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PRACTICE PAPER 1 (2023-24)
INTRODUCTION TO 3-DIMENSIONAL GEOMETRY
(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XI **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. What is the perpendicular distance of the point (6, 7, 8) from xy-plane?
(a) 8 units (b) 7 units (c) 6 units (d) 5 units
Ans: (a) 8 units
Let L be the foot of perpendicular drawn from the point P(6, 7, 8) on the xy-plane.
Then coordinates of L = (6, 7, 0)
$\therefore$ Required distance $PL = \sqrt{(6-6)^2 + (7-7)^2 + (8-0)^2} = 8\,units.$

2. Find the distance between the points P(1, –3, 4) and Q(–4, 1, 2).
(a) $\sqrt{5}$ units (b) $5\sqrt{3}$ units (c) $3\sqrt{5}$ units (d) $2\sqrt{2}$ units
Ans: (c) $3\sqrt{5}$ units
The distance between the points P(1, –3, 4) and Q(–4, 1, 2) is PQ
$= \sqrt{(-4-1)^2 + (1+3)^2 + (2-4)^2} = \sqrt{25+16+4} = \sqrt{45} = 3\sqrt{5}$ units

3. Find the equation of the set of the points P such that its distances from the points A(3, 4, –5) and B(–2, 1, 4) are equal.
(a) 10x + 6y – 18z – 29 = 0 (b) 10x + 18y – 6z – 29 = 0
(c) 5x + 3y – 9z – 29 = 0 (d) 10x + 6y – 18z – 45 = 0
Ans: (a) 10x + 6y – 18z – 29 = 0
Let P(x, y, z) be any point such that $PA = PB \Rightarrow PA^2 = PB^2$
Now $(x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$
$\Rightarrow 10x + 6y - 18z - 29 = 0.$

4. M is the foot of the perpendicular drawn from the point A(6, 7, 8) on the yz-plane. What are the coordinates of point M?
(a) (6, 0, 0) (b) (6, 7, 0) (c) (6, 0, 8) (d) (0, 7, 8)
Ans: (d) (0, 7, 8)
Since M is the foot of perpendicular from A on the yz-plane, so its x-coordinate is zero. Hence, coordinates of M are (0, 7, 8).

5. L is the foot of the perpendicular drawn from a point (3, 5, 6) on x-axis. The coordinates of L are
(a) (3, 0, 0) (b) (0, 6, 0) (c) (0, 0, 5) (d) (0, 5, 6)
Ans: (a) (3, 0, 0)
Since L is the foot of perpendicular from point (3, 5, 6) on x-axis. So, its y and z-coordinates are zero. Hence, the coordinates of L are (3, 0, 0).

6. Determine the point in yz-plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

(a) (0, 1, 3) (b) (1, 0, 3) (c) (0, 2, 3) (d) (0, 3, 1)

Ans: (a) (0, 1, 3)

Since x-coordinate of every point in yz-plane is zero.

Let P(0, y, z) be a point in the yz-plane such that PA = PB = PC.

Now $PA^2 = PB^2$

$\Rightarrow (0-2)^2 + (y-0)^2 + (z-3)^2 = (0-0)^2 + (y-3)^2 + (z-2)^2$

$\Rightarrow z - 3y = 0$ ...(i)

and $PB^2 = PC^2 \Rightarrow y^2 + 9 - 6y + z^2 + 4 - 4z = y^2 + z^2 + 1 - 2z$

$\Rightarrow 3y + z = 6$ ...(ii)

Solving (i) and (ii), we get y = 1, z = 3

Hence, the coordinates of the point P are (0, 1, 3).

7. The two vertices of a triangle are (4, 2, 1) and (5, 1, 4). If the centroid is (5, 2, 3), then the third vertex is

(a) (3, 4, 5) (b) (6, 2, 3) (c) (6, 3, 2) (d) (6, 3, 4)

Ans: (d) (6, 3, 4)

Let the third vertex be (α, β, γ).

$\therefore 5 = \frac{4+5+\alpha}{3} \Rightarrow \alpha = 6$, $2 = \frac{2+1+\beta}{3} \Rightarrow \beta = 3$ and $3 = \frac{1+4+\gamma}{3} \Rightarrow \gamma = 4$

∴ The third vertex is (6, 3, 4).

8. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of a is

(a) 5 (b) ±5 (c) –5 (d) None of these

Ans: (b) ±5

Distance between the points (a, 0, 1) and (0, 1, 2) = $\sqrt{27}$

$\Rightarrow \sqrt{(a-0)^2 + (0-1)^2 + (1-2)^2} = \sqrt{27}$

Squaring both sides, we get

$a^2 + 1 + 1 = 27 \Rightarrow a^2 = 25 \Rightarrow a = \pm 5$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** The foot of perpendicular drawn from the point A(1, 2, 8) on the xy-plane is (1, 2, 0).

Reason (R): Equation of xy-plane is y = 0.

Ans: (c) A is true but R is false.

We know that in xy-plane, z-coordinate is 0.

So, coordinate of foot of perpendicular drawn from point A(1, 2, 8) on xy-plane is (1, 2, 0).

Equation of xy-plane is z = 0

∴ Reason is wrong.

10. **Assertion (A):** The points A(1, –1, 3), B(2, –4, 5) and C(5, –13, 11) are collinear.

Reason (R): If AB + BC = AC, then A, B, C are collinear.

Ans: (a) Both A and R are true and R is the correct explanation of A.

$$|AB| = \sqrt{(1)^2 + (-3)^2 + (2)^2} = \sqrt{1+9+4} = \sqrt{14}$$

$$|BC| = \sqrt{(3)^2 + (-9)^2 + (6)^2} = \sqrt{9+81+36} = 3\sqrt{14}$$

$$|AC| = \sqrt{(4)^2 + (-12)^2 + (8)^2} = \sqrt{16+144+64} = 4\sqrt{14}$$

$\because \quad AB + BC = 4\sqrt{14} = AC$

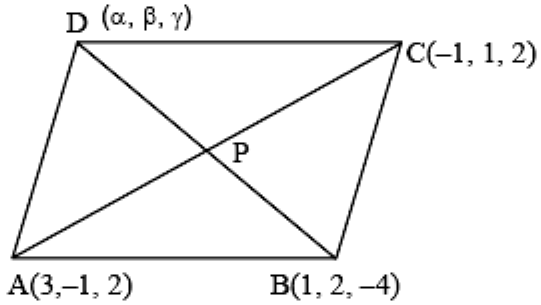
$\therefore$ Points A, B and C are collinear.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Three consecutive vertices of a parallelogram ABCD are A (3, –1, 2), B (1, 2, –4) and C(–1, 1, 2). Find the fourth vertex.

Ans: Let D (α, β, γ) be the fourth vertex, we know diagonals of a parallelogram bisect each other.



$$\therefore \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{\alpha+1}{2}, \frac{\beta+2}{2}, \frac{\gamma-4}{2}\right)$$

$$\Rightarrow \frac{2}{2} = \frac{\alpha+1}{2};\ \frac{0}{2} = \frac{\beta+2}{2};\ \frac{4}{2} = \frac{\gamma-4}{2}$$

$\Rightarrow \alpha + 1 = 2$; $\beta + 2 = 0$; $\gamma - 4 = 4$

$\Rightarrow \alpha = 1$; $\beta = -2$; $\gamma = 8$

$\therefore$ Coordinates of 4th vertex are (1, –2, 8).

12. Find the ratio in which the line segment joining the points (2, 4, 5) and (3, 5, – 4) is divided by the XY-plane.

Ans: Let XY-plane divides the join of points (2, 4, 5) and (3, 5, –4) in the ratio k : 1.

$\therefore$ Point of division is $\left(\frac{3k+2}{k+1}, \frac{5k+4}{k+1}, \frac{-4k+5}{k+1}\right)$

If this lies on XY-plane, then $z = 0$

$$\Rightarrow \frac{-4k+5}{k+1} = 0 \Rightarrow k = \frac{5}{4}$$

$\therefore$ Ratio is $\frac{5}{4}$: 1 or 5 : 4 internally.

13. Find the point on z-axis which is equidistant from (1, 5, 7) and (5, 1, – 4).

Ans: Let point be (0, 0, z) then

$$\sqrt{(0-1)^2 + (0-5)^2 + (z-7)^2} = \sqrt{(0-5)^2 + (0-1)^2 + (z+4)^2}$$

$\Rightarrow 1 + 25 + (z - 7)^2 = 25 + 1 + (z + 4)^2$

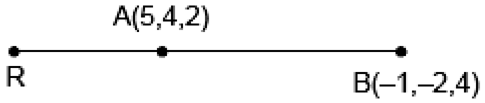
$\Rightarrow z - 7 = \pm (z + 4)$

$\Rightarrow z - 7 = - z - 4 \Rightarrow 2z = 3 \Rightarrow z = \frac{3}{2}$

Therefore, point is $\left(0, 0, \frac{3}{2}\right)$.

