PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD–32
PRACTICE PAPER 2 (2023-24)
INTRODUCTION TO 3-DIMENSIONAL GEOMETRY
(ANSWERS)

SUBJECT: MATHEMATICS — **MAX. MARKS : 40**

CLASS : XI — **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. L is the foot of the perpendicular drawn from a point P (6, 7, 8) on the XY-plane, then the coordinates of point L are

(a) (6, 0, 0) (b) (6, 7, 0) (c) (6, 0, 8) (d) None of these

Ans: (b) Since, L is the foot of perpendicular from P on the XY-plane, z-coordinate will be zero. Hence, coordinates of L are (6, 7, 0).

2. The point (− 2, −3, − 4) lies in the

(a) first octant (b) seventh octant (c) second octant (d) eight octant

Ans: (b) The point (− 2, − 3, − 4) lies in seventh octant.

3. The point on Y-axis which is at a distance $\sqrt{10}$ from the point (1, 2, 3), is

(a) (0, 2, 0) (b) (0, 0, 2) (c) (0, 0, 3) (d) None of these

Ans: (a) Let P be the point on Y-axis. Then, it is of the form P(0, y, 0). Since, the point (1, 2, 3) is at a distance $\sqrt{10}$ from (0, y , 0),

therefore $\sqrt{(1-0)^2+(2-y)^2+(3-0)^2}=\sqrt{10}$

$\Rightarrow y^2 - 4y + 4 = 0 \Rightarrow (y - 2)^2 = 0 \Rightarrow y = 2$

Hence, the required point is (0, 2, 0).

4. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of a is

(a) 5 (b) ± 5 (c) − 5 (d) None of these

Ans: (b) Given, $\sqrt{(a-0)^2+(0-1)^2+(1-2)^2}=\sqrt{27}$

$\Rightarrow 27 = a^2 + 1 + 1 \Rightarrow a^2 = 25 \Rightarrow a = \pm 5$

5. If the point A (3, 2, 2) and B (5, 5, 4) are equidistant from P, which is on X-axis, then the coordinates of P are

(a) (39/4, 2 , 0) (b) (49/4, 2 , 0) (c) (39/4, 0 , 0) (d) (49/4, 0 , 0)

Ans: (d) The point on the X-axis is of form P (x, 0, 0). Since, the points A and B are equidistant from P.

Therefore, $PA^2 = PB^2 \Rightarrow (x - 3)^2 + (0 - 2)^2 + (0 - 2)^2 = (x - 5)^2 + (0 - 5)^2 + (0 - 4)^2$

$\Rightarrow 4x = 25 + 25 + 16 - 17 = 49$

$\Rightarrow x = 49/4$

Therefore the required point is (49/4, 0 , 0)

6. The points (5, –1, 1), (7, – 4, 7), (1, – 6, 10) and (−1, − 3, 4) are

(a) the vertices of a rectangle (b) the vertices of a square
(c) the vertices of a rhombus (d) None of these

Ans: (c) Let A(5, − 1, 1), B(7, − 4, 7), C (1, − 6, 10) and D (− 1, − 3, 4) be the four points of a quadrilateral. Here,

$AB = \sqrt{4 + 9 + 36} = 7, \quad BC = \sqrt{36 + 4 + 9} = 7,$

$CD = \sqrt{4 + 9 + 36} = 7, \quad DA = \sqrt{36 + 4 + 9} = 7$

$AC = \sqrt{16 + 25 + 81} = \sqrt{122}$

$BD = \sqrt{64 + 1 + 9} = \sqrt{74}$

Note that AB = BC= CD =DA and AC¹BD. Therefore,
ABCD is a rhombus.

7. The coordinate of the point P which divides the line joining the points A(−2, 0, 6) and B(10, − 6, − 12) internally in the ratio 5 : 1.

