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PRACTICE PAPER (2023-24)
CHAPTER 08 SEQUENCE AND SERIES (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XI **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. If the 4th and 9th terms of a GP are 54 and 13122 respectively then its 6th term is
(a) 243 (b) 1458 (c) 486 (d) 729
Ans: (c) 486

2. Which term of the GP 5, 10, 20, 40,…is 5120?
(a) 9th (b) 10th (c) 11th (d) 12th
Ans: (c) 11th

3. The 8th term from the end of the GP 3, 6, 12, 24, …, 12288 is
(a) 96 (b) 192 (c) 48 (d) 288
Ans: (a) 96

4. If the nth term of a GP is 2^n , the sum of its first 6 terms is
(a) 124 (b) 126 (c) 190 (d) 254
Ans: (b) 126

5. In a GP it is given that a = 3, an = 96 and Sn = 189.The value of n is
(a) 7 (b) 8 (c) 6 (d) 5
Ans: (c) 6

6. How many terms of the GP 2, 6, 18,…will make the sum 728?
(a) 6 (b) 9 (c) 8 (d) 7
Ans: (a) 6

7. The sum of an infinite series is 8 and its second term is 2. Its common ratio is
(a) 1/2 (b) 1/4 (c) 2/3 (d) 3/4
Ans: (a) 1/2

8. The sum of an infinite geometric series is 15 and the sum of the squares of these terms is 45. The first term of the series is
(a) 6 (b) 7 (c) 5 (d) 9
Ans: (c)

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** The sum of the series $\frac{3}{\sqrt{5}} + \frac{4}{\sqrt{5}} + \sqrt{5} + ...$ 25 terms is $75\sqrt{5}$.

Reason (R): If 27, x, 3 are in GP, then x = ± 4.
Ans: (c) A is true but R is false.

10. **Assertion (A):** The sum of first 6 terms of the GP 4, 16, 64, … is equal to 5460.

Reason (R): Sum of first n terms of the G. P is given by $S_n = \frac{a(r^n - 1)}{r-1}$, where a = first term, r = common ratio and |r| > 1.
Ans: (a) Both A and R are true and R is the correct explanation of A.
Assertion Given GP 4, 16, 64, …

$\therefore a = 4, r = \frac{16}{4} = 4 > 1$

$\therefore S_6 = \frac{4((4)^6 - 1)}{4-1} = \frac{4(4095)}{3} = 5460$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Insert 4 Geometric means between 1 and 243,
Ans: 1, G_1, G_2, G_3, G_4, 243 are in G.P.
$a_6 = 243 \Rightarrow 1.\ r^5 = 243 \Rightarrow r = 3$
$\Rightarrow G_1 = 3, G_2 = 9, G_3 = 27, G_4 = 81$

12. Find the values of *p*, if sum to infinity for the G.P. *p*, 1, $\frac{1}{p}$, ... is $\frac{25}{4}$.

Ans: $\frac{p}{1-\frac{1}{p}} = \frac{25}{4} \Rightarrow \frac{p^2}{p-1} = \frac{25}{4}$
$\Rightarrow 4p^2 = 25p - 25$
$\Rightarrow 4p^2 - 25p + 25 = 0$
$(4p - 5)(p - 5) = 0 \Rightarrow p = \frac{5}{4}, 5$

13. If *a*, *b*, *c*, *d* are in G.P. ; prove that, *a* + *b*, *b* + *c*, *c* + *d* are also in G.P.
Ans: Let *r* be common ratio of the G.P. *a*, *b*, *c*, *d* then $b = ar$, $c = ar^2$ and $d = ar^3$.
$\therefore a + b = a + ar = a(1 + r)$; $b + c = ar + ar^2 = ar(1 + r)$; $c + d = ar^2(r + 1)$
Now, $(b + c)^2 = [ar(1 + r)]^2 = [a(1 + r)][ar^2(1 + r)] = (a + b)(c + d)$
$\therefore a + b, b + c, c + d$ are in G. P.

14. Prove that : $9^{1/3}. 9^{1/9}. 9^{1/27}$ .... = 3
Ans:

$$9^{1/3}.9^{1/9}.9^{1/27}... = 9^{(1/3+1/9+1/27+...)} = 9^{\frac{1/3}{1-1/3}} = 9^{1/2} = 3$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If a, b, c and d are in G.P. Prove that, $(a^n + b^n)$, $(b^n + c^n)$, $(c^n + d^n)$ are in G.P.

Ans: Given, a, b, c and d are in G.P.

Let common ratio be r.

