PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD–32

PRACTICE PAPER (2023-24)

CHAPTER 03 TRIGONOMETRIC FUNCTIONS (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**

CLASS : XI | **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Angle formed by the large hand of a clock in 20 minutes is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

Ans: (d) $\frac{2\pi}{3}$, as angle formed in 20 minutes = 120° = $\frac{2\pi}{3}$.

2. If sin θ + cosec θ = 2, then $\sin^2 \theta + \text{cosec}^2 \theta$ is equal to

(a) 1 (b) 4 (c) 2 (d) 3

Ans: (c) 2

sin θ + cosec θ = 2

On squaring both sides, we get

$\sin^2 \theta + \text{cosec}^2 \theta + 2 \sin \theta \cdot \text{cosec}\, \theta = 4$

$\Rightarrow \sin^2 \theta + \text{cosec}^2 \theta = 4 - 2 = 2 \;\{\because \sin \theta \cdot \text{cosec}\, \theta = 1\}$

3. If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$, then the value of θ + ϕ is

(a) $\frac{\pi}{6}$ (b) π (c) 0 (d) $\frac{\pi}{4}$

Ans: (d) $\frac{\pi}{4}$

we know $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

So $\tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \cdot \tan \phi}$

$\Rightarrow \tan(\theta + \phi) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{5/6}{5/6} = 1 = \tan\frac{\pi}{4} \Rightarrow \theta + \phi = \frac{\pi}{4}$

4. The value of sin (45° + θ) – cos (45° – θ) is

(a) 2 cos θ (b) 2 sin θ (c) 1 (d) 0

Ans: (d) 0

sin (45° + θ) – cos (45° – θ)

= sin (45° + θ) – [sin{90° – (45° – θ)}]

= sin (45° + θ) – sin(45° + θ) = 0

5. The value of tan 75° – cot 75° is

(a) $2\sqrt{3}$ (b) $2 + \sqrt{3}$ (c) $2 - \sqrt{3}$ (d) 1

Ans: (a) $2\sqrt{3}$

$\tan 75° - \cot 75° = \dfrac{\sin 75°}{\cos 75°} - \dfrac{\cos 75°}{\sin 75°}$

$= \dfrac{\sin^2 75° - \cos^2 75°}{\sin 75° \cdot \cos 75°} = \dfrac{-(\cos^2 75° - \sin^2 75°)}{\frac{1}{2}(2\sin 75° \cdot \cos 75°)}$

$= \dfrac{-2\cos 2\times 75°}{\sin 2\times 75°} = \dfrac{-2\cos(180° - 30°)}{\sin(180° - 30°)}$

$= \dfrac{-2(-\cos 30°)}{\sin 30°} = \dfrac{2\cos 30°}{\sin 30°} = \dfrac{2\times\sqrt{3}/2}{1/2} = 2\sqrt{3}$

6. The minimum value of 3 cos x + 4 sin x + 8 is

(a) 5 (b) 9 (c) 7 (d) 3

Ans: (d) 3

let y = 3 cos x + 4 sin x + 8

$\Rightarrow y = 5\left[\dfrac{3}{5}\cos x + \dfrac{4}{5}\sin x\right] + 8$

$= 5\cos(x - a) + 8$, when $\cos a = \dfrac{3}{5}$

Also, $-1 \le \cos(x - a) \le 1$

$\Rightarrow -5 \le 5\cos(x - a) \le 5$

$\Rightarrow 3 \le 5\cos(x - a) + 8 \le 13$

$\Rightarrow 3 \le y \le 13$

So, minimum value is 3.

7. $\cos 2\theta \cos 2\phi + \sin 2(\theta - \phi) - \sin^2(\theta + \phi)$ is equal to

(a) $\sin 2(\theta + \phi)$ (b) $\cos 2(\theta + \phi)$ (c) $\sin 2(\theta - \phi)$ (d) $\cos 2(\theta + \phi)$

Ans: (b) $\cos 2(\theta + \phi)$

$\cos 2\theta \cdot \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$= \cos 2\theta \cos 2\phi + \sin(\theta - \phi + \theta + \phi)\sin(\theta - \phi - \theta - \phi)$

$= \cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi$

$= \cos(2\theta + 2\phi) = \cos 2(\theta + \phi)$

8. If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is

(a) 1 (b) $\dfrac{1}{2}$ (c) 0 (d) –1

Ans: (c) 0

$\sin\theta + \cos\theta = 1$

On squaring both sides, we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 1 \Rightarrow 2\sin\theta\cos\theta = 0$

$\Rightarrow \sin 2\theta = 0$ $\{\because \sin 2\theta = 2\sin\theta \times \cos\theta\}$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. Assertion (A): The ratio of the radii of two circles at the centres of which two equal arcs subtend angles of 30° and 70° is 21:10.
Reason (R): Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.
Ans: (d) A is false but R is true.

