

## Important Questions Class 9 Maths Chapter 2

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**Question 1:** Calculate the value of  $9x^2 + 4y^2$  if  $xy = 6$  and  $3x + 2y = 12$ .

**Answer 1:** Consider the equation  $3x + 2y = 12$

Now, square both sides:

$$(3x + 2y)^2 = 12^2$$

$$\Rightarrow 9x^2 + 12xy + 4y^2 = 144$$

$$\Rightarrow 9x^2 + 4y^2 = 144 - 12xy$$

From the questions,  $xy = 6$

So,

$$9x^2 + 4y^2 = 144 - 72$$

Thus, the value of  $9x^2 + 4y^2 = 72$

**Question 2:** Evaluate the following using suitable identity

$$(102)^3$$

**Answer 2:** We can write 102 as  $100+2$

Using identity,  $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$$(100+2)^3 = (100)^3 + 2^3 + (3 \times 100 \times 2)(100+2)$$

$$= 1000000 + 8 + 600(100 + 2)$$

$$= 1000000 + 8 + 60000 + 1200$$

$$= 1061208$$

**Question 3:** Without any actual division, prove that the following  $2x^4$

$-5x^3 + 2x^2 - x + 2$  is divisible by  $x^2 - 3x + 2$ .

[Hint: Factorise  $x^2 - 3x + 2$ ]

**Answer 3:**  $x^2 - 3x + 2$

$$x^2-2x-1x+2$$

$$x(x-2)-1(x-2)$$

$$(x-2)(x-1)$$

Therefore,  $(x-2)(x-1)$  are the factors.

Considering  $(x-2)$ ,

$$x-2=0$$

$$x=2$$

Then,  $p(x)$  becomes,

$$p(x)=2$$

$$p(x)=2x^4-5x^3+2x^2-x+2$$

$$p(2)=2(2)^4-5(2)^3+2(2)^2-2+2$$

$$=32-40+8$$

$$= -40+40=0$$

Therefore,  $(x-2)$  is a factor.

Considering  $(x-1)$ ,

$$x-1=0$$

$$x=1$$

Then,  $p(x)$  becomes,

$$p(x)=1$$

$$p(x)=2x^4-5x^3+2x^2-x+2$$

$$p(1)=2(1)^4-5(1)^3+2(1)^2-1+2$$

$$=2-5+2-1+2$$

$$=6-6$$

$$=0$$

Therefore,  $(x-1)$  is a factor.

**Question 4: Using the Factor Theorem to determine whether  $g(x)$  is a factor of  $p(x)$  in the following case**

**(i)  $p(x) = 2x^3+x^2-2x-1$ ,  $g(x) = x+1$**

**Answer 4:**  $p(x) = 2x^3+x^2-2x-1$ ,  $g(x) = x+1$

$$g(x) = 0$$

$$\Rightarrow x+1 = 0$$

$$\Rightarrow x = -1$$

$\therefore$  Zero of  $g(x)$  is  $-1$ .

Now,

$$p(-1) = 2(-1)^3+(-1)^2-2(-1)-1$$

$$= -2+1+2-1$$

$$= 0$$

$\therefore$  By the given factor theorem,  $g(x)$  is a factor of  $p(x)$ .

**Question 5: Obtain an example of a monomial and a binomial having degrees of 82 and 99, respectively.**

**Answer 5:** An example of a monomial having a degree of 82 =  $x^{82}$

An example of a binomial having a required degree of 99 =  $x^{99} + 7$

**Question 6: If the two  $x - 2$  and  $x - \frac{1}{2}$  are the given factors of  $px^2$**

**+  $5x + r$ , show that  $p = r$ .**

**Answer 6:** Given,  $f(x) = px^2+5x+r$  and factors are  $x-2$ ,  $x - \frac{1}{2}$

$$g_1(x) = 0,$$

$$x - 2 = 0$$

$$x = 2$$

Substituting  $x = 2$  in place of the equation, we get

$$f(x) = px^2 + 5x + r$$

$$f(2) = p(2)^2 + 5(2) + r = 0$$

$$= 4p + 10 + r = 0 \dots \text{eq.(i)}$$

$$x - \frac{1}{2} = 0$$

$$x = \frac{1}{2}$$

Substituting  $x = \frac{1}{2}$  in place of the equation, we get,

$$f(x) = px^2 + 5x + r$$

$$f\left(\frac{1}{2}\right) = p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$$

