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PRACTICE PAPER (2023-24)

CHAPTER 02 RELATIONS AND FUNCTIONS (ANSWERS)

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**

CLASS : XI — **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If A × B = {(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)}, find A and B, then set B is

(a) {a} (b) {a, b} (c) {1, 2} (d) {1, 2, 3}

Ans: (d) {1, 2, 3}

First entry ∈ set A and second entry ∈ set B

∴ A = {a, b}, B = {1, 2, 3}

2. Range of the function f (x) = $\frac{x}{x+2}$ is

(a) R (b) R – {2} (c) R – {1} (d) R – {–2}

Ans: (c) R – {1}

$$y = \frac{x}{x+2} \Rightarrow xy + 2y = x$$

$$\Rightarrow 2y = x(1-y) \Rightarrow x = \frac{2y}{1-y}$$

$$\therefore\ y \neq 1, \text{Range} = R - \{1\}$$

3. If n(A) = 3, n(B) = 2, then number of non empty relations from set A to set B are

(a) 8 (b) 4 (c) 64 (d) 63

Ans: (d) 63, as n(A × B) = 6

Total relations = 2^6 = 64

Total non-empty relations = 64 – 1 = 63

4. Range of the function f (x) = $\frac{x+4}{|x+4|}$ is

(a) {4} (b) {–4} (c) {–1, 1} (d) any real number

Ans: (c) {–1, 1}

$$|x+4| = \begin{cases} x+4, & x \geq -4 \\ -(x+4), & x < -4 \end{cases}$$

5. If $[x]^2 - 5[x] + 6 = 0$, where [ ] denote the greatest integer function, then

(a) x ∈ [3, 4) (b) x ∈ [2, 3) (c) x ∈ [2, 3) (d) x ∈ [2, 4)

Ans: (d) x ∈ [2, 4), we have $[x]^2 - 5[x] + 6 = 0$

$\Rightarrow [x]^2 - 3[x] - 2[x] + 6 = 0$

$\Rightarrow [x]([x] - 3) - 2([x] - 3) = 0$

$\Rightarrow ([x] - 2)([x] - 3) = 0 \Rightarrow [x] - 2 = 0$ or $[x] - 3 = 0$

$\Rightarrow [x] = 2$ or $[x] = 3 \Rightarrow x \in [2, 3)$ or $x \in [3, 4) \Rightarrow x \in [2, 4)$

6. Domain of $\sqrt{a^2 - x^2}$ (a > 0) is

(a) (–a, a) (b) [–a, a] (c) [0, a] (d) (–a, 0]

Ans: (b) [–a, a], let y = $\sqrt{a^2 - x^2}$ the function y is defined if

$a^2 - x^2 \geq 0 \Rightarrow x^2 - a^2 \leq 0$ or $x^2 \leq a^2$

$-a \leq x \leq a$

So, domain of y = [–a, a]

7. Given set A = {1, 2, 3, ..., 10}. Relation R is defined in set A as R = {(a, b) ∈ A × A : a = 2b}. Then range of relation R is

(a) {2, 4, 6, 8, 10} (b) {1, 3, 5, 7, 9}

(c) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5)} (d) {1, 2, 3, 4, 5}

Ans: (d) {1, 2, 3, 4, 5}, as R = {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5)}

8. Let n(A) = m and n(B) = n. Then the total number of non-empty relations that can be defined from A to B is

(a) m^n (b) $n^m - 1$ (c) mn – 1 (d) $2^{mn} - 1$

Ans: (d) $2^{mn} - 1$, as n(A) = m, n(B) = n ⇒ n(A × B) = mn

So, number of relations = 2^{mn} including void relation f.

Number of non-empty relations = $2^{nm} - 1$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. Assertion (A): Let A = {1, 2} and B = {3, 4}. Then, number of relations from A to B is 16.

Reason (R): If n(A) = p and n(B) = q, then number of relations is 2^{pq}.

Ans: (a) Both A and R are true and R is the correct explanation of A.

10. Assertion (A): The domain of the relation R = {(x + 2, x + 4) : x ∈ N, x < 8} is {3, 4, 5, 6, 7, 8, 9).

Reason (R): The range of the relation R = {(x + 2, x + 4) : x ∈ N, x < 8} is {1, 2, 3, 4, 5, 6, 7}.

Ans: (c) A is true but R is false.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Let f : R → R be given by $f(x) = x^2 + 3$ Find (*i*) {x : $f(x)$ = 28} (*ii*) The pre-images of 39 and 2 under 'f'.

