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PRACTICE PAPER 09 (2023-24)

CHAPTER 10 CIRCLES (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : X **DURATION : 1½ hrs**

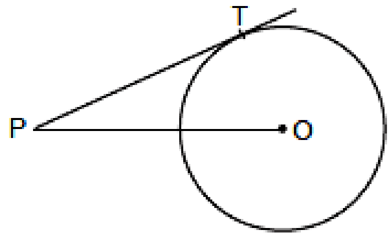
General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

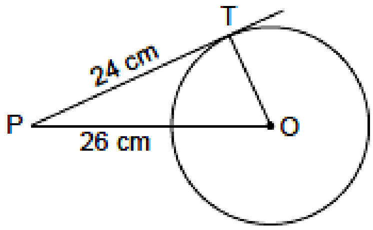
Questions 1 to 10 carry 1 mark each.

1. In the given below figure, point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then the radius of the circle is



(a) 25 cm (b) 26 cm (c) 24 cm (d) 10 cm

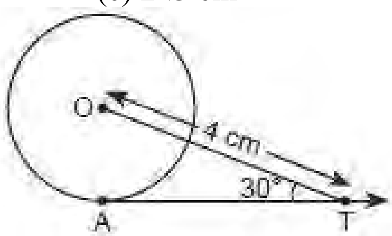
Ans: (d) ∵ OT is radius and PT is tangent



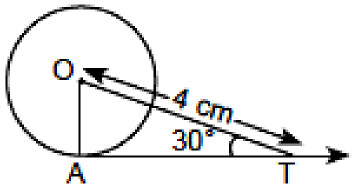
∴ OT ⊥ PT
Now, in ΔOTP,
$OP^2 = PT^2 + OT^2$
$\Rightarrow 26^2 = 24^2 + OT^2$
$\Rightarrow 676 – 576 = OT^2$
$\Rightarrow 100 = OT^2 \Rightarrow 10$ cm = OT

2. In the below figure AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°. Then AT is equal to

(a) 4 cm (b) 2 cm (c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm



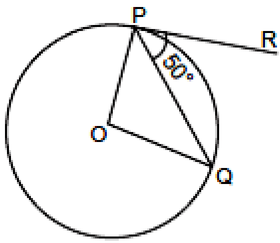
Ans: (c) $2\sqrt{3}$ cm



$\angle OAT = 90^0$ [∵ Tangent is perpendicular to the radius]
In right angled ΔOAT,

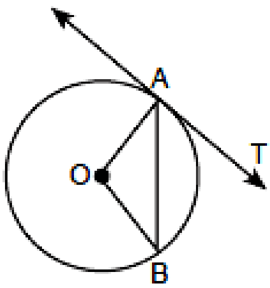
$$\cos 30^0 = \frac{AT}{OT} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4} \Rightarrow AT = 2\sqrt{3}\text{ cm}$$

3. In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠POQ is equal to



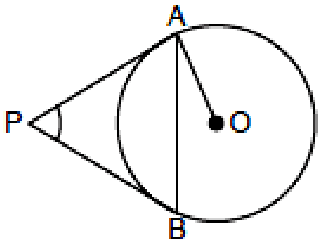
(a) 100° (b) 80° (c) 90° (d) 75°
Ans: (a) 100°
OP ⊥ PR [∵ Tangent and radius are ⊥ to each other at the point of contact]
∠OPQ = 90º – 50º = 40º
OP = OQ [Radii]
∴ ∠OPQ = ∠OQP = 40º
In ΔOPQ,
⇒ ∠POQ + ∠OPQ + ∠OQP = 180º
⇒ ∠POQ + 40º + 40º = 180º
∠POQ = 180° – 80° = 100º.