$$= \frac{p}{4} + \frac{5}{2} + r = 0$$

$$= p + 10 + 4r = 0 \dots \text{eq(ii)}$$

On solving eq(i) and eq(ii),

We get,

$$4p + r = -10 \text{ and } p + 4r = -10$$

the RHS of both equations are the same,

We get,

$$4p + r = p + 4r$$

$$3p = 3r$$

$$p = r.$$

Hence Proved.

**Question 7: Identify constant, linear, quadratic, cubic and quartic polynomials from the following.**

(i)  $-7 + x$

(ii)  $6y$

(iii)  $- ?^3$

(iv)  $1 - y - z^3$

(v)  $x - z^3 + z^4$

(vi)  $1 + x + z^2$

(vii)  $-6z^2$

(viii)  $-13$

(ix)  $-p$

**Answer 7:** (i)  $-7 + x$

The degree of  $-7 + x$  is 1.

Hence, it is a linear polynomial.

(ii)  $6y$

The degree of  $6y$  is 1.

Therefore, it is a linear polynomial.

(iii)  $-z^3$

We know that the degree of  $-z^3$  is 3.

Therefore, it is a cubic polynomial.

(iv)  $1 - y - z^3$

We know that the degree of  $1 - y - z^3$  is 3.

Therefore, it is a cubic polynomial.

(v)  $x - z^3 + z^4$

We know that the degree of  $x - z^3 + z^4$  is 4.

Therefore, it is a quartic polynomial.

(vi)  $1 + x + z^2$

We know that the degree of  $1 + x + z^2$  is 2.

Therefore, it is a quadratic polynomial.

(vii)  $-6x^2$

We know that the degree of  $-6x^2$  is 2.

Therefore, it is a quadratic polynomial.

(viii)  $-13$

We know that  $-13$  is a constant.

Therefore, it is a constant polynomial.

(ix)  $-p$

We know that the degree of  $-p$  is 1.

Therefore, it is a linear polynomial.

**Question 8: Observe the value of the polynomial  $5x - 4x^2 + 3$  at  $x = 2$  and  $x = -1$ .**

**Answer 8:** Let the polynomial be  $f(x) = 5x - 4x^2 + 3$

Now, for  $x = 2$ ,

$$f(2) = 5(2) - 4(2)^2 + 3$$

$$\Rightarrow f(2) = 10 - 16 + 3 = -3$$

Or, the value of the polynomial  $5x - 4x^2 + 3$  at  $x = 2$  is  $-3$ .

Similarly, for  $x = -1$ ,

$$f(-1) = 5(-1) - 4(-1)^2 + 3$$

$$\Rightarrow f(-1) = -5 - 4 + 3 = -6$$

The value of the polynomial  $5x - 4x^2 + 3$  at  $x = -1$  is  $-6$ .

**Question 9: Expanding each of the following, using all the suitable identities:**

(i)  $(x+2y+4z)^2$

(ii)  $(2x-y+z)^2$

(iii)  $(-2x+3y+2z)^2$

(iv)  $(3a - 7b - c)^2$

$$(v) (-2x+5y-3z)^2$$

$$\text{Answer 9: (i) } (x+2y+4z)^2$$

$$\text{Using identity, } (x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

$$\text{Here, } x = x$$

$$y = 2y$$

$$z = 4z$$

$$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2 \times x \times 2y)+(2 \times 2y \times 4z)+(2 \times 4z \times x)$$

$$= x^2+4y^2+16z^2+4xy+16yz+8xz$$

$$(ii) (2x-y+z)^2$$

$$\text{Using identity, } (x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

$$\text{Here, } x = 2x$$

$$y = -y$$

$$z = z$$

$$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2 \times 2x \times -y)+(2 \times -y \times z)+(2 \times z \times 2x)$$

$$= 4x^2+y^2+z^2-4xy-2yz+4xz$$

$$(iii) (-2x+3y+2z)^2$$

$$\text{Using identity, } (x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

