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PRACTICE PAPER (2023-24)

CHAPTER 04 COMPLEX NUMBERS AND QUADRATIC EQUATIONS

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : XI **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. For the quadratic equation $x^2 - 5ix - 6 = 0$, the value of x is

(a) 3, 2 (b) –3i, –2i (c) 3i, 2i (d) none of these

Ans: (c) 3i, 2i

$x^2 - 5ix - 6 = 0$

$\Rightarrow x^2 - 3ix - 2ix - 6 = 0$

$\Rightarrow x(x - 3i) - 2i(x - 3i) = 0$

$\Rightarrow (x - 3i)(x - 2i) = 0$

$\Rightarrow x - 3i = 0$ or $x - 2i = 0$

$\Rightarrow x = 3i$ or $x = 2i$

2. Conjugate of complex number $i^3 - 4$ is

(a) $i^3 + 4$ (b) $4 - i$ (c) $-4 + i$ (d) $-4 - i$

Ans: (c) $-4 + i$

$i^3 - 4 = -i - 4 = -4 - i$

$\therefore \overline{-4 - i} = -4 + i$

3. If a + ib = c + id, then

(a) $a^2 + c^2 = 0$ (b) $b^2 + c^2 = 0$ (c) $b^2 + d^2 = 0$ (d) $a^2 + b^2 = c^2 + d^2$

Ans: (d) $a^2 + b^2 = c^2 + d^2$

$a + ib = c + id$

$\Rightarrow |a + ib| = |c + id|$

$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{c^2 + d^2}$

On squaring both sides, we get $a^2 + b^2 = c^2 + d^2$

4. The value of $(z + 3)(\bar{z} + 3)$ is equivalent to

(a) $|z + 3|^2$ (b) $|z - 3|$ (c) $z^2 + 3$ (d) none of these

Ans: (a) $|z + 3|^2$

let $z = x + iy$, then $\bar{z} = \overline{x + iy} = x - iy$

$(z + 3)(\bar{z} + 3) = (x + iy + 3)(x - iy + 3)$

$= (x + 3)^2 - (iy)^2 = (x + 3)^2 + y^2$

$= |x + 3 + iy|^2 = |z + 3|^2$

5. If $z_1 = a + ib$ and $z_2 = c + id$ are two complex numbers. Then the product z_1z_2 is defined as

(a) ac + bd (b) ac + i(bd) (c) (ac – bd) + i(ad + bc) (d) none of these

Ans: (c) (ac – bd) + i(ad + bc)
z_1 = a + ib and z_2 = c + id
z_1z_2 = (a + ib) (c + id)
= ac + iad + ibc + i^2bd
= (ac – bd) + i(ad + bc) (∵ $i^2 = -1$)

6. Two complex numbers z_1 = a + ib and z_2 = c + id are equal if
(a) a = c (b) b = d (c) a = c and b = d (d) none of these
Ans: (c) a = c and b = d
z_1 = a + ib, Re z_1 = a, Im z_1 = b
For z_2 = c + id, Re z_2 = c, Im z_2 = d
The complex numbers are equal if a = c and b = d.

7. Find the modulus of $\frac{1+i}{1-i}$.
(a) 1 (b) 4 (c) 2 (d) 3
Ans: (a) 1

$$\left|\frac{1+i}{1-i}\right| = \frac{|1+i|}{|1-i|} = \frac{\sqrt{1+1}}{\sqrt{1+1}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

8. Find the magnitude of the following : 12*i* – 5
(a) 12 (b) 17 (c) 7 (d) 13
Ans: (d) 13

$$|12i-5| = \sqrt{(12)^2+(-5)^2} = \sqrt{144+25} = 13$$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** If z is a complex number, then $(\overline{z})^{-1}(\overline{z})$ is equal to 4.

Reason (R): The region of the complex plane for which $\left|\frac{z-a}{z+a}\right|$ =1 [Re(a) ≠ 0] is Y-axis.

Ans: (d) A is false but R is true.

