0+\frac{1}{b}.0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|$$

Squaring, we get

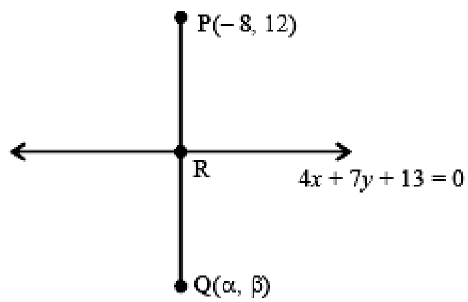
$$p^2=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$$

14. Find the equation of the parabola that satisfies the given conditions Focus (0, –3); directrix $y = 3$.
Ans: Focus = (0, –3), $a = –3$, $x^2 = – 12y$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find image of the point P (–8, 12) with respect to the line mirror $4x + 7y + 13 = 0$.
Ans: Let Q(α, β) be image of P(–8, 12) in the line $4x + 7y + 13 = 0$. Then (*i*) R is mid-point of PQ (*ii*) PQ ⊥ line.



Coordinates of mid-point is

$$R\left(\frac{\alpha-8}{2},\frac{\beta+12}{2}\right)$$

As this point lies on line.

$$4\left(\frac{\alpha-8}{2}\right)+7\left(\frac{\beta+12}{2}\right)+13=0$$

$\Rightarrow 4\alpha – 32 + 7\beta + 84 + 26 = 0$

$\Rightarrow 4\alpha + 7\beta + 78 = 0$ ...(*i*)

Also PQ ⊥ line, therefore

$$\frac{\beta-12}{\alpha+8}\times-\frac{4}{7}=-1$$

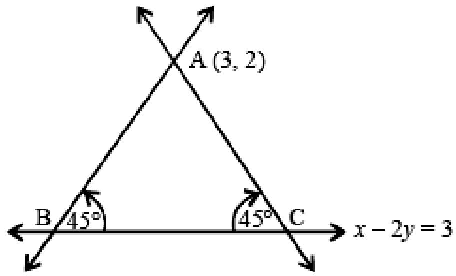
$\Rightarrow 4\beta – 48 = 7\alpha + 56$

$\Rightarrow 7\alpha – 4\beta + 104 = 0$ ...(*ii*)

Solving (*i*) and (*ii*), we get α = –16, β = – 2

Hence, image is (– 16, –2).

16. Find the equation of the line through the point (3, 2) which makes an angle of 45° with the line $x – 2y = 3$.
Ans: Let line through A(3, 2) be $y – 2 = m(x – 3)$ ...(*i*)



Angle between line (i) and the line $x - 2y = 3$ is 45°.

Slope of line $x - 2y = 3$ is $\frac{1}{2}$

$\therefore$ tan (45°) =

$$\left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right| \Rightarrow 1 = \left|\frac{2m-1}{2+m}\right|$$

$$\frac{2m-1}{2+m} = \pm 1$$

$\Rightarrow 2m - 1 = m + 2$ or $2m - 1 = -m - 2$

$\Rightarrow m = 3$ or $3m = -1 \Rightarrow m = -\frac{1}{3}$

Substituting in (i), we get line as

$y - 2 = 3(x - 3)$ or $y - 2 = -\frac{1}{3}(x - 3)$

$\Rightarrow y - 2 = 3x - 9$ or $3y - 6 = -x + 3$

$\Rightarrow 3x - y - 7 = 0$ or $x + 3y - 9 = 0$

are the required equations.

17. Find the equation of hyperbola with foci at (0, ±4) and length of transverse axis is 6.

Ans: Given foci is (0, ± 4)

$\therefore$ Hyperbola is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ ...(i)

Also $2a = 6 \Rightarrow a = 3$

and $c = 4 \Rightarrow \sqrt{a^2 + b^2} = 4$

$\Rightarrow a^2 + b^2 = 16 \Rightarrow 9 + b^2 = 16 \Rightarrow b^2 = 7$.

Substituting for a and b in (i), we get $\frac{y^2}{9} - \frac{x^2}{7} = 1$ as the required equation.

SECTION – D

Questions 18 carry 5 marks.

18. Find the equation of the line passing through the intersection of the lines $x + y + 1 = 0$ and $x - y + 1 = 0$ and whose distance from the origin is 1.

