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## PRACTICE PAPER 11 (2023-24)

## CHAPTER 12 SURFACE AREAS AND VOLUMES

## (ANSWERS)

**SUBJECT: MATHEMATICS** **MAX. MARKS : 40**

**CLASS : X** **DURATION : $1\frac{1}{2}$ hrs**

**General Instructions:**

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

## SECTION – A

**Questions 1 to 10 carry 1 mark each.**

**1.** A tank is made of the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and radius is 30 cm. The total surface area of the tank is:

(a) 30 m (b) 3.3 m (c) 30.3 m (d) 3300 m

Ans: (b) 3.3 m

Total surface area of tank = CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$= 2 \times 22/7 \times 30(145 + 30)\ cm^2$

$= 33000\ cm^2 = 3.3\ m^2$

**2.** A cone, a hemisphere and cylinder are of the same base and of the same height. The ratio of their volumes is

(a) 1 : 2 : 3 (b) 2 : 1 : 3 (c) 3 : 1 : 2 (d) 3 : 2 : 1

Ans: (a) 1 : 2 : 3

Let the base radii of them be r and height h.

Then ratio of volumes

cone : hemisphere : cylinder

$$= \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h = \frac{1}{3}\pi r^2 r : \frac{2}{3}\pi r^3 : \pi r^2 r \ (\because r = h)$$

$$= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = \frac{1}{3} : \frac{2}{3} : 1 = 1 : 2 : 3$$

**3.** Volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is

(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

Ans: (d) 16 : 9

Let the radius of two spheres be r1 and r2.

Given, the ratio of the volume of two spheres = 64 : 27

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$$

Let the surface areas of the two spheres be S1 and S2.

$$\therefore \frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

**4.** The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is

(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 5 : 1

Ans: (d) 5 : 1

$$\frac{\text{Total surface area}}{\text{Lateral surface area}} = \frac{2\pi r(h+r)}{2\pi rh} = \frac{h+r}{h} = \frac{(20+80)}{20} = \frac{100}{20} = \frac{5}{1}$$

**5.** The ratio of the volumes of two spheres is 8 : 27. The ratio between their surface areas is

(a) 2 : 3 (b) 4 : 27 (c) 8 : 9 (d) 4 : 9

**Ans: (d) 4 : 9**

$$\frac{\text{Volume of sphere with radius } r}{\text{Volume of sphere with radius } R} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27} \Rightarrow \frac{r^3}{R^3} = \frac{8}{27} \Rightarrow \left(\frac{r}{R}\right)^3 = \left(\frac{2}{3}\right)^3 \Rightarrow \frac{r}{R} = \frac{2}{3}$$

$$\text{Ratio between their surface areas} = \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

**6.** The radius (in cm) of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is

(a) 4.2 (b) 2.1 (c) 8.1 (d) 1.05

Ans: (b) 2.1

Edge of the cube = 4.2 cm

Diameter of base of largest possible cone = 4.2 cm

$\therefore$ Radius = $\frac{4.2}{2}$ = 2.1 cm

**7.** A cube whose edge is 20 cm long, has circles on each of its faces painted black. What is the total area of the unpainted surface of the cube if the circles are of the largest possible areas?

(a) 90.72 $cm^2$ (b) 256.72 $cm^2$ (c) 330.3 $cm^2$ (d) 514.28 $cm^2$

Ans: (d) 514.28 $cm^2$

Diameter of largest circle = 20 cm.

$\therefore$ Area of circle = $100\pi$ $cm^2$

$\therefore$ Area of 6 circles = $6 \times 100\pi = 600\pi$ $cm^2$

($\because$ there are six faces in a cube)

Also, Area of cube = $6 \times (20)^2 = 2400$ $cm^2$

Area of unpainted surface = $2400 - 600\pi = 2400 - 600 \times \frac{22}{7}$ = 514.28 $cm^2$.

