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## PRACTICE PAPER (2023-24)
## CHAPTER 10 CONIC SECTIONS (ANSWERS)

**SUBJECT: MATHEMATICS** **MAX. MARKS : 40**
**CLASS : XI** **DURATION : $1\frac{1}{2}$ hrs**

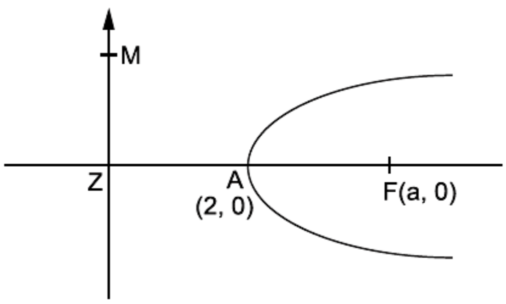
**General Instructions:**
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

## SECTION – A
**Questions 1 to 10 carry 1 mark each.**

**1.** The centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is
(a) (–3, 2) (b) (3, 2) (c) (3, –2) (d) (–3, –2)
Ans: (c) (3, –2)
Given circle is $x^2 + y^2 - 6x + 4y - 12 = 0$.
This is of the form $x^2 + y^2 + 2gx + 2fy + c = 0$, where
$(2g = -6 \text{ and } 2f = 4) \Rightarrow (g = -3 \text{ and } f = 2)$.
$\therefore$ centre is $(-g, -f) = (3, -2)$.

**2.** In the parabola $y^2 = -12x$, the focus and the equation of directrix are respectively
(a) F( 3, 0), x = −3 (b) F(−3, 0), x = 3
(c) F(−3, 0), x = −3 (d) none of these
Ans: (b) F(−3, 0), x = 3
Given equation is $y^2 = -4ax$, where a = 3.
$\therefore$ focus is $F(-a, 0) = F(-3, 0)$.
And, the directrix is $x = a \Rightarrow x = 3$.

**3.** If A(2, 0) is the vertex and the y-axis is the directrix of a parabola, then its focus is
(a) F(2, 0) (b) F(−2, 0) (c) F(4, 0) (d) F(−4, 0)
Ans: (c) F(4, 0)
Let ZM be the directrix, where coordinates of Z are (0, 0).
Let the focus be F( a, 0).




Then, vertex A is the midpoint of ZF.
$\therefore \frac{a+0}{2} = 2 \Rightarrow a = 4$
$\therefore$ focus is F(4, 0).

**4.** The focal distance of a point P on the parabola $y^2 = 12x$ is 4. The abscissa of P is
(a) –1 (b) 1 (c) –2 (d) 2

Ans: (b) 1
Given parabola is $y^2 = 4ax$, where a = 3.
Its directrix is $x = -a \Rightarrow x = -3 \Rightarrow x + 3 = 0$.
Focal distance of $P(x_1, y_1)$ = distance of $P(x_1, y_1)$ from $x + 3 = 0$

$$= \frac{x_1 + 3}{\sqrt{1^2}} = \frac{x_1 + 3}{1} = x_1 + 3$$

$\therefore x_1 + 3 = 4 \Rightarrow x_1 = 1$.

**5.** The vertices of an ellipse are (±5, 0) and its foci are (±4, 0). The equation of the ellipse is

(a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (b) $\frac{x^2}{9} + \frac{y^2}{25} = 1$ (c) $\frac{x^2}{25} + \frac{y^2}{16} = 1$ (d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

Ans: (a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Since the vertices of the given ellipse are on the $x$-axis, so it is a horizontal ellipse.

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 > b^2$.

Its vertices are $(\pm a, 0) = (\pm 5, 0) \Rightarrow a = 5$.

Its foci are $(\pm c, 0) = (\pm 4, 0) \Rightarrow c = 4$.

$c^2 = (a^2 - b^2) \Rightarrow b^2 = (a^2 - c^2) = (25 - 16) = 9 \Rightarrow b = 3$.

$\therefore$ required equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

**6.** The vertices of a hyperbola are (±2, 0) and its foci are (±3, 0). The equation of the hyperbola is

(a) $\frac{x^2}{2} - \frac{y^2}{3} = 1$ (b) $\frac{x^2}{3} - \frac{y^2}{4} = 1$ (c) $\frac{x^2}{4} - \frac{y^2}{5} = 1$ (d) none of these

Ans: (c) $\frac{x^2}{4} - \frac{y^2}{5} = 1$

Since the vertices of the given hyperbola are of the form $(\pm a, 0)$, so it is a horizontal hyperbola.

Let the required equation be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Then, its vertices are $(\pm a, 0) = (\pm 2, 0) \Rightarrow a = 2$.

Let its foci be $(\pm c, 0) = (\pm 3, 0) \Rightarrow c = 3$.