10. Assertion (A): cosec x is negative in third and fourth quadrants.
Reason (R): cot x decreases from 0 to -∞ in first quadrant and increases from 0 to ∞ in third quadrant.
Ans: (c) A is true but R is false.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If $\alpha + \beta = \dfrac{\pi}{4}$, then find the value of $(1 + \tan\alpha)(1 + \tan\beta)$

Ans:

$\alpha + \beta = \dfrac{\pi}{4}$ (given)

$\Rightarrow \tan(\alpha + \beta) = \tan\dfrac{\pi}{4} \Rightarrow \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$

$\Rightarrow \tan\alpha + \tan\beta + \tan\alpha\tan\beta = 1$ ...(*i*)

Now $(1 + \tan\alpha)(1 + \tan\beta)$

$= 1 + \tan\alpha + \tan\beta + \tan\alpha\tan\beta$

$= 1 + 1 = 2$ {using (*i*)}

12. If $\sin x = \dfrac{3}{5}, \dfrac{\pi}{2} < x < \pi$, then find the value of cos *x*, tan *x*, sec *x* and cot *x*.

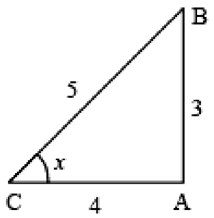
Ans:

$\sin x = \dfrac{3}{5}$ and $\dfrac{\pi}{2} < x < \pi$

$\Rightarrow$ *x* belongs to second quadrant.

$\cos x = -\dfrac{4}{5}$, $\tan x = -\dfrac{3}{4}$, $\sec x = -\dfrac{5}{4}$

$\cot x = -\dfrac{4}{3}$



13. A wheel makes 270 revolutions in one minute. Through how many radians does it turn in one second ?

Ans: Number of revolutions in one second = $\dfrac{270}{60} = \dfrac{9}{2}$

Angle traced in one revolution = 2π radians.

Angle traced in one second = $\dfrac{9}{2} \times 2\pi$ radius = 9π raidus

14. Prove: $\cos 24° + \cos 55° + \cos 125° + \cos 204° + \cos 300° = \dfrac{1}{2}$.

Ans: $\cos 24° + \cos 55° + \cos(180° - 55°) + \cos(180° + 24°) + \cos(360° - 60°)$
$\cos 24° + \cos 55° - \cos 55° - \cos 24° + \cos 60°$

$= \cos 60° = \dfrac{1}{2}$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If $\sin x = \frac{3}{5}$, $\cos y = \frac{-12}{13}$ and x, y both lie in the second quadrant, find the values of $\sin(x + y)$

Ans: Given, $\sin x = \frac{3}{5}$, $\cos y = \frac{-12}{13}$ and x, y both lie in the second quadrant.

We know that $\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} \Rightarrow \cos x = \pm\frac{4}{5}$

Since, x lies in 2nd quadrant, $\cos x$ is (–ve).

$\therefore \cos x = \frac{4}{5}$

Also, $\sin^2 y = 1 - \cos^2 y = 1 - \left(\frac{-12}{13}\right)^2 = \frac{25}{169} \Rightarrow \sin y = \pm\frac{5}{13}$

Since, y lies in 2nd quadrant, $\sin y$ is (+ve)

$\therefore \sin y = \frac{5}{13}$

$\sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y$

$= \frac{3}{5} \times \left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right) \times \frac{5}{13} = \frac{-36}{65} - \frac{20}{65} = \frac{-56}{65}$

16. Prove that $\frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 4x \sin 3x} = \tan 2x$

Ans:

$$\text{LHS} = \frac{2\sin 8x \cos x - 2\sin 6x \cos 3x}{2\cos 2x \cos x - 2\sin 4x \sin 3x}$$

$$= \frac{[\sin(8x + x) + \sin(8x - x)] - [\sin(6x + 3x) + \sin(6x - 3x)]}{[\cos(2x + x) + \cos(2x - x)] - [\cos(4x - 3x) - \cos(4x + 3x)]}$$