**8.** The radii of 2 cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Then, the ratio of their volumes is:

(a) 19 : 20 (b) 20 : 27 (c) 18:25 (d) 17:23

Ans: (b) 20 : 27

Let $r_1$ and $r_2$ be the two radii and $h_1$ and $h_2$ be the corresponding two heights of the two cylinders. Then

$\frac{r_1}{r_2} = \frac{L}{3}$ and $\frac{h_1}{h_2} = \frac{5}{3}$ (Given)

$\therefore \frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$

**In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

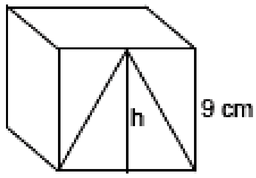
**9.** **Assertion (A):** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 $cm^2$.
**Reason (R):** If r be the radius and h be the height of the cylinder, then total surface area = $(2\pi rh + 2\pi r^2)$.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

**10.** **Assertion (A):** If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.
**Reason (R):** If r be the radius and h the slant height of the cone, then slant height = $\sqrt{(h^2 + r^2)}$
Ans: (d) Assertion (A) is false but reason (R) is true.

## SECTION – B

**Questions 11 to 14 carry 2 marks each.**

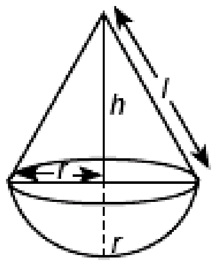
**11.** Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm? [Use $\pi = 22/7$]
Ans: Radius of cone = $\frac{1}{2} \times$ edge of cube = $\frac{9}{2}$ cm
Height of cone, $h = 9$ cm




Volume of cone = $\frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 9$
= 190.928 $cm^3$ = 190.93 $cm^3$ (approx.)

**12.** Two cubes each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid.
Ans: Length of resulting cuboid, $l$ = 4 cm + 4 cm = 8 cm, breadth, $b$ = 4 cm, height, h = $4$ cm
Surface area of cuboid = $2 (l \times b + b \times h + h \times l)$
= 2 (8 × 4 + 4 × 4 + 8 × 4) = 160 $cm^2$

**13.** A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.
Ans: Let radius of the base be $r$




Height of conical part = $h$
Slant height of conical part = $l$
$\therefore\ l = \sqrt{h^2 + r^2}$ ...(i)

ATQ $2\pi r^2 = \pi rl \Rightarrow l = 2r$

$\therefore$ Equation (i) becomes $2r = \sqrt{h^2 + r^2}$

$\Rightarrow 4r^2 = h^2 + r^2 \Rightarrow h^2 = 3r^2 \Rightarrow \frac{r^2}{h^2} = \frac{1}{3}$

$\Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}} \Rightarrow r : h = 1 : \sqrt{3}$

**14.** A solid cube is cut into two cuboids of equal volumes. Find the ratio of the total surface area of the given cube and that of one of the cuboids.

Ans: Let edge of the cube be $2x$ is cut into two cuboids.

Dimension of each cuboid are $2x$, $2x$, $x$.

Total surface area of the cube = 6 (edge)$^2$ = $6(2x)^2 = 6 \times 4x^2 = 24x^2$

Total surface area of one of the cuboid

$= 2(lb + bh + lh)$

$= 2(2x \times 2x + 2x \times x + 2x \times x)$

$= 2(4x^2 + 2x^2 + 2x^2) = 16x^2$

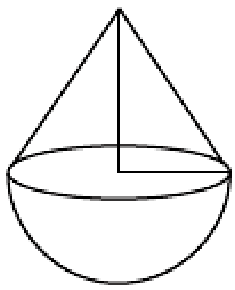
Total surface area of cube : Total surface area of one cuboid = $24x^2 : 16x^2 = 3 : 2$

## SECTION – C

**Questions 15 to 17 carry 3 marks each.**

**15.** A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of base of the cone is 21 cm and its volume is $\frac{2}{3}$ of the volume of the hemisphere, calculate the height of the cone and the surface area of the toy. [Use $\pi = \frac{22}{7}$]