4. In figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠AOB = 100°, then ∠BAT is equal to



(a) 100° (b) 40° (c) 50° (d) 90°
Ans: (c) 50°
∠AOB = 100°
∠OAB = ∠OBA (∵ OA and OB are radii)
Now, in ΔAOB,
∠AOB + ∠OAB + ∠OBA = 180° (Angle sum property of Δ)
⇒ 100° + x + x = 180°
[Let ∠OAB = ∠OBA = x]
⇒ 2x = 180° – 100°
⇒ 2x = 80° ⇒ x = 40°
Also, ∠OAB + ∠BAT = 90° [∵ OA is radius and TA is tangent at A]
⇒ 40° + ∠BAT = 90° ⇒ ∠BAT = 50°

5. In the figure PA and PB are tangents to the circle with centre O. If ∠APB = 60°, then ∠OAB is



(a) 30° (b) 60° (c) 90° (d) 15°

Ans: (a) 30°

Given ∠APB = 60°

∵ ∠APB + ∠PAB + ∠PBA = 180°

⇒ ∠APB + x + x = 180° [∵ PA = PB ∴ ∠PAB = ∠PBA = x (say)]

⇒ 60° + 2x = 180°

⇒ 2x = 180° – 60°

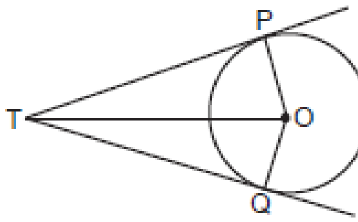
⇒ 2x = 120° ⇒ x = $\frac{120^0}{2}$ = 60°

Also, ∠OAP = 90° ⇒ ∠OAB + ∠PAB = 90°

⇒ ∠OAB + 60° = 90°

⇒ ∠OAB = 30°

6. In the given figure, TP and TQ are two tangents to a circle with centre O, such that ∠POQ = 110°. Then ∠PTQ is equal to



(a) 55° (b) 70° (c) 110° (d) 90°

Ans: (b) 70°

In quadrilateral POQT,

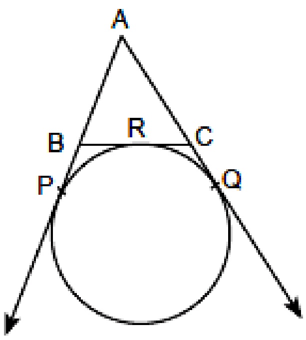
∠PTQ + ∠TPO + ∠TQO + ∠POQ = 360°

⇒ ∠PTQ + 90° + 90° + 110° = 360°

⇒ ∠PTQ + 290° = 360°

⇒ ∠PTQ = 360° – 290° = 70°

7. In figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then the length of AP (in cm) is



(a) 7.5 (b) 15 (c) 10 (d) 9

Ans: (a) 7.5

AP = AQ

⇒ AB + BP = AC + CQ

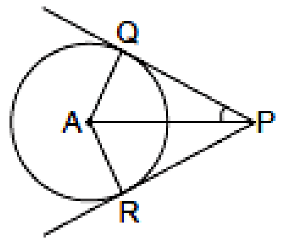
⇒ 5 + BP = 6 + CQ

BP = 1 + CQ

BP = 1 + CR

(∵ CQ = CR)
BP = 1 + (BC – BR)
BP = 1 + (4 – BP) (∵ BR = BP)
$2BP = 5 \Rightarrow BP = \frac{5}{2} = 2.5$ cm
Now, AP = AB + BP = 5 + 2.5 = 7.5 cm

8. In figure, PQ and PR are tangents to a circle with centre A. If ∠QPA = 27°, then ∠QAR equals to

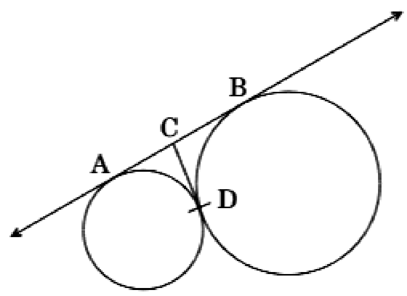


(a) 63° (b) 153° (c) 126° (d) 117°
Ans: (c) 126°
∠QPA = ∠RPA [∵ ΔAQP ≅ ΔARP (RHS congruence rule)]
⇒ ∠RPA = 27°
∴ ∠QPR = ∠QPA + ∠RPA = 27° + 27° = 54°
Now, ∠QAR + ∠AQP + ∠ARP + ∠QPR = 360°
⇒ ∠QAR = 90° + 90° + 54° = 360°
⇒ ∠QAR = 360° – 234° = 126°