$\therefore \quad b^2 = (c^2 - a^2) = (3^2 - 2^2) = (9 - 4) = 5$.

$\therefore \quad a^2 = 2^2 = 4, b^2 = 5$.

Hence, the required equation is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

**7.** The end points of a diameter of a circle are A(2, − 3) and B(−3, 5). The equation of the circle is
(a) $x^2 + y^2 + 2x - y - 21 = 0$ (b) $x^2 + y^2 + x - 2y - 21 = 0$
(c) $x^2 + y^2 + x - 2y + 21 = 0$ (d) none of these
Ans: (b) $x^2 + y^2 + x - 2y - 21 = 0$
Given end points are $(x_1, y_1) = (2, -3)$ and $(x_2, y_2) = (-3, 5)$.
$\therefore$ equation of the circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$\Rightarrow (x - 2)(x + 3) + (y + 3)(y - 5) = 0$
$\Rightarrow x^2 + y^2 + x - 2y - 21 = 0$.

**8.** If the parabola $y^2 = 4ax$ passes through the point P( 3, 2), then the length of its latus rectum is

(a) 1/3 (b) 2/3 (c) 4/3 (d) 4

Ans: (a) 1/3

Since the point P(3, 2) lies on $y^2 = 4ax$, we have

$4a \times 3 = 2^2 \Rightarrow a = 1/3$

$\therefore$ latus rectum = 4a = 4 × 1/3 = 4/3

**For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.**

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

**9.** Parabola is symmetric with respect to the axis of the parabola.

**Assertion (A):** If the equation has a term $y^2$, then the axis of symmetry is along the x-axis.

**Reason (R):** If the equation has a term $x^2$, then the axis of symmetry is along the x-axis.

Ans: (c) A is true but R is false.

**10.** Let the centre of an ellipse is at (0,0)

**Assertion (A):** If major axis is on the y-axis and ellipse passes through the points (3,2) and (1,6), then the equation of ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$.

**Reason (R):** $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is an equation of ellipse if major axis is along y-axis.

Ans: (a) Both A and R are true and R is the correct explanation of A.

## SECTION – B

**Questions 11 to 14 carry 2 marks each.**

**11.** Find the coordinates of the focus, the equation of directrix, vertex and length of latus rectum for the parabola $y^2 = -12x$.

Ans: Given parabola is $y^2 = -12x$

$\Rightarrow 4a = -12 \Rightarrow a = -3$

$\therefore$ Focus is (–3, 0);

Directrix is $x + a = 0 \Rightarrow x - 3 = 0$

$\therefore$ Vertex is (0, 0);

Length of latus rectum = $4a$ = 12

**12.** Find the coordinates of the foci, the vertices, the eccentricity, the length of latus rectum of the hyperbola : $16x^2 - 9y^2 = 144$.

Ans: Given hyperbola is $16x^2 - 9y^2 = 144$

$\Rightarrow \frac{x^2}{9}-\frac{y^2}{16}=1 \Rightarrow a^2 = 9, b^2 = 16 \Rightarrow a = 3, b = 4$

Vertices are ($\pm a$, 0), i.e. (± 3, 0)

$c = \sqrt{a^2+b^2} = \sqrt{9+16} = 5$

Foci are (± 5, 0)

Eccentricity ($e$) = $\frac{\sqrt{a^2+b^2}}{a} = \frac{c}{a} = \frac{5}{3}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times16}{3} = \frac{32}{3}$

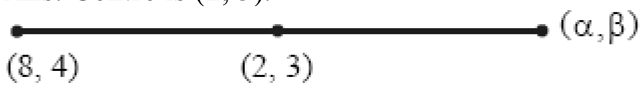
**13.** Find the eccentricity of the ellipse if its latus rectum is equal to one half of its minor axis.

Ans: Latus rectum = $\frac{1}{2}$ (minor axis)

$$\Rightarrow \frac{2b^2}{a} = \frac{1}{2}(2b) \Rightarrow b = \frac{a}{2}$$

Now, apply $b^2 = a^2(1 - e^2)$, we get $e = \frac{\sqrt{3}}{2}$

**14.** If one end of the diameter of a circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (8, 4), show that coordinates of the other end are (– 4, 2).
Ans: Centre is (2, 3).