10. **Assertion (A):** If $\sqrt{a+ib}$ = x + iy, then $\sqrt{a-ib}$ = x – iy.
Reason (R): A complex number z is said to be purely imaginary, if Re(z) = 0.
Ans: (b) Both A and R are true but R is not the correct explanation of A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If $z_1 = 2 + i$, $z_2 = 2 - 3i$, $z_3 = 4 + 5i$, evaluate $\text{Re}\left(\frac{z_1.\overline{z_2}}{z_3}\right)$

Ans: Consider $\frac{z_1.\overline{z_2}}{z_3} = \frac{(2+i)\overline{(2-3i)}}{4+5i}$

$$=\frac{(2+i)(2+3i)(4-5i)}{16+25}=\frac{44}{41}+\frac{27}{41}i$$

$$\text{Re}\left(\frac{z_1 \cdot \overline{z}_2}{z_3}\right)=\frac{44}{41}$$

12. Express in the form of $a + ib$: $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

Ans:

We have $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

$$=\left\{\frac{(1+i)-2+8i}{(1-4i)(1+i)}\right\}\times\left(\frac{3-4i}{5+i}\right)=\left(\frac{-1+9i}{1+i-4i+4}\right)\times\left(\frac{3-4i}{5+i}\right)=\frac{(-1+9i)(3-4i)}{(5-3i)(5+i)}$$

$$=\frac{-3+4i+27i+36}{25+5i-15i+3}=\frac{33+31i}{28-10i}=\left(\frac{33+31i}{28-10i}\right)\times\left(\frac{28+10i}{28+10i}\right)=\frac{924-310+(330+868)i}{784-100(-1)}$$

$$=\frac{614+1198i}{884}=\frac{307}{442}+\frac{599}{442}i$$

13. Find the solution of $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$ over complex numbers.

Ans:

Consider, $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

$$D=(-\sqrt{2})^2-4\times\sqrt{3}\times3\sqrt{3}$$

$$=2-36=-34<0$$

$$\therefore \quad x=\frac{-(-\sqrt{2})\pm\sqrt{-34}}{2\sqrt{3}}$$

$$=\frac{\sqrt{2}\pm\sqrt{34}i}{2\sqrt{3}}$$

14. Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$.

Ans:

$$\text{Let } z=\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}=\frac{6+9i-4i+6}{2-i+4i+2}$$

$$=\frac{12+5i}{4+3i}=\left(\frac{12+5i}{4+3i}\right)\times\left(\frac{4-3i}{4-3i}\right)=\frac{48-36i+20i+15}{4^2-(3i)^2}=\frac{63-16i}{16+9}=\frac{63}{25}-\frac{16}{25}i$$

$$\therefore \text{Conjugate of } z=\frac{63}{25}+\frac{16}{25}i.$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If α and β are different complex numbers with | β | = 1, then find $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$.

Ans:

Given $|\beta| = 1 \Rightarrow |\beta|^2 = 1$

$\Rightarrow \beta\bar{\beta} = 1$ ...(*i*)

Consider, $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\overline{\left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)}$

$= \frac{(\beta-\alpha)(\bar{\beta}-\bar{\alpha})}{(1-\bar{\alpha}\beta)(1-\alpha\bar{\beta})}$

$= \frac{\beta\bar{\beta}-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}}{1-\bar{\alpha}\beta-\alpha\bar{\beta}+\alpha\bar{\alpha}\beta\bar{\beta}}$

$= \frac{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}}{1-\bar{\alpha}\beta-\bar{\alpha}\beta+\alpha\bar{\alpha}}$ [Using (*i*)]

$\Rightarrow \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = 1 \Rightarrow \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$

16. Find the values of x and y, if $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$

Ans: Consider, $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$

$\Rightarrow (3-i)\{(1+i)x-2i\} + (3+i)\{(2-3i)y+i\} = i(3+i)(3-i)$

$\Rightarrow (3-i)(1+i)x - 2i(3-i) + (3+i)(2-3i)y + i(3+i) = i(9+1)$

$\Rightarrow (3+2i+1)x - 6i - 2 + (6-7i+3)y + 3i - 1 = 10i$

$\Rightarrow (4+2i)x + (9-7i)y = 3 + 13i$

$\Rightarrow (4x+9y) + (2x-7y)i = 3 + 13i$

$\Rightarrow 4x + 9y = 3$ ...(*i*)

and $2x - 7y = 13$ ...(*ii*)

Multiplying (*ii*) by 2 and subtracting from (*i*), we get

$9y + 14y = 3 - 26 \Rightarrow 23y = -23$

$\Rightarrow y = -1$

Substituting $y = -1$ in (*i*) we get

$4x - 9 = 3 \Rightarrow 4x = 12 \Rightarrow x = 3$

$\therefore x = 3, y = -1$

17. If x + iy = $\sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Ans:

$x+iy=\sqrt{\frac{a+ib}{c+id}} \Rightarrow (x+iy)^2 = \frac{a+ib}{c+id}$

$\Rightarrow |x+iy|^2 = \frac{|a+ib|}{|c+id|}$

$\Rightarrow \left(\sqrt{x^2+y^2}\right)^2 = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

SECTION – D

Questions 18 carry 5 marks.