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm then the radius of the circle is 7 cm.
Reason (R): A tangent to a circle is perpendicular to the radius through the point of contact
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. Assertion (A): In the below figure, AB and CD are common tangents to circles which touch each other at D. If AB = 8 cm, then the length of CD is 4 cm

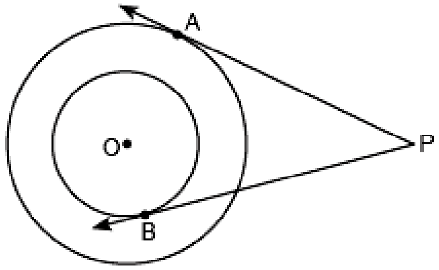


Reason (R): A tangent to a circle is perpendicular to the radius through the point of contact
Ans: (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

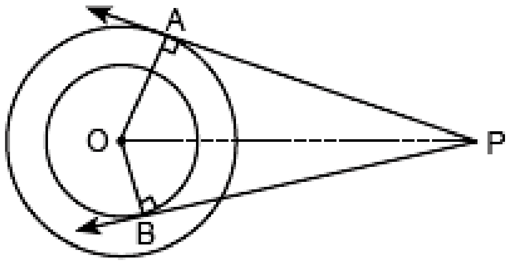
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the below figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.

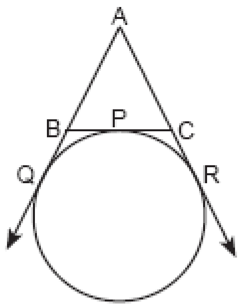


Ans: PA = 12 cm, OA = 5 cm, OB = 3 cm



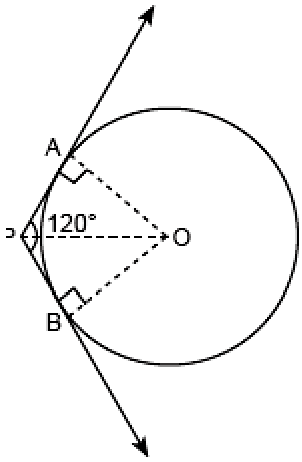
$OP^2 = OA^2 + AP^2 = OB^2 + BP^2$
$\Rightarrow 25 + 144 = 9 + BP^2$
$\Rightarrow 169 - 9 = BP^2$
$\Rightarrow BP = \sqrt{160}$ cm = 12.65 cm. (Approx.)

12. In figure, a circle touches the side BC of ΔABC at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of ΔABC.



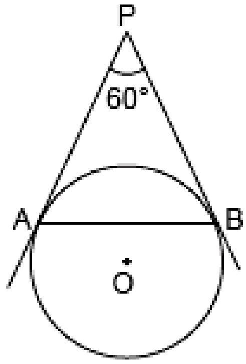
Ans: AQ and AR are tangents from the same point
AQ = AR = 5 cm ...(i) [Tangents from the same external points are equal]
BQ and BP are tangents from same point
⇒ BQ = BP ...(ii)
CP and CR are also tangents from the same point
⇒ CP = CR ...(iii)
In Δ ABC, Perimeter of Δ ABC = AB + BC + AC = AB + BP + CP + AC
AB + BQ + CR + AC = AQ + AR [From (ii) and (iii)]
= 5 cm + 5 cm = 10 cm [From (i)]
Perimeter of Δ ABC = 10 cm

13. Two tangents PA and PB are drawn to the circle with centre O, such that ∠APB = 120°. Prove that OP = 2AP.
Ans: Consider Δs PAO and PBO



PA = PB [Tangents to a circle, from a point outside it, are equal.]
OP = OP [Common]
$\angle$OAP = $\angle$OBP = 90°
$\therefore$ ΔOAP ≅ ΔOBP [RHS]
$\therefore$ $\angle OPA = \angle OPB = \frac{1}{2}\angle APB = \frac{1}{2} \times 120° = 60°$.
In right angled ΔOAP, $\frac{AP}{OP} = \cos 60° = \frac{1}{2} \Rightarrow OP = 2AP$.