Ans: The line passing through the intersection of the given lines is

$(x + y + 1) + \lambda(x - y + 1) = 0$ ...(i)

$\Rightarrow (1 + \lambda)x + (1 - \lambda)y + (1 + \lambda) = 0$

Its distance from the origin is 1

$\Rightarrow 1 = \left|\frac{1+\lambda}{\sqrt{2+2\lambda^2}}\right|$

$\Rightarrow 2 + 2\lambda^2 = (1 + \lambda)^2 \Rightarrow \lambda = 1$

Putting value of λ in (i), we get $2x + 2 = 0$
$\Rightarrow x + 1 = 0$.

OR

Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line $y = x - 1$

Ans: Let circle be $(x - h)^2 + (y - k)^2 = 9$ ... (i)

Circle (i), passes through the point (7, 3).

$\therefore (7 - h)^2 + (3 - k)^2 = 9$ ... (ii)

Also centre (h, k) lies on line $y = x - 1$

$\Rightarrow k = h - 1$ ... (iii)

From (ii) and (iii), we have; $(7- h)^2 + (4 - h)^2 = 9$

$\Rightarrow 49 - 14h + h^2 + 16 - 8h + h^2 = 9$

$\Rightarrow 2h^2 -22h + 56 = 0 \Rightarrow h^2 -11h + 28 = 0$

$\Rightarrow (h - 7)(h - 4) = 0 \Rightarrow h = 7$ or $h = 4$

From (iii), when $h = 7$, $k = 6$ and when $h = 4$, $k = 3$

Substituting in (i), we get, circle as

$(x - 7)^2 + (y - 6)^2 = 9$ or $(x - 4)^2 + (y - 3)^2 = 9$

$\Rightarrow x^2 + y^2 - 14x - 12y + 76 = 0$

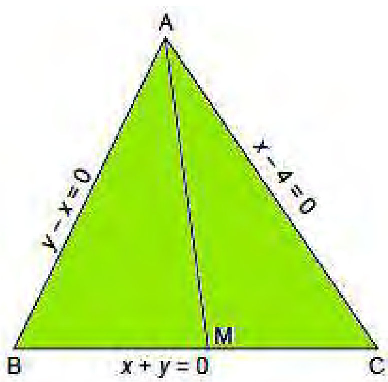
or $x^2 + y^2 - 8x - 6y + 16 = 0$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Three friends are sitting in a triangular park ABC. They marked the sides AB, BC and CA are represented by lines y – x = 0, x + y = 0 and x – 4 = 0 respectively

One friend drawn the partition through A to M. The partition AM divides the triangle into two equal parts



On the bases of the information answer the following.

(i) Find the coordinates of vertices A, B and C.

(ii) Find the area of triangular park.

(iii) Find the equation of line AM.

(iv) If a fountain is at the equidistant from corners of the park then what are the coordinates of point where fountain is located.

Ans: (i) Coordinates are A(4, 4)

Coordinates are B(0, 0)

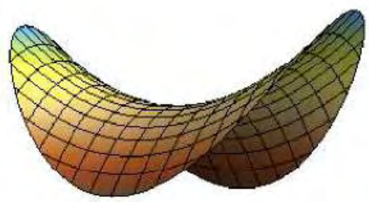
Coordinates are C(4, –4)

(ii) Area of triangle = 16 sq units

(iii) Equation of line AM is 3x – y = 8

(iv) Coordinates of fountain

20. Aditya, the student of class XI was studying in his house. He felt hungry and found that his mother was not at home. So, he went to the nearby shop and purchased a packet of chips. While eating the chips, he observed that one piece of the chips is in the shape of hyperbola. Consider the vertices of hyperbola at (±5, 0) and foci at (±7, 0).




51. (i) Find the equation of hyperbolic curve formed by given piece of chips.
52. (ii) Find the length of conjugate axis of given curve formed by given piece of chips.
53. (iii) Find the eccentricity of hyperbolic curve formed by given piece of chips
54. (iv) What is the length of latus rectum of given hyperbolic curve?

Ans: (i) We have, a = 5 and ae = 7

Now, $b^2 = a^2e^2 - a^2 = 49 - 25 = 24$.

So, equation is $\frac{x^2}{25} - \frac{y^2}{24} = 1$

(ii) Length of conjungate axis = 2b = 2 × 2√6 = 4√6

(iii) Eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{24}{25}} = \sqrt{\frac{49}{25}} = \frac{7}{5}$

(iv) Length of latus rectum = $\frac{2b^2}{a} = \frac{48}{5} = 9.6$