$\therefore$ numbers in G.P. are a, $b = ar$, $c = ar^2$, $d = ar^3$

If $(a^n + b^n)$, $(b^n + c^n)$, $(c^n + d^n)$ are in G.P.

then $$\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n}$$

$$\Rightarrow \quad \frac{(ar)^n + (ar^2)^n}{a^n + (ar)^n} = \frac{(ar^2)^n + (ar^3)^n}{(ar)^n + (ar^2)^n}$$

$$\Rightarrow \quad \frac{a^n r^n[1 + r^n]}{a^n[1 + r^n]} = \frac{a^n r^{2n}[1 + r^n]}{a^n r^n[1 + r^n]}$$

$\Rightarrow r^n = r^n$. true. Hence in G.P.

16. Find the sum of 'n' terms of the series : 0.5 + 0.55 + 0.555 + ... n terms.

Ans: $S_n = 0.5 + 0.55 + 0.555 + \ldots$ n terms

$= 5[0.1 + 0.11 + 0.111 + \ldots$ n terms$]$

$= \frac{5}{9}$ $[0.9 + 0.99 + 0.999 + \ldots$ n terms$]$

$S_n = \frac{5}{9}$ $[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \ldots$ n terms$]$

$= \frac{5}{9}$ $[n - \{0.1 + 0.01 + 0.001 + \ldots$ n terms$\}]$

$$= \frac{5}{9}\left[n - \frac{0.1\{1 - (0.1)^n\}}{1 - 0.1}\right]$$

$$= \frac{5}{9}\left[n - \frac{1}{9}\{1 - (0.1)^n\}\right] = \frac{5}{81}[9n - 1 + (0.1)^n]$$

17. If the pth, qth and rth term of a G.P. are a, b, c respectively, prove that : $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$.

Ans: Let A be the first term and R the common ratio of a given G.P.

Given $a_p = a$, $a_q = b$, $a_r = c$,

$\therefore a = AR^{p-1}$; $b = AR^{q-1}$; $c = AR^{r-1}$

Consider $a^{q-r}\, b^{r-p}\, c^{p-q}$

$= (AR^{p-1})^{q-r}\, (AR^{q-1})^{r-p}\, (AR^{r-1})^{p-q}$

$= A^{q-r}\, R^{(p-1)\,(q-r)} \,.\, A^{r-p}\, R^{(q-1)\,(r-p)}\, A^{p-q}\, R^{(r-1)\,(p-q)}$

$= A^{q-r+r-p+p-q}\, R^{(p-1)\,(q-r)+(q-1)\,(r-p)+(r-1)\,(p-q)}$

$= A^0\, R^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q}$

$= A^0\, R^0 = 1$

SECTION – D

Questions 18 carry 5 marks.

18. The sum of two numbers is '6' times their geometric mean, show that the numbers are in the ratio $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$.

Ans:

Let the two numbers be a and b, $(a > b)$

We have, $a + b = 6\sqrt{ab}$ ...(i)

Also, $(a-b)^2 = (a+b)^2 - 4ab$

$= (6\sqrt{ab})^2 - 4ab = 32ab$

$\Rightarrow$ $a - b = \sqrt{32ab}$ $(a > b)$

$= 4\sqrt{2}\sqrt{ab}$ ...(ii)

From (i) and (ii), we have

$$\frac{a+b}{a-b} = \frac{6\sqrt{ab}}{4\sqrt{2}\sqrt{ab}} = \frac{3}{2\sqrt{2}}$$

Applying componendo and dividendo, we get

$$\Rightarrow \quad \frac{a+b+a-b}{a+b-a+b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$\Rightarrow \quad \frac{2a}{2b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$\Rightarrow \quad a : b = 3+2\sqrt{2} : 3-2\sqrt{2}.$$

OR

If S_n denotes the sum of n terms of a G.P., prove that $(S_{10} - S_{20})^2 = S_{10}(S_{30} - S_{20})$.

Ans:

$(S_{10} - S_{20})^2$

$$= \left\{\frac{a(1-r^{10})}{1-r} - \frac{a(1-r^{20})}{1-r}\right\}^2 = \frac{a^2}{(1-r)^2}\{r^{20} - r^{10}\}^2$$

$$= \frac{a^2}{(1-r)^2}(r^{40} + r^{20} - 2r^{30}) \quad \text{... (i)}$$

$S_{10}(S_{30} - S_{20})$

$$= \frac{a(1-r^{10})}{1-r}\left\{\frac{a(1-r^{30})}{1-r} - \frac{a(1-r^{20})}{1-r}\right\}$$