14. In figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and $\angle$APB = 60°. Find the length of chord AB.

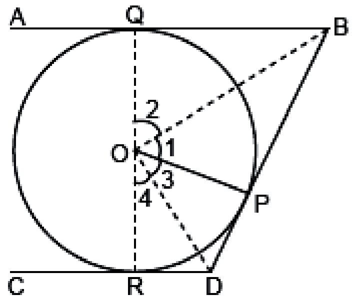


Ans: AP = BP [tangents from external point P]
$\therefore$ $\angle$PAB = $\angle$PBA [Angles opposite to equal sides]
Now $\angle APB + \angle PAB + \angle PBA = 180°$
$\Rightarrow 60° + 2\angle PAB = 180°$
$\Rightarrow \angle PAB = 60°$
$\therefore$ ΔAPB is an equilateral Δ
AB = AP = 5 cm

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.
Ans: AB and CD are two tangents to a circle and AB || CD.
Tangent BD intercepts an angle BOD at the centre.

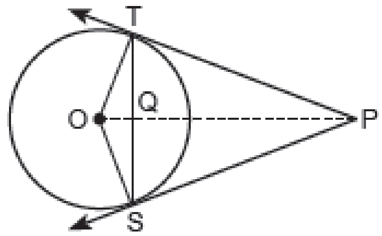


OP ⊥ BD.
[A tangent at any point of a circle is perpendicular to the radius through the point of contact.]
In right angled Δs OQB and OPB,
$\angle 1 = \angle 2$,
Similarly in right angled Δs OPD and ORD
$\angle 3 = \angle 4$

$\therefore \angle BOD = \angle 1 + \angle 3 = \frac{1}{2}\,[2\angle 1 + 2\angle 3)] = \frac{1}{2}\,(\angle 1 + \angle 1 + \angle 3 + \angle 3)$

$= \frac{1}{2}\,(\angle 1 + \angle 2 + \angle 3 + \angle 4) = \frac{1}{2}\,(180°) = 90°.$

16. In figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius *r*. If OP = 2*r*, show that ∠OTS = ∠OST = 30°.



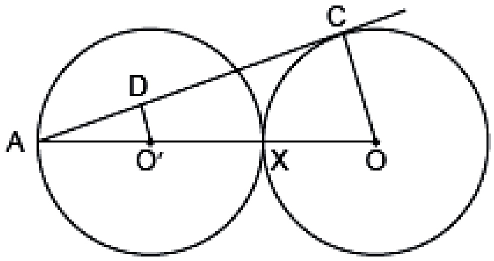
Ans: In ΔOTS OT = OS [radii]
⇒ ∠OTS = ∠OST ...(*i*)

$\therefore$ In right ΔOTP, $\frac{OT}{OP} = \sin \angle TPO$

$\Rightarrow \frac{r}{2r} = \sin \angle TPO \Rightarrow \sin \angle TPO = \frac{1}{2} = \sin 30° \Rightarrow \angle TPO = 30°$

Similarly ∠OPS = 30°
⇒ ∠TPS = 30° + 30° = 60°
Also ∠TPS + ∠SOT = 180°
⇒ ∠SOT = 120°
In ΔSOT, ∠SOT + ∠OTS + ∠OST = 180°
⇒ 120° + 2 ∠OTS = 180°
⇒ ∠OTS = 30° ...(*ii*)
From (*i*) and (*ii*)
∠OTS = ∠OST = 30°

17. In figure, two equal circles, with centres O and O′, touch each other at X. OO′ produced meets the circle with centre O′ at A. AC is tangent to the circle with centre O, at the point C. O′D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.