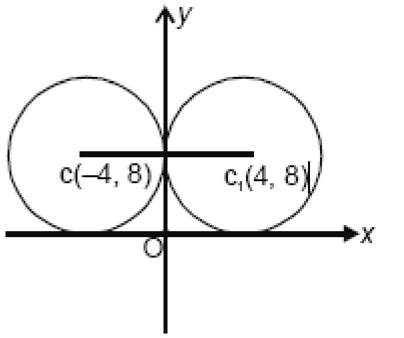

$$\frac{8+\alpha}{2} = 2; \frac{4+\beta}{2} = 3$$

$\Rightarrow \alpha = -4, \beta = 2$, i.e. (– 4, 2)

## SECTION – C

**Questions 15 to 17 carry 3 marks each.**

**15.** Find the equation of the image of the circle $x^2 + y^2 + 8x - 16y + 64 = 0$ in the line mirror $x = 0$.
Ans: The equation of the given circle is $x^2 + y^2 + 8x - 16y + 64 = 0$




$\Rightarrow x^2 + 8x + 16 + (y^2 - 16y + 64) = 16$
$\Rightarrow (x + 4)^2 + (y - 8)^2 = 4^2$
$\Rightarrow \{x - (4)\}^2 + (y - 8)^2 = 4^2$
Clearly, its centre in the line mirror has its centre (4, 8) and radius 4. So its equation is $(x - 4)^2 + (y - 8)^2 = 4^2$
$\Rightarrow x^2 + y^2 - 8x - 16y + 64 = 0.$

**16.** Find the equation of the ellipse whose foci are at (± 5, 0) and x = $\frac{36}{5}$ as one of the directrix.

Ans: Let ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(*i*)
Foci are $(\pm ae, 0) = (\pm 5, 0) \Rightarrow ae = 5$ ...(*ii*)
One of the directrix is $x = \frac{36}{5} \Rightarrow \frac{a}{e} = \frac{36}{5}$ ...(*iii*)
From (*ii*) and (*iii*), we get
$a^2 = 36$ and $e = \frac{5}{6}$
Also $e = \frac{\sqrt{a^2 - b^2}}{a} \Rightarrow a^2e^2 = a^2 - b^2$

$\Rightarrow 25 = 36 - b^2 \Rightarrow b^2 = 11$

Substituting $a^2$, $b^2$ in ($i$), we get equation of ellipse as $\frac{x^2}{36}+\frac{y^2}{11}=1$

**OR**

Find the equation of an ellipse, the distance between whose foci is 5 units and the distance between the directrices is 20 units.

Ans: Distance between the foci is $2ae = 5$, Distance between the directrices, $= \frac{2a}{e} = 20$

$\Rightarrow (2ae)\left(\frac{2a}{e}\right) = (5)(20) \Rightarrow 4a^2 = 100$

$\Rightarrow a^2 = 25 \Rightarrow a = 5$

Also $2ae = 5 \Rightarrow 2(5)e = 5 \Rightarrow e = \frac{1}{2}$

Now, $b^2 = a^2(1 - e^2) = 5^2\left(1-\frac{1}{4}\right)=\frac{75}{4}$

The equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{25}+\frac{4y^2}{75}=1$.

**17.** The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$. Find equation of the hyperbola if its eccentricity is 2.

Ans: The given ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1 \Rightarrow a^2 = 25$ and $b^2 = 9$

Let $e$ be the eccentricity of the ellipse.

$\Rightarrow b^2 = a^2(1 - e^2) \Rightarrow 9 = 25(1 - e^2)$

$\Rightarrow e = \frac{4}{5} \Rightarrow ae = \frac{4}{5} . 5 = 4$

Foci for the ellipse are $(\pm ae, 0)$ i.e., $(\pm 4, 0)$

$\Rightarrow$ The foci of the hyperbola are $(\pm 4, 0)$

The eccentricity of the hyperbola is 2.

$\Rightarrow a.2 = 4 \Rightarrow a = 2$

Now, $b^2 = a^2(e^2 - 1) = 4(4 - 1) = 12$.

$\Rightarrow$ The required equation of the hyperbola is $\frac{x^2}{4}-\frac{y^2}{12}=1 \Rightarrow 3x^2 - y^2 = 12$.

## SECTION – D

**Questions 18 carry 5 marks.**

**18.** Find the equation of circle which passes through (2, –2) and (3, 4) and whose centre lies on the line $x + y = 2$.

Ans: Let circle be $(x - h)^2 + (y - k)^2 = r^2$ ... ($i$)

Circle passes through (2, – 2) and (3, 4)

$(2 - h)^2 + (-2 - k)^2 = r^2$ ... ($ii$)

$(3 - h)^2 + (4 - k)^2 = r^2$ ... ($iii$)

Also centre $(h, k)$ lies on the line $x + y = 2$

$\Rightarrow h + k = 2$ ... ($iv$)

From ($ii$) and ($iii$), we get

$4 - 4h + h^2 + 4 + 4k + k^2 = 9 - 6h + h^2 + 16 - 8k + k^2$

$\Rightarrow 2h + 12k = 17$ ... ($v$)

Solving (*iv*) and (*v*) ,we get $k = \frac{13}{10}, h = \frac{7}{10}$

Substituting in (*iii*), we get $\left(3 - \frac{7}{10}\right)^2 + \left(4 - \frac{13}{10}\right)^2 = r^2$