14. Find the coordinates of a point which divides the line segment joining the points (5, 4, 2) and (– 1, – 2, 4) in the ratio 2 : 3 externally.

Ans:



2 : 3 externally

Coordinates of R are $\left(\frac{-2-15}{2-3}, \frac{-4-12}{2-3}, \frac{8-6}{2-3}\right)$ i.e. (17, 16, –2).

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Determine the point on XY-plane which is equidistant from three points A (2, 0, 3), B (0, 3, 2) and C (0, 0, 1).

Ans: We know that z-coordinate of every point on XY-plane is zero. So, let P (x, y, 0) be a point on XY-plane such that PA = PB = PC.

Now PA = PB $\Rightarrow PA^2 = PB^2$

$\Rightarrow (x - 2)^2 + (y - 0)^2 + (0 - 3)^2 = (x - 0)^2 + (y - 3)^2 + (0 - 2)^2$

$\Rightarrow 4x - 6y = 0$ or $2x - 3y = 0$ ... (i)

PB = PC $\Rightarrow PB^2 = PC^2$

$\Rightarrow (x - 0)^2 + (y - 3)^2 + (0 - 2)^2 = (x - 0)^2 + (y - 0)^2 + (0 - 1)^2$

$\Rightarrow - 6y + 12 = 0 \Rightarrow y = 2$ ... (ii)

Putting $y = 2$ in (i), we obtain $x = 3$. Hence, the required point is (3, 2, 0).

16. Find the equation of the set of points P such that $PA^2 + PB^2 = 2k^2$, where A and B are the points (3, 4, 5) and (– 1, 3, – 7) respectively.

Ans: Let point P be (x, y, z). Given points are A(3, 4, 5) and B(–1, 3, –7) given that

$PA^2 + PB^2 = k^2$

$\Rightarrow [(x - 3)^2 + (y - 4)^2 + (z - 5)^2] + [(x + 1)^2 + (y - 3)^2 + (z + 7)^2] = k^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 + z^2 - 10z + 25 + x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 + 14z + 49 = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 - k^2 = 0$ is the required equation.

17. Show that the points (–2, 6, –2), (0, 4, –1), (–2, 3, 1) and (–4, 5, 0) are vertices of a square.

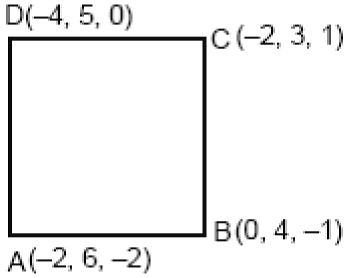
Ans: Let A (–2, 6, –2), B (0, 4, –1), C (–2, 3, 1) and D (–4, 5, 0) be the given points. We have

AB = $\sqrt{(0+2)^2 + (4-6)^2 + (-1+2)^2} = \sqrt{9} = 3$

BC = $\sqrt{(-2-0)^2 + (3-4)^2 + (1+1)^2} = \sqrt{9} = 3$

CD = $\sqrt{(-4+2)^2 + (5-3)^2 + (0-1)^2} = \sqrt{9} = 3$

AD = $\sqrt{(-4+2)^2 + (5-6)^2 + (0+2)^2} = \sqrt{9} = 3$



Since AB = BC = CD = DA. So ABCD is a square or a rhombus.

Now, AC = $\sqrt{(-2+2)^2+(3-6)^2+(1+2)^2} = \sqrt{18}$

BD = $\sqrt{(-4-0)^2+(5-4)^2+(0+1)^2} = \sqrt{18}$

Since, Diagonal AC = Diagonal BD.

Hence, ABCD is a square.

SECTION – D

Questions 18 carry 5 marks.

18. (a) Using section formula, prove that the three points (– 2, 3, 5), (1, 2, 3) and (7, 0, – 1) are collinear.
(b) Centroid of a triangle with vertices (*a*, 1, 3), (– 2, *b*, – 5) and (4, 7, *c*), is origin. Find the values of *a*, *b*, *c*.

Ans: (a) Let points be A (– 2, 3, 5), B (1, 2, 3) and C (7, 0, –1).

Let B divides the join of A and C in the ratio *k* : 1.

