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PRACTICE PAPER 08 (2023-24)

CHAPTER 08 & 09 TRIGONOMETRY (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**
CLASS : X | **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If 3 cot θ = 2, then the value of tan θ

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{\sqrt{13}}$ (d) $\frac{2}{\sqrt{13}}$

Ans: (b) $\frac{3}{2}$

$3\cot\theta = 2 \Rightarrow \cot\theta = \frac{2}{3} \Rightarrow \tan\theta = \frac{3}{2}$

2. If sin θ – cos θ = 0, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

Ans: (b) $\frac{1}{2}$

$\sin\theta - \cos\theta = 0 \Rightarrow (\sin\theta - \cos\theta)^2 = 0$

$\Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 0$

$\Rightarrow -2\sin\theta\cos\theta = -1 \Rightarrow 2\sin\theta\cos\theta = 1$

$\Rightarrow \sin\theta\cos\theta = \frac{1}{2} \Rightarrow \sin^2\theta\cos^2\theta = \frac{1}{4}$

$\sin^4\theta + \cos^4\theta = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= (1)^2 - 2 \times \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}$

3. If ΔABC is right angled at C, then the value of sin (A + B) is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Ans: (b) 1
ΔABC is right angled at C,
∴ A + B + C = 180°
A + B = 180° – 90° = 90° (∵ ∠C = 90°)
sin (A + B) = sin 90° = 1

4. If cosecA – cotA = $\frac{4}{5}$, then cosec A =

(a) $\frac{47}{40}$ (b) $\frac{59}{40}$ (c) $\frac{51}{40}$ (d) $\frac{41}{40}$

Ans: (d) $\frac{41}{40}$

$\text{cosec A} - \text{cot A} = \frac{4}{5}$

Also $\text{cosec}^2 \text{A} - \text{cot}^2 \text{A} = 1$

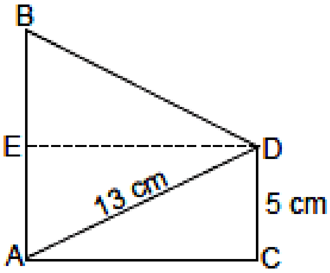
$\Rightarrow (\text{cosec A} - \text{cot A})(\text{cosec A} + \text{cot A}) = 1$

$\Rightarrow \frac{4}{5}(\text{cosec A} + \text{cot A}) = 1$

$\Rightarrow \text{cosec A} + \text{cot A} = \frac{5}{4}$

Adding both we get, cosecA = $\frac{41}{40}$

5. In the given figure, if AB = 14 cm, then the value of tan B is:



(a) $\frac{4}{3}$ (b) $\frac{14}{3}$ (c) $\frac{5}{3}$ (d) $\frac{13}{3}$

Ans: (a) $\frac{4}{3}$

$AC = \sqrt{13^2 - 5^2} = 12$

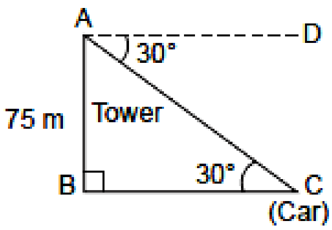
DE = AC = 12 cm, BE = 14 cm – 5 cm = 9 cm

In ΔBED, $\tan B = \frac{DE}{BE} = \frac{12}{9} = \frac{4}{3}$.

6. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30°. The distance of the car from the base of the tower (in m) is:

(a) $25\sqrt{3}$ (b) $50\sqrt{3}$ (c) $75\sqrt{3}$ (d) 150

Ans: (c) $75\sqrt{3}$



In ΔABC, $\tan 30^0 = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BC} \Rightarrow BC = 75\sqrt{3}$

7. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0° < \theta < 90°$, find the value of θ.