$[\because 2\sin A \cos B = \sin(A + B) + \sin(A - B), \text{ etc.}]$

$$= \frac{(\sin 9x + \sin 7x) - (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) - (\cos x - \cos 7x)}$$

$$= \frac{(\sin 7x - \sin 3x)}{(\cos 3x + \cos 7x)}$$

$$= \frac{2\cos\left(\frac{7x + 3x}{2}\right)\sin\left(\frac{7x - 3x}{2}\right)}{2\cos\left(\frac{7x + 3x}{2}\right)\cos\left(\frac{7x - 3x}{2}\right)} \quad \left[\because \sin C - \sin D = 2\cos\frac{(C + D)}{2}\sin\frac{(C - D)}{2}\right]$$

$$= \frac{\cos 5x \sin 2x}{\cos 5x \cos 2x} = \frac{\sin 2x}{\cos 2x} = \tan 2x = \text{RHS}.$$

17. Prove that, $\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$.

Ans: LHS $= \frac{1}{2}\cos 20° \cos(60° - 20°)\cos(60° + 20°)$

$= \frac{1}{2}\cos 20° [\cos^2 20° - \sin^2 60°]$

$= \frac{1}{2}\cos 20° [\cos^2 20° - \frac{3}{4}]$

$= \frac{1}{8}$ [4 $\cos^3 20° - 3\cos 20°$]

$= \frac{1}{8}\cos 60° = \frac{1}{16}$ = RHS [using $\cos 3x = 4\cos^3 x - 3\cos x$]

SECTION – D

Questions 18 carry 5 marks.

18. In a triangle ABC, prove that, $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2\left(1+\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$

Ans:

$\text{LHS} = \cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}$

$=\frac{1}{2}(1+\cos A)+\frac{1}{2}(1+\cos B)+\frac{1}{2}(1+\cos C)=\frac{3}{2}+\frac{1}{2}(\cos A+\cos B+\cos C)$

$=\frac{3}{2}+\frac{1}{2}\left[2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)+\cos C\right]$

$=\frac{3}{2}+\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)+\frac{1}{2}\left(1-2\sin^2\frac{C}{2}\right)\ \left[\because \left(\frac{A+B}{2}\right)=\left(\frac{\pi}{2}-\frac{C}{2}\right) \text{ and } \cos C=1-2\sin^2\frac{C}{2}\right]$

$=2+\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-\sin^2\frac{C}{2}\ =2+\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\sin\frac{C}{2}\right]$

$=2+\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\sin\left\{\frac{\pi}{2}-\left(\frac{A+B}{2}\right)\right\}\right]$

$=2+\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right]$

$=2+\sin\frac{C}{2}\left(2\sin\frac{A}{2}\sin\frac{B}{2}\right)\ =2\left(1+\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)=\text{RHS.}$

OR

If α, β are the roots of $a\cos\theta + b\sin\theta = c$, show that, $\cos(\alpha+\beta) = \frac{a^2-b^2}{a^2+b^2}$.

Ans: α, β are roots of the equation $a\cos\theta + b\sin\theta = c$

$\Rightarrow a\cos\alpha + b\sin\theta = c$ and $a\cos\alpha + b\sin\beta = c$

On subtracting we get,

$a(\cos\alpha - \cos\beta) + b(\sin\alpha - \sin\beta) = 0$

$\Rightarrow -2a\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}+2b\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}=0$

$\Rightarrow \sin\frac{\alpha-\beta}{2}\left(-2a\sin\frac{\alpha+\beta}{2}+2b\cos\frac{\alpha+\beta}{2}\right)=0\ \Rightarrow -2a\sin\frac{\alpha+\beta}{2}+2b\cos\frac{\alpha+\beta}{2}=0$

$\Rightarrow 2a\sin\frac{\alpha+\beta}{2}=2b\cos\frac{\alpha+\beta}{2}\ \Rightarrow \tan\frac{\alpha+\beta}{2}=\frac{b}{a}$ …. (i)

Also, $\cos(\alpha+\beta)=\dfrac{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)}$

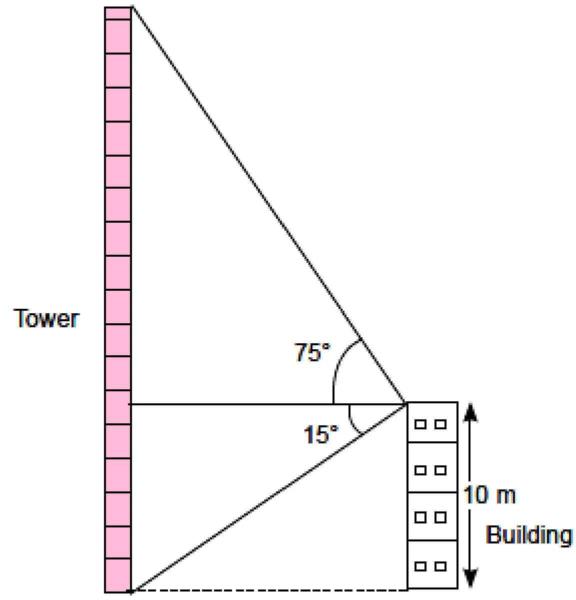
Substitute value from (*i*), we get

$\cos(\alpha+\beta) = \dfrac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}}\ \frac{a^2-b^2}{a^2+b^2}$.