(a) (8, 5, 9) (b) (–8, 5, 9) (c) (8, –5, –9) (d) None of these

Ans: (c) Let P(x, y, z) be the required point. Then, P divides AB in the ratio 5 : 1. So,

$$P(x, y, z) = \left(\frac{5\times10 + 1\times-2}{5+1}, \frac{5\times-6 + 1\times0}{5+1}, \frac{5\times-12 + 1\times6}{5+1}\right)$$

= (8, − 5, − 9)

8. The ratio in which the line joining (2, 4, 5) and (3, 5, –4) is divided by the YZ-plane, is

(a) 2 : 3 (b) 3 : 2 (c) –2 : 3 (d) 4 : –3

Ans: (c) Let the point R divides the line joining the points P(2, 4, 5) and Q(3, 5, − 4) in the ratio m : n. Then, the coordinates of R are

$$\left(\frac{3m + 2n}{m + n}, \frac{5m + 4n}{m + n}, \frac{-4m + 5n}{m + n}\right)$$

For YZ-plane, x-coordinates will be zero.

$$\therefore \quad \frac{3m + 2n}{m + n} = 0 \Rightarrow \frac{m}{n} = \frac{-2}{3}$$

Hence, m : n = − 2 : 3

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** The points A(3, –1, 2), B(1, 2, – 4), C(–1, 1, 2) and D(1, –2, 8) are the vertices of a parallelogram.

Reason (R): Coordinates of mid-point of a line joining the points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Ans: (a) Both A and R are true and R is the correct explanation of A.

$$\text{Mid-point of } AC = \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = (1, 0, 2)$$

$$\text{Mid-point of } BD = \left(\frac{1+1}{2}, \frac{2-2}{2}, \frac{-4+8}{2}\right) = (1, 0, 2)$$

$\because$ Mid-points of AC and BD coincides.

$\therefore$ $ABCD$ is a parallelogram.

10. Assertion (A): The distance between the points $(1 + \sqrt{11}, 0, 0)$ and $(1, -2, 3)$ is $2\sqrt{6}$ units.
Reason (R): Distance between any two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is,
$|AB| = \sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2 + (z_2 + z_1)^2}$
Ans: (c) A is true but R is false
Let $A \equiv (1+\sqrt{11}, 0, 0)$ and $B \equiv (1, -2, 3)$

$$\therefore \ AB = \sqrt{(1-1-\sqrt{11})^2 + (-2-0)^2 + (3-0)^2}$$

$$= \sqrt{11+4+9} = \sqrt{24} = 2\sqrt{6} \text{ units}$$

$\therefore$ Assertion is correct but Reason is wrong.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the point on y-axis which is equidistant from the point A(3, 2, 2) and B(5, 5, 4).
Ans: The point on the y-axis is of the form P(0, y, 0). Since the points A(3, 2, 2) and B (5, 5, 4) are equidistant from P.

$$\therefore \ PA = PB \quad \Rightarrow \quad PA^2 = PB^2$$

$$\Rightarrow (0-3)^2 + (y-2)^2 + (0-2)^2 = (0-5)^2 + (y-5)^2 + (0-4)^2$$

$$\Rightarrow 6x = 25 + 25 + 16 - 17 \ \ i.e., \ y = \frac{49}{6}$$

Thus, the point $P\left(0, \frac{49}{6}, 0\right)$, on the y-axis is equidistant from A and B.

12. Show that the points A(0, 7, 10), B(–1, 6, 6) and C(–4, 9, 6) form an isosceles right-angled triangle.
Ans: Given, A(0, 7, 10), B(–1, 6, 6) and C(–4, 9, 6)
$\therefore$ Using distance formula, we get,

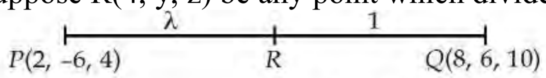
$$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} = \sqrt{36} = 6$$

Clearly, AB = BC and $AB^2 + BC^2 = 18 + 18 = 36 = AC^2$.
Hence, triangle ABC is an isosceles right-angled triangle.