(a) 30° (b) 45° (c) 60° (d) 90°

Ans: (a) 30°

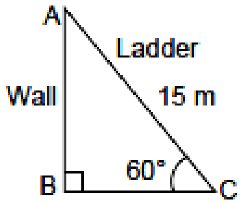
$\sqrt{3}\ \sin\theta - \cos\theta = 0 \Rightarrow \sqrt{3}\ \sin\theta = \cos\theta$

$\Rightarrow \sqrt{3} = \dfrac{\cos\theta}{\sin\theta} \Rightarrow \cot\theta = \sqrt{3} = \cot 30^0 \Rightarrow \theta = 30°$

8. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is

(a) $15\sqrt{3}$ m (b) $15\sqrt{3}/2$ m (c) 15/2 m (d) 15 m

Ans: (b) $15\sqrt{3}/2$ m



In ΔABC, $\sin 60^0 = \dfrac{AB}{AC} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{AB}{15} \Rightarrow AB = \dfrac{15\sqrt{3}}{2} m$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$ then the value of $x + y = 3$.

Reason (R): For any value of θ, $\sin^2\theta + \cos^2\theta = 1$

Ans: We know that for any value of θ, $\sin^2\theta + \cos^2\theta = 1$

So, Reason is correct.

For assertion: We have $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$

Then, $x + y = 2\sin^2\theta + 2\cos^2\theta + 1$

$= 2(\sin^2\theta + \cos^2\theta) + 1$

$= 2 \times 1 + 1\ \ [\because \sin^2\theta + \cos^2\theta = 1]$

$= 2 + 1 = 3$.

Hence, Assertion is also correct.

Correct option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. **Assertion (A):** Sin A is the product of Sin and A.

Reason (R): The value of sinθ increases as θ increases.

Ans: For assertion: Sin A is not the product of Sin and A.

It is the Sine of ∠A.

∴ Assertion is not correct.

For reason: The value of sinθ increases as θ increases in interval of $0° < \theta < 90°$

So, Reason is correct.

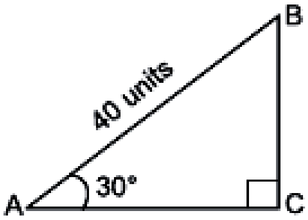
Correct option is (d) Assertion (A) is false but reason (R) is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. ABC is a right triangle, right angled at C. If A = 30° and AB = 40 units, find the remaining two sides of ΔABC.

Ans: Since ∠A + ∠B + ∠C = 180°



$30° + \angle B + 90° = 180° \Rightarrow \angle B = 60°$

Now, $\cos A = \frac{AC}{AB}$

$\Rightarrow \cos 30° = \frac{AC}{40} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AC}{40}$

$\Rightarrow$ AC = $20\sqrt{3}$ units.

and, $\sin A = \frac{BC}{AB} \Rightarrow \sin 30^0 = \frac{1}{2} = \frac{BC}{40} \Rightarrow$ BC = 20 units

12. Find the value of x if $\tan 3x = \sin 45°. \cos 45° + \sin 30°$.

Ans: $\tan 3x = \sin 45° . \cos 45° + \sin 30°$

$\Rightarrow \tan 3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \tan 3x = 1 = \tan 45°$

$\Rightarrow 3x = 45° \Rightarrow x = 15°$

13. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, show that $q(p^2 - 1) = 2p$.

Ans: $\sin\theta + \cos\theta = p$, $\sec\theta + \text{cosec}\,\theta = q$

LHS $= q(p^2 - 1) = (\sec\theta + \text{cosec}\,\theta)\,[(\sin\theta + \cos\theta)^2 - 1]$

$= \left[\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right] [\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta - 1]$

$= \left[\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right] [1 + 2\cos\theta\sin\theta - 1]$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\cos\theta\sin\theta$

$= 2(\sin\theta + \cos\theta) = 2p$ $(\because \sin\theta + \cos\theta = p)$

LHS = RHS. Hence proved.

14. Evaluate: $\frac{5\sin^2 30^0 + \cos^2 45^0 + 4\tan^2 60^0}{2\sin 30^0 \cos 60^0 + \tan 45^0}$

Ans: $\frac{5\sin^2 30^0 + \cos^2 45^0 + 4\tan^2 60^0}{2\sin 30^0 \cos 60^0 + \tan 45^0}$

$= \frac{5\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\sqrt{3}\right)^2}{2 \times \frac{1}{2} \times \frac{1}{2} + 1} = \frac{\frac{5}{4} + \frac{1}{2} + 12}{\frac{3}{2}} = \frac{7 + 48}{4 \times \frac{3}{2}} = \frac{55}{6} = 9\frac{1}{6}$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \frac{p^2 - 1}{p^2 + 1}$.