$$= \frac{a^2}{(1-r)^2}(1-r^{10})(r^{20} - r^{30})$$

$$= \frac{a^2}{(1-r)^2}(r^{20} - r^{30} - r^{30} + r^{40})$$

$$= \frac{a^2}{(1-r)^2}(r^{40} + r^{20} - 2r^{30}) \quad \text{... (ii)}$$

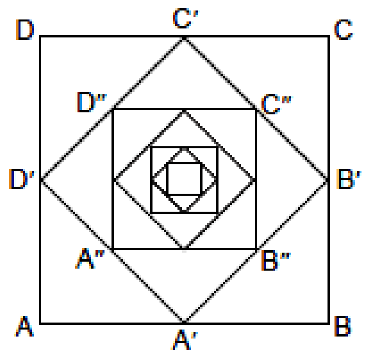
From (i) and (ii) we get

$(S_{10} - S_{20})^2 = S_{10}(S_{30} - S_{20})$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Geometrical mathematics has helped in art integration in the formation of designs of different patterns. Let us consider a square pattern. The mid points of whose sides are again joined to form another square, the mid points of whose sides are again joined to form another square and the process continues infinity. The pattern looks like.



If side of original square is 100 cm. Answer the following:

(a) What is the side of square A′B′C′D′?

(b) Find the area of square A″B″C″D″.

(c) Find the Perimeter of square A′B′C′D′

(d) Find the Sum of areas of squares if process continuous infinity.

Ans:

(a) Let side be x

$\therefore x^2 = (50)^2 + (50)^2 = 2(50)^2$

$\Rightarrow \quad x = 50\sqrt{2}$ cm

(b) Let side = A″ B″ C″ D″ = y cm

$$\therefore \quad \text{Area} = y^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = \frac{1}{2}x^2$$

$$= \frac{1}{2}\cdot 2(50)^2 = 2500 \text{ cm}^2$$

(c)

Side of A′ B′ C′ D′ = $50\sqrt{2}$ cm

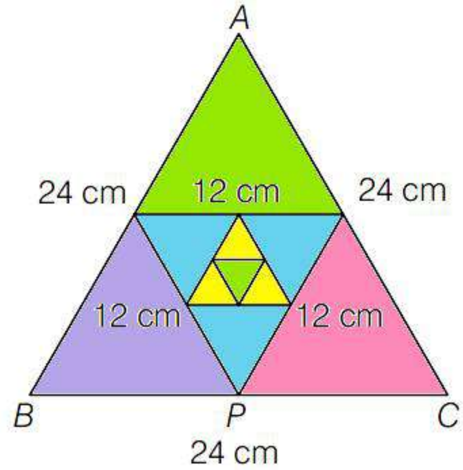
Perimeter = $4 \times 50\sqrt{2} = 200\sqrt{2}$ cm

(d) Sum of areas of infinite squares

$$= 10000 + 5000 + 2500 + \ldots$$

$$= \frac{10000}{1-\frac{1}{2}} = 20000 \text{ cm}^2.$$

20. In Rangoli competition in school, Preeti made Rangoli in the equilateral shape. Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



Based on above information, answer the following questions.
(a) Find the side of the 5th triangle is (in cm) (1)
(b) Find the sum of perimeter of first 6 triangle is (in cm) (2)

OR

(b) Find the area of all the triangle is (in sq cm). (2)
(c) Find the sum of perimeter of all triangle is (in cm). (1)
Ans: (a) Side of first triangle is 24.
Side of second triangle is 24/2 = 12
Similarly, side of second triangle is 12/2 = 6
∴ a = 24, r = 12/24 = 1/2

∴ Side of the fifth triangle, $a_5 = ar^4 = 24 \times \left(\frac{1}{2}\right)^4 = \frac{24}{16} = 1.5cm$

(b) Perimeter of first triangle = 24 × 3 = 72
Perimeter of second triangle = 72/2 = 36
Similarly, perimeter of third triangle = 36/2 =18
∴ a = 72 r = 36/72 = 1/2

∴ Sum of perimeter of first 6 triangle, $S_6 = \frac{a(1-r^6)}{1-r} = \frac{72\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}} = \frac{72 \times 63 \times 2}{2^6} = \frac{567}{4}cm$

OR

(b) Area of first triangle = $\frac{\sqrt{3}}{4}(24)^2$

Area of second triangle = $\frac{\sqrt{3}}{4}(12)^2$

Similarly, Area of third triangle = $\frac{\sqrt{3}}{4}(6)^2$

∴ a = $\frac{\sqrt{3}}{4}(24)^2$, r = 1/4

Sum of the areas of all triangles $= \frac{a}{1-r} = \frac{\frac{\sqrt{3}}{4}(24)^2}{1-\frac{1}{4}} = \frac{\sqrt{3}}{3}(24)^2 = 192\sqrt{3}cm$

(c) The sum of perimeter of all triangle 3(24 + 12 + 6 + ...) is

$$3\left(\frac{a}{1-r}\right)=3\left(\frac{24}{1-\frac{1}{2}}\right)=144cm$$