Ans: AC is tangent

$\therefore$ CO $\perp$ AC

Also O′D $\perp$ AC

$\therefore$ O′D $\parallel$ OC

Now OX = XO′ = O′A

$\Rightarrow$ AO = 3AO $\Rightarrow$ $\frac{AO'}{AO} = \frac{1}{3}$ ...(i)

In ΔAO′D, ΔAOC

$\angle$ADO = $\angle$ACO [each 90°]; $\angle$A = $\angle$A

$\therefore$ ΔAO′D ~ ΔAOC

$\therefore$ $\frac{AO'}{AO} = \frac{DO'}{CO}$

$\Rightarrow$ $\frac{DO'}{CO} = \frac{1}{3}$ [Using (i)]

SECTION – D

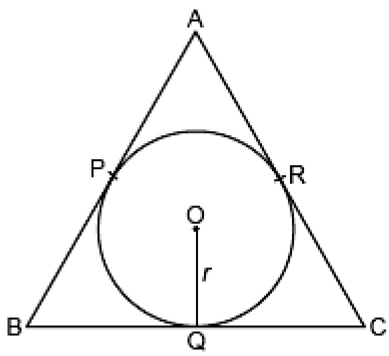
Questions 18 carry 5 marks.

18. In figure, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively.

Prove that

(i) AB + CQ = AC + BQ

(ii) Area (ΔABC) = $\frac{1}{2}$ (perimeter of ΔABC) × r

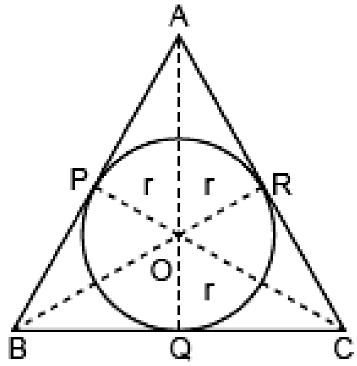


Ans:

(i) AP = AR [Tangents from A] ...(i)

Similarly, BP = BQ ...(ii)

CR = CQ ...(iii)



Now,

$\because$ AP = AR

$\Rightarrow$ (AB – BP) = (AC – CR)

$\Rightarrow$ AB + CR = AC + BP

$\Rightarrow$ AB + CQ = AC + BQ [Using eq. (ii) and (iii)]

(ii) Let AB = x, BC = y, AC = z

$\therefore$ Perimeter of ΔABC = $x + y + z$ ...(iv)

Area of ΔABC = $\frac{1}{2}$ [area of ΔAOB + area of ΔBOC + area of ΔAOC]

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2} \times AB \times OP + \frac{1}{2} \times BC \times OQ + \frac{1}{2} \times AC \times OR$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2} x \times r + \frac{1}{2} y \times r + \frac{1}{2} z \times r$

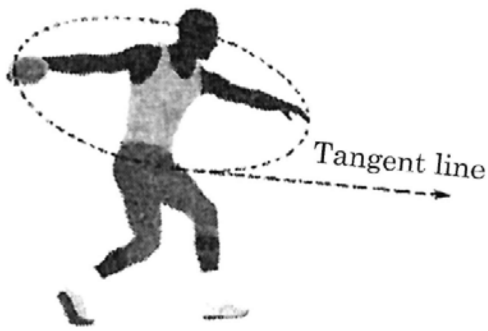
$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2} (x + y + z) \times r$

$\Rightarrow$ Area of ΔABC = $\frac{1}{2}$ (Perimeter of ΔABC) × r [Using (iv)]

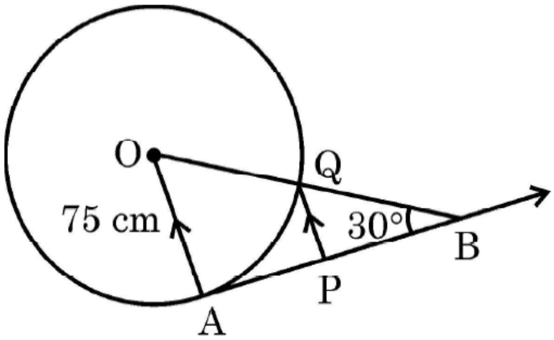
SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and ∠ABO = 30°. PQ is parallel to OA.