Ans: $\dfrac{p^2-1}{p^2+1}=\dfrac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$

$=\dfrac{(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+(\tan^2\theta+1)+2\sec\theta\tan\theta}=\dfrac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+\sec^2\theta+2\sec\theta\tan\theta}$

$=\dfrac{2\tan^2\theta+2\sec\theta\tan\theta}{2\sec^2\theta+2\sec\theta\tan\theta}=\dfrac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\sec\theta+\tan\theta)}$

$=\dfrac{2\tan\theta}{2\sec\theta}=\tan\theta\times\cos\theta=\dfrac{\sin\theta}{\cos\theta}\times\cos\theta=\sin\theta$

16. Prove that: $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\dfrac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\dfrac{2\sec^2\theta}{\tan^2\theta-1}$

Ans: LHS = $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\dfrac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$=\dfrac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$=\dfrac{\sin^2\theta+\cos^2\theta+2\sin\theta.\cos\theta+\sin^2\theta+\cos^2\theta+2\sin\theta.\cos\theta}{\sin^2\theta-\cos^2\theta}$

$=\dfrac{\sin^2\theta+\cos^2\theta+\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=\dfrac{1+1}{\sin^2\theta-\cos^2\theta}=\dfrac{2}{\sin^2\theta-\cos^2\theta}$

$=\dfrac{\dfrac{2}{\cos^2\theta}}{\dfrac{\sin^2\theta}{\cos^2\theta}-\dfrac{\cos^2\theta}{\cos^2\theta}}=\dfrac{2\sec^2\theta}{\tan^2\theta-1}=\text{RHS}$

17. Prove that: $\dfrac{1}{\operatorname{cosec}\theta-\cot\theta}-\dfrac{1}{\sin\theta}=\dfrac{1}{\sin\theta}-\dfrac{1}{\operatorname{cosec}\theta+\cot\theta}$

Ans: This can be written as $\dfrac{1}{\operatorname{cosec}\theta-\cot\theta}+\dfrac{1}{\operatorname{cosec}\theta+\cot\theta}=\dfrac{1}{\sin\theta}+\dfrac{1}{\sin\theta}=\dfrac{2}{\sin\theta}$

LHS = $\dfrac{1}{\operatorname{cosec}\theta-\cot\theta}+\dfrac{1}{\operatorname{cosec}\theta+\cot\theta}=\dfrac{\operatorname{cosec}\theta+\cot\theta+\operatorname{cosec}\theta-\cot\theta}{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta)}$

$=\dfrac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-\cot^2\theta}=\dfrac{2\operatorname{cosec}\theta}{1}=2\operatorname{cosec}\theta=\dfrac{2}{\sin\theta}=\text{RHS}$

SECTION – D

Questions 18 carry 5 marks.

18. If cosec θ – sin θ = m and sec θ – cos θ = n, prove that $(m^2n)^{2/3}+(mn^2)^{2/3}=1$.

Ans: cosec θ – sin θ = m

$\Rightarrow\dfrac{1}{\sin\theta}-\sin\theta=m\Rightarrow\dfrac{1-\sin^2\theta}{\sin\theta}=m\Rightarrow\dfrac{\cos^2\theta}{\sin\theta}=m$ ...(*i*)

Also, sec θ – cos θ = n

$\Rightarrow\dfrac{1}{\cos\theta}-\cos\theta=n\Rightarrow\dfrac{1-\cos^2\theta}{\cos\theta}=n\Rightarrow\dfrac{\sin^2\theta}{\cos\theta}=n$ ...(*ii*)

Now, LHS = $(m^2n)^{2/3}+(mn^2)^{2/3}$

$=\left\{\left(\dfrac{\cos^2\theta}{\sin\theta}\right)^2\left(\dfrac{\sin^2\theta}{\cos\theta}\right)\right\}^{2/3}+\left\{\left(\dfrac{\cos^2\theta}{\sin\theta}\right)\left(\dfrac{\sin^2\theta}{\cos\theta}\right)^2\right\}^{2/3}$