Ans: Radius of the base of cone = 21 cm



Volume of cone = $\frac{2}{3}$ volume of hemisphere

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{2}{3}\pi r^3$

$\Rightarrow h = \frac{4}{3}r = \frac{4}{3} \times 21 = 28$ cm

$l^2 = h^2 + r^2 = (28)^2 + (21)^2 = 784 + 441 = 1225$

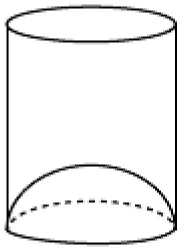
$l = \sqrt{1225} = 35$ cm

Surface area of the toy = $\pi rl + 2\pi r^2$

$= \frac{22}{7} \times 21 \times 35$ cm$^2$ + $2 \times 22 \times 21 \times 21$ cm$^2$

= 2310 cm$^2$ + 2772 cm$^2$ = 5082 cm$^2$

**16.** A juice seller serves his customers using a glass as shown in figure. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. [$\pi = 3.14$]



Ans: Here, inner diameter of the cylindrical glass = 5 cm
height of the glass = 10 cm
Apparent capacity of glass = $\pi r^2 h$

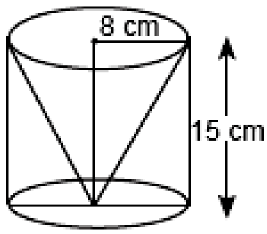
$= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10 \text{ cm}^3 = 196.25 \text{ cm}^3$

Actual capacity of glass = apparent capacity – volume of hemispherical part

$= 196.25 \text{ cm}^3 - \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^2 \text{ cm}^3$

$= 196.25 \text{ cm}^3 - 32.70 \text{ cm}^3 = 163.55 \text{ cm}^3$

**17.** From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take $\pi$ = 3.14]
Ans: Here, height of cylinder = 15 cm and diameter = 16 cm. So, radius, r = 8 cm

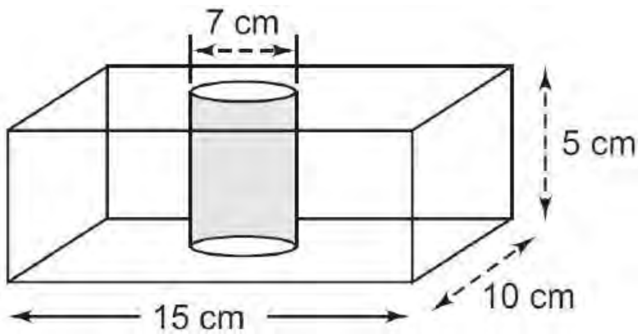



Total surface area of the solid = $2\pi rh + \pi rl + \pi r^2$
$= 2\pi \times 8 \times 15 + \pi \times 8 \times l + \pi(8)^2$
$= 240\pi + 8\pi \times 17 + 64\pi$ [$l = \sqrt{8^2 + 15^2} = 17$ cm]
$= \pi[240 + 136 + 64] = 440\pi = 440 \times 3.14 \text{ cm}^2 = 1381.6 \text{ cm}^2$

## SECTION – D

**Questions 18 carry 5 marks.**

**18.** A rectangular metal block has length 15 cm, breadth 10 cm and height 5 cm. From this block, a circular hole of diameter 7 cm is drilled out. Find: (i) the volume of the remaining solid (ii) the surface area of the remaining solid.
Ans: (i) The volume of the remaining solid
= Volume of rectangular block – Volume of the circular hole



$= (15\times10\times5)-\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times5$ cm³

= 750 cm³ – 192.5 cm³ = 557.5 cm³

(ii) The surface area of the remaining solid

= Total surface area of the block – 2 (area of circle of the hole) + curved surface of circular hole (cylinder)

$= 2(l \times b + b \times h + h \times l) - 2(\pi r^2) + 2\pi rh$

$= 2(15\times10+10\times5+5\times15)-2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times5+2\times\frac{22}{7}\times\frac{7}{2}\times5$

= 550 cm² – 77 cm² + 110 cm² = 583 cm².