$\therefore \left(\frac{-2+7k}{k+1}, \frac{3}{k+1}, \frac{5-k}{k+1}\right) = (1, 2, 3)$

$\Rightarrow \frac{-2+7k}{k+1} = 1; \quad \frac{3}{k+1} = 2; \quad \frac{5-k}{k+1} = 3$

$\Rightarrow -2+7k = k+1; \; 3 = 2k+2; \quad 5-k = 3k+3$

$\Rightarrow \quad 6k = 3; \quad 2k = 1; \quad 4k = 2$

$\Rightarrow k = \frac{1}{2}; \quad k = \frac{1}{2}; \quad k = \frac{1}{2}$

As $k = \frac{1}{2}$ in all the cases, therefore, B divides the join of AC in the ratio 1 : 2. Hence A, B, C are collinear.

(b) $\left(\frac{a-2+4}{3}, \frac{1+b+7}{3}, \frac{3-5+c}{3}\right) = (0, 0, 0)$

$\Rightarrow a = -2, b = -8, c = 2$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A parent decorates his son's room for his birthday celebration. During the decoration he arranged the balloons hanging from the ceiling. Now assuming balloons as points in space, they lie on the same line and the co-ordinate for three balloons (points) are A(3, 2, -4), B(5, 4, -6) and C(9, 8, -10).

On the basis of above information, answer the following:
(i) What is the distance between points A and B ? (1)
(ii) What is the distance between points B and C? (1)
(iii) In what ratio does the point B(5, 4, -6) divides line segment AC? (2)
Ans: (i) Given coordinates are A(3, 2, - 4) and B(5, 4, - 6).

$$\therefore \quad AB = \sqrt{(5-3)^2 + (4-2)^2 + (-6+4)^2}$$
$$= \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

(ii) Given coordinates are B(5, 4, - 6) and C(9, 8, - 10).

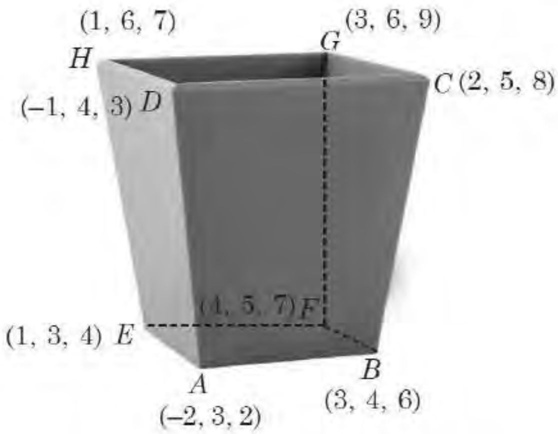
$$\therefore \; BC = \sqrt{(9-5)^2 + (8-4)^2 + (-10+6)^2} = \sqrt{16+16+16} = 4\sqrt{3} \text{ units}$$

(iii) Let B divides AC in the ratio k : 1.
Using section formula, we have

$$x = \frac{kx_2 + x_1}{k+1} \qquad \Rightarrow \quad 5 = \frac{k \times 9 + 1 \times 3}{k+1}$$
$$\Rightarrow \quad 5k + 5 = 9k + 3 \qquad \Rightarrow \quad 2 = 4k$$
$$\therefore \quad k = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ Ratio is 1 : 2 internally.

20. Ravi makes a plan to gift his friend a hand made pen-stand of the trapezoidal shape given in the figure:



The vertices of the pen-stand are A(–2, 3, 2), B(3, 4, 6), C(2, 5, 8), D(–1, 4, 3), E(1, 3, 4) F(4, 5, 7), G(3, 6, 9) and H(1, 6, 7).
On the basis of above information, answer the following:
(i) Find the coordinates of the point which divides the line segment EH in 2 : 1. (1)
(ii) Find the length of foot of perpendicular drawn from C on y-axis. (1)
(iii)Find the ratio in which the line segment joining the vertices E and F is divided by yz-plane externally. (2)
Ans: (i) Let P be the point which divides EH in 2 : 1.
∴ The coordinates of P are

$$\left(\frac{2+1}{2+1}, \frac{12+3}{2+1}, \frac{14+4}{2+1}\right) = (1, 5, 6)$$

(ii) As we know, on y-axis x = 0, z = 0

∴ The coordinates of foot of perpendicular drawn from C are (0, 5, 0)

(iii) Let YZ plane divides the line segment joining E(1, 3, 4) and F(4, 5, 7) in h : 1 at P(x, y, z)

∴ Coordinates of $P = \left(\frac{4h+1}{h+1}, \frac{5h+3}{h+1}, \frac{7h+4}{h+1}\right)$

Since P is in YZ plane

∴ Its x coordinates is zero.

∴ $4h + 1 = 0$

$$h = \frac{-1}{4}$$

∴ YZ-plane divides EF externally in the ratio 1 : 4