$\Rightarrow r^2 = \left(\frac{23}{10}\right)^2 + \left(\frac{27}{10}\right)^2 = \frac{529}{100} + \frac{729}{100} = \frac{1258}{100}$

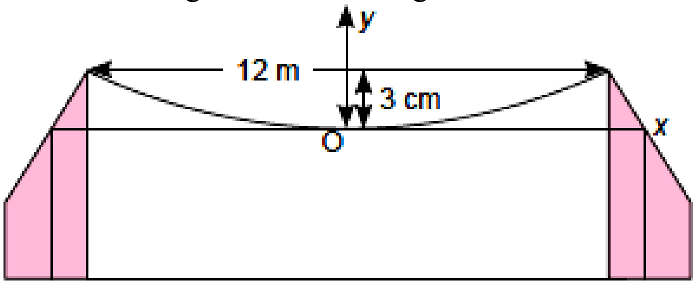
Substituting for *h*, *k*, *r* in (*i*), we get

Equation of circle as $\left(x - \frac{7}{10}\right)^2 + \left(y - \frac{13}{10}\right)^2 = \frac{1258}{100}$

## SECTION – E (Case Study Based Questions)

**Questions 19 to 20 carry 4 marks each.**

**19.** A beam is supported at its ends by supports which are 12 m apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and defected beam is in the shape of parabola. Now considering the centre of beam is at origin as shown in figure. Answer the following:




(i) Write the form of the equation of parabola. (1)
(ii) Find the focus of parabola. (1)
(iii) Find the length of latus rectum of parabola. (1)
(iv) How far from the centre is the deflection 1 cm? (1)
Ans: (i) Equation of parabola is $x^2 = 4ay$

(ii) Point $\left(6, \frac{3}{100}\right)$ lies on parabola

$\therefore\ 36 = 4 \times a \times \frac{3}{100} \Rightarrow a = \frac{3600}{12} = 300$

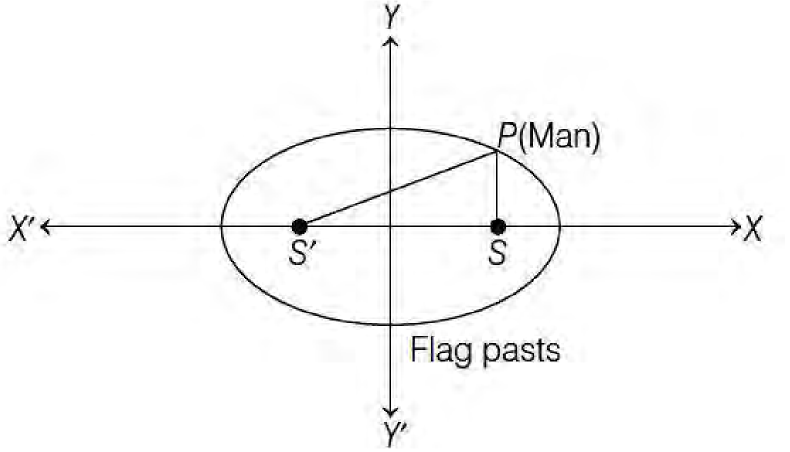
Focus = (0, 300)
(iii) Length of latus rectum = 4a = 4 × 300 = 1200 m

(iv) Where the deflection is 1 cm. Let the coordinates of point be $\left(k, \frac{3}{100}\right)$

$x^2 = 4ay \Rightarrow k^2 = 4 \times 300 \times 2/100$
$\Rightarrow k^2 = 24 \Rightarrow k = 2\sqrt{6}$
$\therefore$ At distance of $2\sqrt{6}$ m from centre deflection is 1 cm.

**20.** A man running on a race course notices that sum ofits distances from two flag posts from him is always 10m and the distance between the flag posts is 8 m. He notes that he can read the messages of value system 'Honesty' and 'Respect for other' on the poles whichever side he moves, then answer the following questions which are based on above it.
(i) Find the value of a for the standard equation of path. (1)
(ii) Find the value of b for the standard equation of path. (1)
(iii) Find the Equation of path. (1)
(iv) Find the value of (2 a + b). (1)

Ans: Since, sum of distances of any point on race course from two fixed points is always constant, so path traced by the man will be ellipse




(i) Given that, SP + S'P = 10 ⇒ 2a = 10 ⇒ a = 5

(ii) Since, the coordinate of S and S' are(c, 0)and(−c, 0).

Therefore, the distance between S and S' is 2c = 8 ⇒ c = 4

∴ $c^2 = a^2 - b^2$ [∵ a = 5]

⇒ $16 = 25 - b^2 \Rightarrow b^2 = 25 - 16 = 9 \Rightarrow b = 3$

(iii) Since a = 5 and b = 3,therefore the equation of the path is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

(iv) Here, a = 5 and b = 3

∴ 2a + b = 2 (5) + 3 = 13