13. A point R with x-coordinate 4 lies on the line segment joining the points P(2, –6, 4) and Q(8, 6, 10). Find the coordinates of the point R.
Ans: Suppose R(4, y, z) be any point which divides PQ in the ratio λ : 1 internally.



Then, the coordinates of R are $\left(\frac{8\lambda+2}{\lambda+1}, \frac{6\lambda-6}{\lambda+1}, \frac{10\lambda+4}{\lambda+1}\right)$

Since x-coordinate of R is 4.

$$\therefore \ \frac{8\lambda+2}{\lambda+1} = 4 \Rightarrow 8\lambda + 2 = 4\lambda + 4 \Rightarrow 4\lambda = 2 \Rightarrow \lambda = \frac{1}{2}.$$

$$\therefore \quad y = \frac{6\left(\frac{1}{2}\right) - 6}{\frac{1}{2}+1} = -2 \text{ and } z = \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1} = 6$$

Hence, the coordinates of R are (4, –2, 6).

14. Find the coordinates of the point which divides the join of P(2, –1, 4) and Q(4, 3, 2) in the ratio 2 : 3 externally.

Ans: Let R(x, y, z) be any point which divides PQ externally in the ratio 2 : 3, then

$$x = \frac{2\times 4 - 3\times 2}{2-3}, y = \frac{2\times 3 - 3\times(-1)}{2-3},$$

$$z = \frac{2\times 2 - 3\times 4}{2-3} \quad \left[\because\ (x,y,z) = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)\right]$$

$\Rightarrow x = -2, y = -9, z = 8$

So, the coordinates of R are (–2, –9, 8).

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the ratio in which the join of A(2, 1, 5) and B(3, 4, 3) is divided by the plane 2x + 2y − 2z = 1. Also, find the coordinates of the point of division.

Ans: Suppose the given plane intersects AB at a point C and let the required ratio be λ : 1.

Then, the coordinates of C are

$$\left(\frac{3\lambda + 2}{\lambda + 1}, \frac{4\lambda + 1}{\lambda + 1}, \frac{3\lambda + 5}{\lambda + 1}\right). \quad \ldots (1)$$

Since C lies on the plane $2x + 2y - 2z = 1$, this point must satisfy the equation of the plane.

$$\therefore \quad 2\left(\frac{3\lambda + 2}{\lambda + 1}\right) + 2\left(\frac{4\lambda + 1}{\lambda + 1}\right) - 2\left(\frac{3\lambda + 5}{\lambda + 1}\right) = 1 \quad \text{or} \quad \lambda = \frac{5}{7}.$$

So, the required ratio is $\frac{5}{7}$: 1, i.e., 5 : 7.

Putting $\lambda = \frac{5}{7}$ in (i), the required point of division is $C\left(\frac{29}{12}, \frac{9}{4}, \frac{25}{6}\right)$.

16. Find the points of trisection of the segment joining the points A(1, 0, –6) and B(–5, 9, 6).

Ans: Let P and Q be the points of trisection of the segment [AB], then P divides [AB] in the ratio 1 : 2 and Q divides [AB] in the ratio 2 : 1.

$$\therefore \quad P \equiv \left(\frac{1\times(-5) + 2\times 1}{1+2}, \frac{1\times 9 + 2\times 0}{1+2}, \frac{1\times 6 + 2\times(-6)}{1+2}\right),$$

i.e., P is (–1, 3, –2)

$$\text{and } Q \equiv \left(\frac{2\times(-5) + 1\times 1}{2+1}, \frac{2\times 9 + 1\times 0}{2+1}, \frac{2\times 6 + 1\times(-6)}{2+1}\right),$$

i.e., Q is (–3, 6, 2).

Hence, the required points of trisection are P(–1, 3, –2) and Q(–3, 6, 2).

17. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Ans: Let A(x, y, z) be any point which is equidistant from points B(1, 2, 3) and C(3, 2, –1).