**OR**

Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹120 per $m^2$, find the amount shared by each school to set up the tents.

**Ans:** Radius of the base of cylinder (r) = 2.8 m = Radius of the base of the cone (r)

Height of the cylinder (h)=3.5 m

Height of the cone (H)=2.1 m.

Slant height of conical part $(l)=\sqrt{(r^2 + H^2)} = \sqrt{[(2.8)^2 + (2.1)^2]} = \sqrt{(7.84 + 4.41)} = \sqrt{12.25} = 3.5$ m

Area of canvas used to make tent = CSA of cylinder + CSA of cone = $2\pi rh + \pi rl$

$= \pi r(2h + l) = \frac{22}{7}\times2.8\times(7+3.5)=22\times0.4\times10.5=92.4m^2$

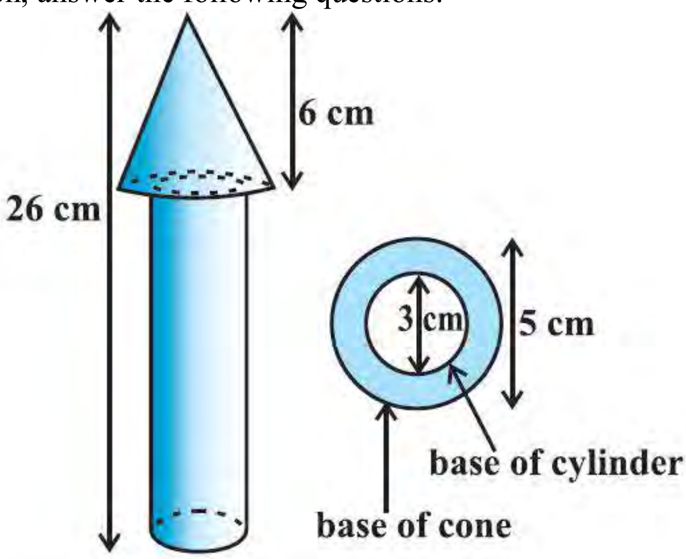
Cost of 1500 tents at ₹120 per sq.m = 1500 × 120 × 92.4 = 1,66,32,000

Share of each school to set up the tents = 16632000/50 = ₹3,32,640

## SECTION – E (Case Study Based Questions)

**Questions 19 to 20 carry 4 marks each.**

**19.** In a toys manufacturing company, wooden parts are assembled and painted to prepare a toy. One specific toy is in the shape of a cone mounted on a cylinder. For the wood processing activity center, the wood is taken out of storage to be sawed, after which it undergoes rough polishing, then is cut, drilled and has holes punched in it. It is then fine polished using sandpaper. For the retail packaging and delivery activity center, the polished wood sub-parts are assembled together, then decorated using paint. The total height of the toy is 26 cm and the height of its conical part is 6 cm. The diameters of the base of the conical part is 5 cm and that of the cylindrical part is 3 cm. On the basis of the above information, answer the following questions:




(a) If its cylindrical part is to be painted yellow, find the surface area need to be painted. [1]

(b) If its conical part is to be painted green, find the surface area need to be painted. [2]

OR

(b) Find the volume of the wood used in making this toy. [2]

(c) If the cost of painting the toy is 3 paise per sq cm, then find the cost of painting the toy. (Use π = 3.14) [1]

Ans: Let the radius of cone be r, slant height of cone be l, height of cone be h, radius of cylinder be r′ and height of cylinder be h′.