Ans: (*i*) $28 = x^2 + 3 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

(*ii*) $39 = x^2 + 3 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$;

$2 = x^2 + 3 \Rightarrow x^2 = -1$, not possible

12. Determine the domain and range of the relation R defined by R = {(x + 1, x + 5) : x ∈ (0, 1, 2, 3, 4, 5)}

Ans: Relation R is {(1, 5), (2, 6), (3, 7), (4, 8), (5, 9), (6, 10)}

Domain of R = {1, 2, 3, 4, 5, 6} ; Range of R = {5, 6, 7, 8, 9, 10}

13. Find the domain of each of the following functions given by : $f(x) = \frac{x^3 - x + 3}{x^2 - 1}$

Ans: $f(x) = \frac{x^3 - x + 3}{x^2 - 1}$,

For domain, $x^2 - 1 \neq 0 \Rightarrow x \neq \pm 1$

$\therefore$ Domain = R – {–1, 1}

14. Find the range of the following functions given by : $f(x) = \frac{|x - 4|}{x - 4}$

Ans: $y = \frac{(x-4)}{(x-4)} or \frac{-(x-4)}{(x-4)}$, i.e. 1 or –1

$\therefore$ range {–1, 1}

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the domain and the range of the function : $f(x) = \sqrt{x^2 - 4}$

Ans: Given, $f(x) = \sqrt{x^2 - 4}$; For D_f., $f(x)$ must be a real number.

$\Rightarrow \sqrt{x^2 - 4}$ must be a real number. $\Rightarrow x^2 - 4 \geq 0 \Rightarrow (x + 2)(x - 2) \geq 0$

$\Rightarrow$ Either $x \leq -2$ or $x \geq 2$. $\Rightarrow D_f = (-\infty, -2] \cup [2, \infty)$.

For R_f, let $y = \sqrt{x^2 - 4}$ ... (*i*)

As square root of a real number is always non-negative, $y \geq 0$.

On squaring (*i*), we get $y^2 = x^2 - 4 \Rightarrow x^2 = y^2 + 4$ but $x^2 \geq 0\ \forall\ x \in D_f$.

$\Rightarrow y^2 + 4 \geq 0 \Rightarrow y^2 \geq -4$, which is true $\forall\ y \in R$,

Also, $y \geq 0$. $\Rightarrow R_f = [0, \infty)$.

16. Find the domain and range of the real function $f(x) = \sqrt{9 - x^2}$

Ans: Given function is $f(x) = \sqrt{9 - x^2}$

For domain of 'f', $9 - x^2 \geq 0$

$\Rightarrow 9 \geq x^2 \Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3$

$\therefore$ Domain is $\{x \in R \mid -3 \leq x \leq 3\}$, i.e. [–3, 3]

For range : $f(x) = \sqrt{9 - x^2} \Rightarrow y = \sqrt{9 - x^2}$

$\sqrt{9 - x^2}$ is always +ve

$\Rightarrow y$ is always +ve.

$\Rightarrow y^2 = 9 - x^2 \Rightarrow x^2 = 9 - y^2$

$\Rightarrow x = \sqrt{9 - y^2}$

For x to exist $9 - y^2 \geq 0 \Rightarrow y^2 \leq 9 \Rightarrow -3 \leq y \leq 3$

As $y \geq 0$

$\therefore$ Range = [0, 3]

17. If A = $\{x : x \in W, x < 2\}$, B = $\{x : x \in N, 1 < x < 5\}$, C = {3, 5} find

(*i*) A × (B ∩ C) (*ii*) A × (B ∪ C)

Ans: A = $\{x : x \in W, x < 2\}$ = {0, 1},

B = $\{x : x \in N, 1 < x < 5\}$ = {2, 3, 4};

C = {3, 5}

(*i*) A × (B ∩ C) = {0, 1} × {3} = {(0, 3), (1,3)}

(*ii*) A × (B ∪ C) = {0, 1} × {2, 3, 4, 5}

= {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)}

SECTION – D

Questions 18 carry 5 marks.