It is given that $AB = AC$

$$\sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2} = \sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$$

$\Rightarrow \quad (x-1)^2 + (y-2)^2 + (z-3)^2$

$= (x-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow \quad x^2 + 1 - 2x + z^2 + 9 - 6z = x^2 + 9 - 6x + z^2 + 1 + 2z$

$\Rightarrow -2x - 6z + 10 = -6x + 2z + 10$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

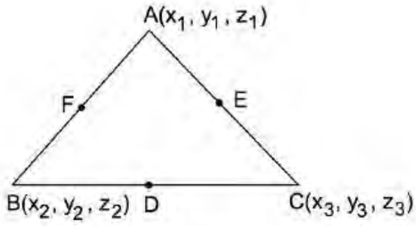
$\Rightarrow 4x - 8z = 0 \Rightarrow x - 2z = 0$

SECTION – D

Questions 18 carry 5 marks.

18. The midpoints of the sides of a triangle are (1, 5, −1), (0, 4, −2) and (2, 3, 4). Find its vertices.

Ans: Let A(x_1, y_1, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3) be the vertices of the given triangle, and let D(1, 5, −1), E(0, 4, − 2) and F(2, 3, 4) be the midpoints of the sides BC, CA and AB respectively. Then,



$$\frac{x_2+x_3}{2}=1;\ \frac{y_2+y_3}{2}=5;\ \frac{z_2+z_3}{2}=1;$$
$$\frac{x_3+x_1}{2}=0;\ \frac{y_3+y_1}{2}=4;\ \frac{z_3+z_1}{2}=-2;$$
$$\frac{x_1+x_2}{2}=2;\ \frac{y_1+y_2}{2}=3 \text{ and } \frac{z_1+z_2}{2}=4.$$

Thus, $x_2+x_3=2;\ x_3+x_1=0;\ x_1+x_2=4;$

$y_2+y_3=10;\ y_3+y_1=8;\ y_1+y_2=6;$

$z_2+z_3=-2;\ z_3+z_1=-4;\ z_1+z_2=8.$

Adding first three equations, we get

$2(x_1+x_2+x_3)=6$ or $x_1+x_2+x_3=3.$

Thus, $x_1=1,\ x_2=3$ and $x_3=-1.$

Adding next three equations, we get

$2(y_1+y_2+y_3)=24$ or $y_1+y_2+y_3=12.$

$\therefore\ y_1=2;\ y_2=4$ and $y_3=6.$

Adding last three equations, we get

$2(z_1+z_2+z_3)=2$ or $z_1+z_2+z_3=1.$

$\therefore\ z_1=3,\ z_2=5$ and $z_3=-7.$

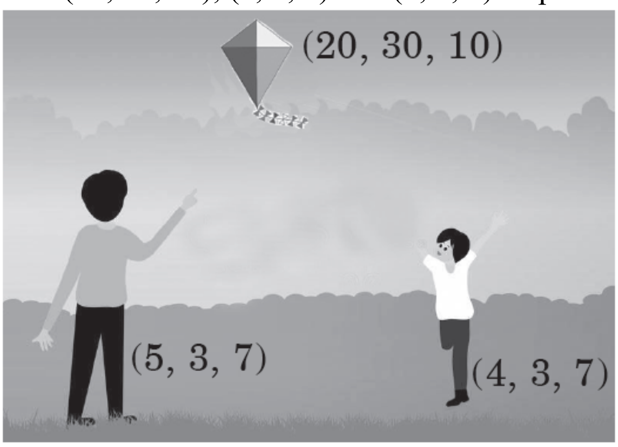
Hence, the vertices of the given triangle are

$A(1, 2, 3);\ B(3, 4, 5)$ and $C(-1, 6, -7).$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Raj and his father were walking in a large park. They saw a kite flying in the sky. The position of kite, Raj and Raj's father are at (20, 30, 10), (4, 3, 7) and (5, 3, 7) respectively.