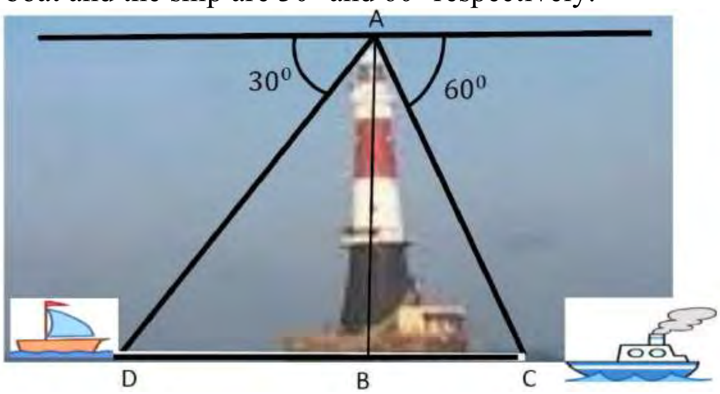
$$= \left(\frac{\cos^4\theta.\sin^2\theta}{\sin^2\theta.\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta.\sin^4\theta}{\sin\theta.\cos^2\theta}\right)^{2/3}$$

$= (\cos^3 \theta)^{2/3} + (\sin^3 \theta)^{2/3} = \cos^2 \theta + \sin^2 \theta = 1 = \text{RHS}$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A lighthouse is a tall tower with light near the top. These are often built on islands, coasts or on cliffs. Lighthouses on water surface act as a navigational aid to the mariners and send warning to boats and ships for dangers. Initially wood, coal would be used as illuminators. Gradually it was replaced by candles, lanterns, electric lights. Nowadays they are run by machines and remote monitoring. Prongs Reef lighthouse of Mumbai was constructed in 1874-75. It is approximately 40 meters high and its beam can be seen at a distance of 30 kilometres. A ship and a boat are coming towards the lighthouse from opposite directions. Angles of depression of flash light from the lighthouse to the boat and the ship are 30^0 and 60^0 respectively.



(i) Which of the two, boat or the ship is nearer to the light house. Find its distance from the lighthouse? (2)

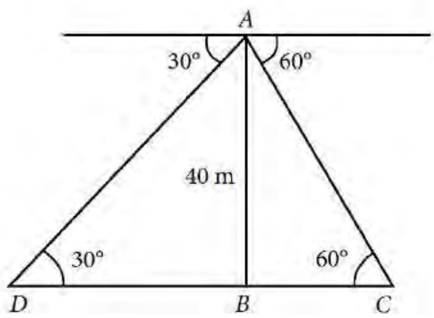
(ii) Find the time taken by the boat to reach the light house if it is moving at the rate of 2 km per hour. (2)

OR

(ii) The ratio of the height of a light house and the length of its shadow on the ground is √3 : 1 . What is the angle of elevation of the sun? (2)

Ans:

(i) Here, height of lighthouse $(AB) = 40$ m (Given)



In ΔACB, $\tan 60° = \dfrac{AB}{BC}$

$\Rightarrow \sqrt{3} = \dfrac{40}{BC} \Rightarrow BC = \dfrac{40}{\sqrt{3}} = \dfrac{40\sqrt{3}}{3}$ m

Also, in ΔADB, $\tan 30° = \dfrac{AB}{BD}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{40}{DB} \Rightarrow DB = 40\sqrt{3}$ m

Thus, ship is nearer to the light house.

(ii) Boat moving at the speed of 2 km/hr *i.e.*, $\dfrac{2000}{60}$ m/min.

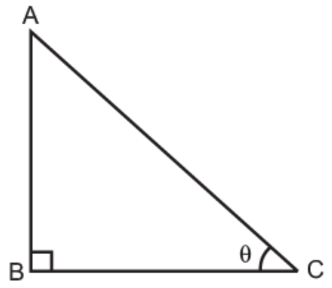
$\therefore$ Time taken to cover the distance $= \dfrac{\text{Distance } DB}{\text{Speed}} = \dfrac{60}{2000} \times 40\sqrt{3} = 2.078$ minutes