18. (a) Relations R_1 and R_2 are defined on the set Z of integers as follows :
$(x, y) \in R_1 \Leftrightarrow x^2 + y^2 = 25$; $(y, x) \in R_2 \Leftrightarrow x^2 + y^2 = 25$
Express R_1 and R_2 as the sets of ordered pairs and hence find their respective domains.
(b) A relation R is defined from a set A = {2, 3, 4, 5} to a set B = {3, 6, 7, 10} as follows : $(x, y) \in R \Leftrightarrow x$ divides y. Express R as a set of ordered pairs and determine the domain and range of R.
Ans: (a) $(x, y) \in R \Leftrightarrow x^2 + y^2 = 25$
$\Leftrightarrow y = \pm\sqrt{25 - x^2}$
We observe that : $x = 0 \Rightarrow y = \pm 5$
$x = \pm 3 \Rightarrow y = \sqrt{25-9} = \pm 4,$
$x = \pm 4, y = \sqrt{25-16} = \pm 3$
$x = \pm 5, \Rightarrow y = \sqrt{25-25} = 0.$
R_1 = {(0, 5), (0, – 5), (3, 4), (– 3, 4), (3, – 4), (– 3, – 4), (4, 3), (– 4, 3), (4, – 3), (– 4, – 3), (5, 0), (– 5, 0)}
R_2 = {(5, 0), (– 5, 0), (4, 3), (4, – 3), (– 4, 3), (– 4, – 3), (3, 4), (3, – 4), (– 3, 4), (– 3, – 4), (0, 5), (0, – 5)}
Domain (R_1) = {0, 3, – 3, 4, – 4, 5, – 5} = Domain (R_2)
(b) $a|b$ stands for 'a divides b'. For the elements of the given sets A and B, we find that 2 |6, 2|10, 3|3, 3|6 and 5|10.
∴ (2, 6) ∈ R, (2, 10) ∈ R, (3, 3) ∈ R, (3, 6) ∈ R and (5, 10) ∈ R.
Thus, R = {(2, 6), (2, 10), (3, 3), (3, 6), (5, 10)}
Clearly, Domain (R) = {2, 3, 5} and range (R) = {3, 6, 10}.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Maths teacher started the lesson Relations and Functions in Class XI. He explained the following topics:
Ordered Pairs: The ordered pair of two elements a and b is denoted by (a, b) : a is first element (or first component) and b is second element (or second component).
Two ordered pairs are equal if their corresponding elements are equal. i.e., (a, b) = (c, d) ⇒ a = c and b = d
Cartesian Product of Two Sets: For two non-empty sets A and B, the cartesian product A x B is the set of all ordered pairs of elements from sets A and B.
In symbolic form, it can be written as A x B= {(a, b) : a ∈ A, b ∈ B}
Based on the above topics, answer the following questions.
(i) If (a – 3, b + 7) = (3, 7), then find the value of a and b
(ii) If (x + 6, y – 2) = (0, 6), then find the value of x and y
(iii) If (x + 2, 4) = (5, 2x + y), then find the value of x and y
(iv) Find x and y, if (x + 3, 5) = (6, 2x + y).
Ans:
(i) We know that, two ordered pairs are equal, if their corresponding elements are equal.
(a – 3, b + 7) = (3, 7)
⇒ a – 3 = 3 and b + 7 = 7 [equating corresponding elements]
⇒ a = 3 + 3 and b = 7 – 7 ⇒ a= 6 and b = 0
(ii) (x + 6, y – 2) = (0, 6)
⇒ x + 6 = 0 ⇒ x = -6 and y – 2 = 6 ⇒ y = 6 + 2 = 8
(iii) (x + 2, 4) = (5, 2x + y)
⇒ x + 2 =5 ⇒ x = 5 – 2 = 3 and 4 = 2x + y ⇒ 4 = 2 x 3 + y ⇒ y = 4 – 6 = -2
(iv) $x + 3 = 6, 2x + y = 5 \Rightarrow x = 3, y = 1$

20. Maths teacher explained the topics:

Method to Find the Sets When Cartesian Product is Given

For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say).

Number of Elements in Cartesian Product of Two Sets

If there are p elements in set A and q elements in set B, then there will be pq n(A) = p and n(B) = q, then n(A x B) = pq

Based on the above two topic, answer the following questions.

(i) If A x B = {(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)}. Then, find A and B

(ii) If the set A has 3 elements and set B has 4 elements, then find the number of elements in A x B

(iii) A and B are two sets given in such a way that A x B contains 6 elements. If three elements of A x B are (1, 3), (2, 5) and (3, 3), then find A, B

(iv) The cartesian product P x P has 16 elements among which are found (a, 1) and (b, 2). Then, find the set P

Ans: (i) Here, first element of each ordered pair of A x B gives the elements of set A and corresponding second element gives the elements of set B.

∴ A = {a, b} and B = {1,3,2}

(ii) Given, n(A) = 3 and n(B) = 4.

∴ The number of elements in A x B is n(A x B) = n(A) x n(B) = 3 × 4 = 12

(iii) A = {1, 2, 3} and B = {3, 5}

∵ A x B = {1, 2, 3} x {3, 5} = {(1,3), (1,5), (2,3), (2,5), (3,3), (3,5)}

(iv) Given $n(P \times P) = 16$

$\Rightarrow n(P).n(P) = 16 \Rightarrow n(P) = 4$

Now, as (a, 1) ∈ P

⇒ a ∈ P and 1 ∈ P

and as (b, 2) ∈ P

⇒ b ∈ P and 2 ∈ P

⇒ a, b, 1, 2 ∈ P

Hence P has exactly four elements.