On the basis of above information, answer the following:

(i) Find the distance between Raj and kite. Also find the distance between Raj's father and kite. (2)

(ii) Find the form of the co-ordinates of points in the XY-plane (1)

(iii) If co-ordinates of kite, Raj and Raj's father form a triangle, then find the centroid of it. (1)

Ans: (i) Distance between Raj and kite

$= \sqrt{(20-4)^2 + (30-3)^2 + (10-7)^2}$

$= \sqrt{16^2 + 27^2 + 3^2}$

$= \sqrt{256 + 729 + 9} = \sqrt{994}$

Distance between Raj's father and kite

$= \sqrt{(20-5)^2 + (30-3)^2 + (10-7)^2}$

$= \sqrt{15^2 + 27^2 + 3^2}$

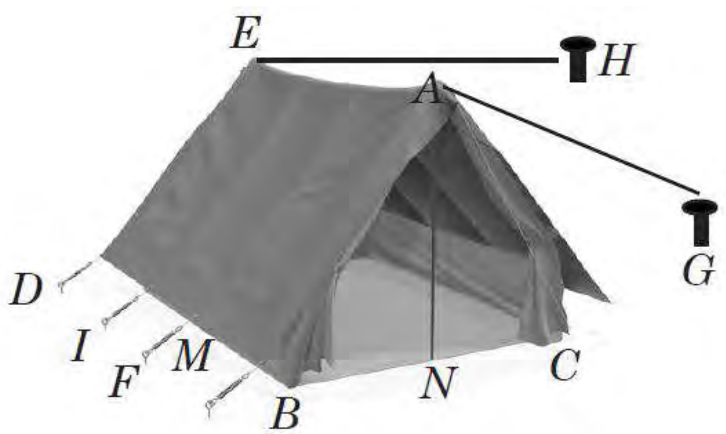
$= \sqrt{225 + 729 + 9} = \sqrt{963}$

(ii) For XY-plane, z = 0 ⇒ The co-ordinates are of the form (x, y, 0).

(iii)

$$\text{Centroid} = \left(\frac{20+4+5}{3}, \frac{30+3+3}{3}, \frac{10+7+7}{3}\right)$$
$$= (9.67, 12, 8)$$

20. Deepak and his friends went for camping for 2 or 3 days. There they set up a tent which is triangular in shape. The vertices of the tent are A(4, 5, 9), B(3, 2, 5), C(5, 2, 5), D(–3, 2, –5) and E(–4, 5, –9) respectively. The vertex A is tied up by the rope at the ends F and G and the vertex E is tied up at the ends I and H.



On the basis of above information, answer the following:

(i) If M denotes the position of their bags inside the tent and it is just in middle of the vertices B and D, then find the coordinates of M. (1)

(ii) Find the length AE. (1)

(iii) If the length of the rope by which E is tied up with H is 5√2 units, then the equation denotes the set of point of H (2)

Ans: (i) As, M is the middle point of B(3, 2, 5) and D(–3, 2, –5)

∴ The coordinates of *M* are

$$\left(\frac{3-3}{2}, \frac{2+2}{2}, \frac{5-5}{2}\right) = (0,2,0)$$

(ii) The length AE is

$= \sqrt{(-4-4)^2 + (5-5)^2 + (-9-9)^2}$

$= \sqrt{64 + 0 + 324}$

$= \sqrt{388} = 2\sqrt{97}$ units .

(iii) As, the distance of H(*x*, *y*, *z*) from E(–4, 5, –9) is 5√2 units .

∴ EH = 5√2

$\Rightarrow \sqrt{(x+4)^2+(y-5)^2+(z+9)^2} = 5\sqrt{2}$

Squaring both sides, we get

$(x + 4)^2 + (y - 5)^2 + (z + 9)^2 = 25 \times 2$

$x^2 + y^2 + z^2 + 8x - 10y + 18z + 122 = 50$

$\Rightarrow \quad x^2 + y^2 + z^2 + 8x - 10y + 18z + 72 = 0$