SECTION – E (Case Study Based Questions)

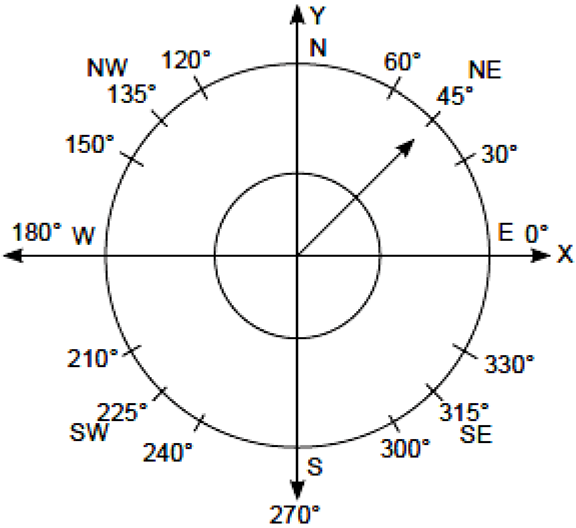
Questions 19 to 20 carry 4 marks each.

19. From the top of a tower of 10 m high building the angle of elevation of top of a tower is 75° and the angle of depression of foot of the tower is 15°. If the tower and building are on the same horizontal surfaces.



(i) Find the value of tan 15°. (2)
(ii) Find the value of cos 75°. (2)
Ans: (i) By the trigonometry formula, we know, tan (A – B) = (tan A – tan B) /(1 + tan A tan B)
Therefore, we can write, tan(45 – 30)° = tan 45° – tan 30°/1+tan 45° tan 30°
Now putting the values of tan 45° and tan 30° from the table we get;
tan(45 – 30)° = (1 – 1/√3)/ (1 + 1.1/√3)
tan (15°) = √3 – 1/ √3 + 1
Hence, the value of tan (15°) is √3 – 1/√3 + 1.
(ii) Using the formula for cos(A + B) = cos(A)cos(B) – sin(A)sin(B),
we can find the value of cos 75°.
cos(75°) = cos(30° + 45°)
Now, we know that cos 30° = √3/2, cos 45° = √2/2, sin 30° = 1/2, and sin 45° = √2/2 from the special values of trigonometric functions.
cos(75°) = cos(30°)cos(45°) – sin(30°)sin(45°)
Substituting the known values:
cos(75°) = (√3)/2)(√2)/2) – (1/2)(√2)/2)
Simplifying:
cos(75°) = (√6)/4) – (√2)/4)
Combining the terms:
cos(75°) = (√6 – √2))/4 = (√3-1) / 2√2

20. The below figure shows the compass. The East direction is along the positive X-axis (0° angle) and North direction is along the +ve Y-axis (90° angles). Initially the pointer is pointed towards North-East direction. Pointer is deflected in a magnetic field by some angle.



On the basis of above answer the following.

(i) If pointer move in anticlockwise direction by an angle of 90°, then find the value of sine of angle made by pointer from East direction. (1)

(ii) If pointer moves an angle of 165° from its initial position in anticlockwise direction, then find the value of cosine of angle made by pointer from East direction. (1)

(iii)If the sine and cosine of angle made by pointer with East direction is $-\frac{1}{\sqrt{2}}$ then find where the pointer pointed? (1)

(iv) How much angle will pointer move in anticlock wise direction if tangent of angle made by pointer with x-axis is – 1? (1)

Ans: (i) Angle made by pointer with East direction = 45° + 90° = 135°

$\therefore \sin 135° = \sin (180° - 45°) = \sin 45° = \frac{1}{\sqrt{2}}$

(ii) Angle made by pointer with East direction = 45° + 165° = 210°

$\cos 210° \quad = \cos (180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(iii) sine and cosine both are – ve so quadrant is III and we known that $\sin 45° = \frac{1}{\sqrt{2}}$. Given that,

$\sin \theta \quad = -\frac{1}{\sqrt{2}}$

$\Rightarrow \sin \theta = \sin (180° + 45°)$

$\Rightarrow \theta = 225°$

⇒ South West direction

(iv) If $\tan \theta = -1$

$\theta = 135°$ or $315°$

Initially the pointer is at 45°.

So angle moved by pointer is

$= 135° - 45° = 90°$

Or $315° - 45° = 270°$

$\Rightarrow \quad 90°$ or $270°$