OR

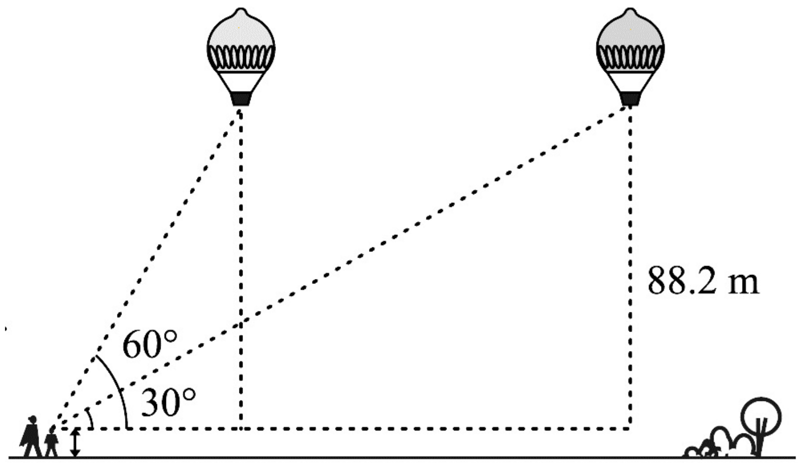
(ii) Let height of light house be AB and its shadow be BC.

In ΔABC, $\tan\theta = \dfrac{AB}{AC}$

But $\frac{AB}{AC} = \frac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3} = \tan 60^0 \Rightarrow \theta = 60^0$



20. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After 30 seconds, the angle of elevation reduces to 30° (see the below figure).



Based on the above information, answer the following questions. (Take √3 =1.732)
(i) Find the distance travelled by the balloon during the interval. (2)
(ii) Find the speed of the balloon. (2)

OR

(ii) If the elevation of the sun at a given time is 30°, then find the length of the shadow cast by a tower of 150 feet height at that time. (2)

Ans: (i) In the figure, let C be the position of the observer (the girl).
A and P are two positions of the balloon.
CD is the horizontal line from the eyes of the (observer) girl.
Here PD = AB = 88.2 m − 1.2 m = 87 m

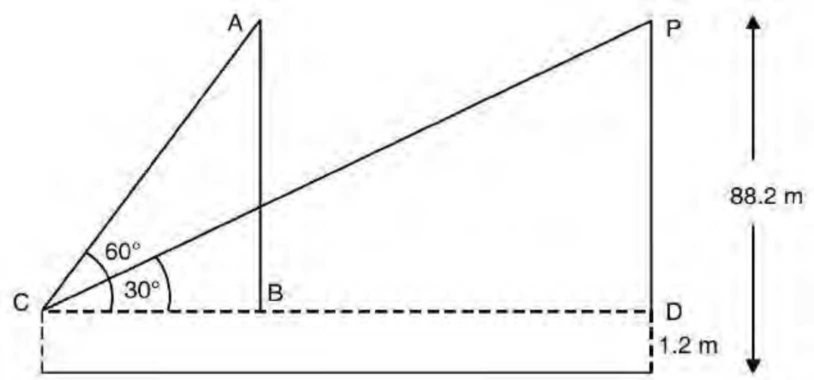
In ΔABC, we have $\frac{AB}{BC} = \tan 60^0 \Rightarrow \frac{87}{BC} = \sqrt{3}$

$\Rightarrow BC = \frac{87}{\sqrt{3}} = 29\sqrt{3}$

In ΔPDC, we have $\frac{PD}{CD} = \tan 30^0 \Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}}$

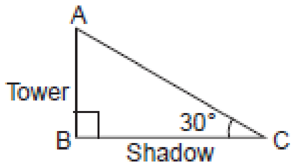
$\Rightarrow CD = 87\sqrt{3}$

Now, BD = CD = BC = $87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}$ m



Thus, the required distance between the two positions of the balloon = 58 √3 m
= 58 x 1.732 = 100.46 m (approx.)
(ii) Speed of the balloon = Distance/time = 100.46/30 = 3.35 m/s (approx.)

OR



In right ΔABC

$\frac{AB}{BC} = \tan 30°$

$\Rightarrow \frac{150}{BC} = \frac{1}{\sqrt{3}} \Rightarrow BC = 150\sqrt{3}$ feet