Based on the above, information:
(a) Find the length of AB. (1)
(b) Find the length of OB. (1)
(c) Find the length of AP. (2)

OR

(c) Find the length of PQ. (2)

Ans: (a) $\tan 30^0 = \frac{1}{\sqrt{3}} = \frac{75}{AB} \Rightarrow AB = 75\sqrt{3}\ \ cm$

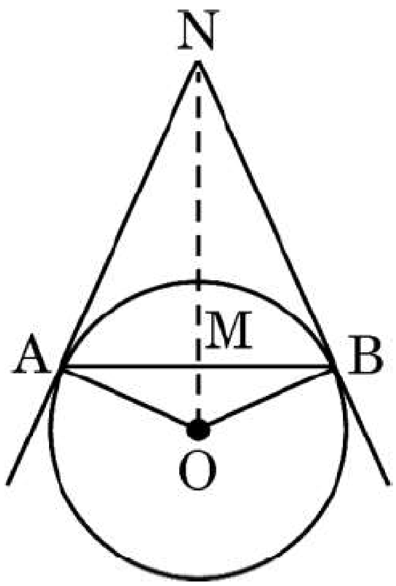
(b) $\sin 30^0 = \frac{1}{2} = \frac{75}{OB} \Rightarrow OB = 150\ \ cm$

(c) QB = 150 – 75 = 75 cm
$\Rightarrow$ Q is mid point. of OB
Since PQ || AO therefore P is mid point of AB
Hence AP = $\frac{75\sqrt{3}}{2}$ cm.

OR

$QB = 150 - 75 = 75cm$
$Now, \Delta BQP \sim \Delta BOA$
$\Rightarrow \frac{QB}{OB} = \frac{PQ}{OA} \Rightarrow \frac{1}{2} = \frac{PQ}{75} \Rightarrow PQ = \frac{75}{2}\ \ cm$

20. Circles play an important part in our life. When a circular object is hung on the wall with a cord at nail N, the cords NA and NB work like tangents. Observe the figure, given that ∠ANO = 30° and OA = 5 cm.

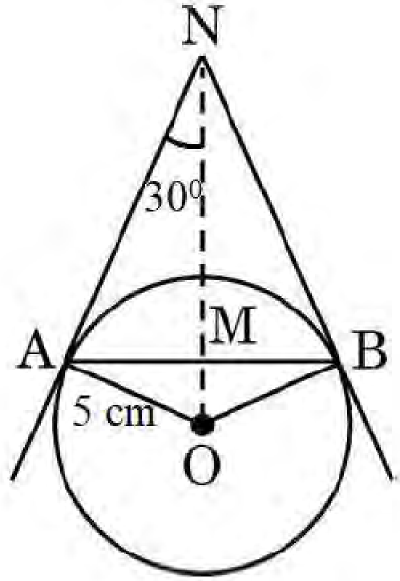


Based on the above, answer the following questions:
(a) Find the distance AN.
(b) Find the measure of ∠AOB.
(c) Find the total length of cords NA, NB and the chord AB.

OR

(c) If ∠ANO is 45°, then name the type of quadrilateral OANB.

Ans: (a) $\tan 30^0 = \frac{5}{AN} \Rightarrow AN = 5\sqrt{3}\ \ cm$



(b) ∠BNO = 30^0 ⇒ ∠ BNA = 60^0

∴ ∠ AOB = $180^0 - 60^0 = 120^0$

(c) AN = 5 3 and in ΔANB, ∠ANB = 60^0 and NA = NB

∴ ∠NAB = ∠NBA = 60^0 or ΔNAB is an equilateral triangle.

Hence, AB = 5 3 cm.

AN + NB + AB = $3 \times 5\sqrt{3} = 15\sqrt{3}$ cm.

OR

(c) ∠ ANO = 45^0 ⇒ ∠ AOB = 90^0

∴ Each angle of quad. AOBN is 90^0.

Also, OA = OB.

∴ OANB is a square.