18. (a) If $\frac{(1+i)^2}{2-i} = x + iy$, then find the value of $x + y$.

(b) If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, then find (x, y).

Ans: (a)

$$\frac{(1+i)^2}{2-i} = x + iy, \quad \frac{1+i^2+2i}{2-i} = x+iy$$

$$\Rightarrow \frac{2i(2+i)}{4+1} = x + iy$$

$$\Rightarrow \frac{4i-2}{5} = x+iy \Rightarrow -\frac{2}{5}+\frac{4}{5}i = x+iy$$

$$\Rightarrow \quad x = -\frac{2}{5} \text{ and } y = \frac{4}{5}$$

(b) Consider, $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$

$$\Rightarrow \left[\frac{(1+i)^2}{2}\right]^3 - \left[\frac{(1-i)^2}{2}\right]^3 = x + iy$$

$\Rightarrow (i)^3 - (-i)^3 = x + iy$

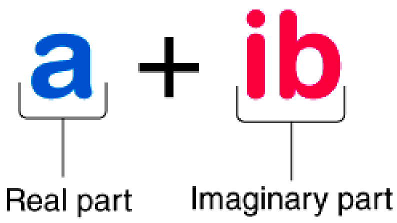
$\Rightarrow -i - i = x + iy$

$\Rightarrow 0 - 2i = x + iy$, $x = 0$, $y = -2$; $(0, -2)$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Complex numbers are the numbers that are expressed in the form of a+ib where, a, b are real numbers and 'i' is an imaginary number called "iota". The value of i = (√-1). For example, 2 + 3i is a complex number, where 2 is a real number (Re) and 3i is an imaginary number (Im).
An imaginary number is usually represented by 'i' or 'j', which is equal to √-1. Therefore, the square of the imaginary number gives a negative value. Since, i = √-1, so, $i^2 = -1$
The complex number is basically the combination of a real number and an imaginary number. The complex number is in the form of **a + ib,** where a = real number and ib = imaginary number. Also, a,b belongs to real numbers and i = √-1.
Hence, a complex number is a simple representation of addition of two numbers, i.e., real number and an imaginary number. One part of it is purely real and the other part is purely imaginary.



Based on the above information, answer the following questions.

(a) Express $\frac{3-i}{5+6i}$ in the form $(a + ib)$. (2)

(b) Express $i^{15} - 3i^7 + 2i^{109} + i^{100} - i^{17} + 5i^3$. in the form $(a + ib)$. (2)

Ans: (a)

$$\frac{(3-i)(5-6i)}{25+36} = \frac{15-18i-5i+6i^2}{61} = \frac{9-23i}{61}$$

$$= \frac{9}{61} - \frac{23}{61}i$$

(b) $i^{15} - 3i^7 + 2i^{109} + i^{100} - i^{17} + 5i^3$
$= (i^2)^7.i - 3\ (i^2)^3\ i + 2\ (i^2)^{54}.\ i + (i^2)^{50} - (i^2)^8.\ i + 5\ (i^2).i$
$= (-1)^7.\ i - 3\ (-1)^3\ i + 2\ (-1)^{54}.\ i + (-1)^{50} - (-1)^8.\ i + 5\ (-1)i$
$= -i + 3i + 2i + 1 - i - 5i = 1 - 2i$

20. A conjugate of a complex number is another complex number that has the same real part as the original complex number, and the imaginary part has the same magnitude but opposite sign. If we multiply a complex number with its conjugate, we get a real number.

A complex number z is purely real if and only if $\bar{z}$ = z and is purely imaginary if and only if $\bar{z}$ = -z.
Based on the above information, answer the following questions.
(a) Find the conjugate of the following : $(2 + i)^2$ (1)
(b) Find the multiplicative inverse of $(4 - 3i)$. (1)
(c) Express $(3 + 4i)\ (6 - 3i)\ (5 + i)$ in the form $(a + ib)$. (2)
Ans: (a)

$$\overline{(2+i)^2} = \overline{(4+i^2+4i)} = \overline{3+4i} = 3-4i$$

(b) Multiplicative inverse of $(4 - 3i) = \dfrac{1}{4-3i}$

$$= \frac{4+3i}{16+9} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

(c) $(3 + 4i)(6 - 3i)(5 + i)$
$= (3 + 4i)\ (30 + 6i - 15i - 3i^2)$
$= (3 + 4i)(33 - 9i) = 99 - 27i + 132i - 36i^2$
$= 135 + 105i$