Then r = 2.5 cm, h = 6 cm, r′ = 1.5 cm, h′ = 26 – 6 = 20 cm and

Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{2.5^2 + 6^2} = \sqrt{6.25 + 36} = \sqrt{42.25} = 6.5cm$

(a) Area to be painted yellow = CSA of the cylinder + area of one base of the cylinder

$= 2\pi r'h' + \pi(r')^2 = \pi r'(2h' + r') = (3.14 \times 1.5)(2 \times 20 + 1.5)\ cm^2$

$= 4.71 \times 41.5\ cm^2$

$= 195.465\ cm^2$

(b) Area to be painted green = CSA of the cone + base area of the cone – base area of the cylinder

$= \pi rl + \pi r^2 - \pi(r')^2 = \pi[(2.5 \times 6.5) + (2.5)^2 - (1.5)^2]\ cm^2$

$= \pi[20.25]\ cm^2 = 3.14 \times 20.25\ cm^2$

$= 63.585\ cm^2$

**OR**

Volume of wood used in making the toy = Volume of cone + Volume of cylinder

$= \frac{1}{3}\pi r^2 h + \pi r'^2 h' = \pi\left[\frac{1}{3}r^2 h + r'^2 h'\right] = 3.14\left[\frac{1}{3} \times 2.5 \times 2.5 \times 6 + 1.5 \times 1.5 \times 20\right]$

$= 3.14(12.5 + 45) = 180.55 cm^3$

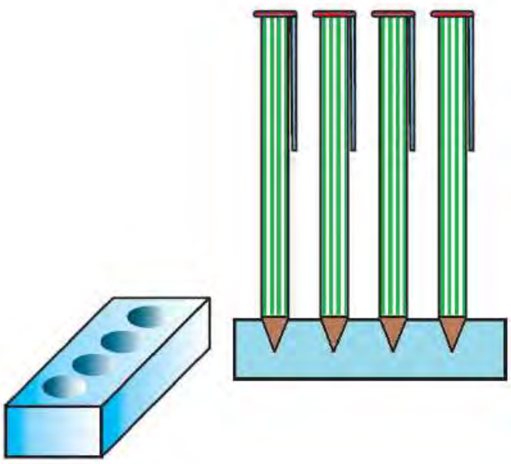
(c) Total area of painting = 195.465 + 63.585 = 259.05 $cm^2$

Cost of painting 1 $cm^2$ = Re. 0.03

Total cost of painting = Rs. 0.03 x 256.05

= Rs. 7.77

**20.** A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



Based on the above information, answer the following questions.

(i) Find the volume of four conical depressions in the entire stand [2]

(ii) Find the volume of wood in the entire stand [2]

**OR**

(ii) Three cubes each of side 15 cm are joined end to end. Find the total surface area of the resulting cuboid. [2]

Ans: (i) Dimensions of the cuboid are 15 cm, 10 cm and 3.5 cm.

∴ Volume of the cuboid = 15 × 10 × 35/10 $cm^3$

= 15 × 35 $cm^3$

= 525 $cm^3$

Since each depression is conical with base radius (r) = 0.5 cm and depth (h) = 1.4 cm,
∴ Volume of each depression (cone)

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{5}{10}\right)^2 \times \frac{14}{10} \text{ cm}^3$$

Since there are 4 depressions,
∴ Total volume of 4 depressions

$$= 4 \times \frac{1}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{14}{10} \text{ cm}^3 = \frac{4}{3} \times \frac{11}{10} \text{ cm}^3 = \frac{44}{30} \text{ cm}^3$$

(ii) Volume of the wood in entire stand
= [Volume of the wooden cuboid] − [Volume of 4 depressions]

$$= 525 \text{ cm}^3 - \frac{44}{30} \text{ cm}^3 = \frac{15750 - 44}{30} \text{ cm}^3 = \frac{15706}{30} \text{ cm}^3 = \mathbf{523.53 \ cm^3.}$$

**OR**

(ii) New length (l) = 15+15+15 = 45cm,
New breadth (b) = 15cm,
New height (h) = 15cm,
Total surface of the cuboid = 2(lb + bh + hl)
= 2( 45 x 15 + 15 x 15 + 15 x 45)
= 2 × 1575 = 3,150 $cm^2$