$$\text{Here, } x = -2x$$

$$y = 3y$$

$$z = 2z$$

$$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2 \times -2x \times 3y)+(2 \times 3y \times 2z)+(2 \times 2z \times -2x)$$

$$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$$

$$(iv) (3a-7b-c)^2$$

$$\text{Using identity } (x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

Here,  $x = 3a$

$$y = -7b$$

$$z = -c$$

$$\begin{aligned}(3a - 7b - c)^2 &= (3a)^2 + (-7b)^2 + (-c)^2 + (2 \times 3a \times -7b) + (2 \times -7b \times -c) + (2 \times -c \times 3a) \\ &= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca\end{aligned}$$

$$(v) (-2x + 5y - 3z)^2$$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here,  $x = -2x$

$$y = 5y$$

$$z = -3z$$

$$\begin{aligned}(-2x + 5y - 3z)^2 &= (-2x)^2 + (5y)^2 + (-3z)^2 + (2 \times -2x \times 5y) + (2 \times 5y \times -3z) + (2 \times -3z \times -2x) \\ &= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx\end{aligned}$$

**Question 10:** If the polynomials  $az^3 + 4z^2 + 3z - 4$  and  $z^3 - 4z +$  leave the same remainder when divided by  $z - 3$ , find the value of  $a$ .

**Answer 10:** Zero of the polynomial,

$$g_1(z) = 0$$

$$z - 3 = 0$$

$$z = 3$$

Hence, zero of  $g(z) = -2a$

$$\text{Let } p(z) = az^3 + 4z^2 + 3z - 4$$

Now, substituting the given value of  $z = 3$  in  $p(z)$ , we get,

$$p(3) = a(3)^3 + 4(3)^2 + 3(3) - 4$$

$$\Rightarrow p(3) = 27a + 36 + 9 - 4$$

$$\Rightarrow p(3) = 27a + 41$$

$$\text{Let } h(z) = z^3 - 4z + a$$

Now, by substituting the value of  $z = 3$  in  $h(z)$ , we get,

$$h(3) = (3)^3 - 4(3) + a$$

$$\Rightarrow h(3) = 27 - 12 + a$$

$$\Rightarrow h(3) = 15 + a$$

As per the question,

The two polynomials,  $p(z)$  and  $h(z)$ , leave the same remainder when divided by  $z - 3$

$$\text{So, } h(3) = p(3)$$

$$\Rightarrow 15 + a = 27a + 41$$

$$\Rightarrow 15 - 41 = 27a - a$$

$$\Rightarrow -26 = 26a$$

$$\Rightarrow a = -1$$

**Question 11: Compute the perimeter of a rectangle whose area is  $25x^2 - 35x + 12$ .**

**Answer 11:** Area of rectangle =  $25x^2 - 35x + 12$

We know the area of a rectangle = length  $\times$  breadth

So, by factoring  $25x^2 - 35x + 12$ , the length and breadth can be obtained.

$$25x^2 - 35x + 12 = 25x^2 - 15x - 20x + 12$$

$$\Rightarrow 25x^2 - 35x + 12 = 5x(5x - 3) - 4(5x - 3)$$

$$\Rightarrow 25x^2 - 35x + 12 = (5x - 3)(5x - 4)$$

Thus, the length and breadth of a rectangle are  $(5x - 3)(5x - 4)$ .

So, the perimeter =  $2(\text{length} + \text{breadth})$

Therefore, the perimeter of the given rectangle =  $2[(5x - 3) + (5x - 4)]$

$$= 2(5x - 3 + 5x - 4)$$

$$= 2(10x - 7)$$

$$= 20x - 14$$

Hence, the perimeter of the rectangle =  $20x - 14$

**Question 12:**  $2x^2+y^2+z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

**Answer 12:** Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that,  $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$$

$$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + (2x \cdot -\sqrt{2}x \cdot y) + (2 \cdot y \cdot 2\sqrt{2}z) + (2 \cdot 2\sqrt{2}x \cdot -\sqrt{2}z)$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$$

**Question 13:** If  $x + 2a$  is a factor of  $x^5 - 4a^2x^3 + 2x + 2a + 3$ , find  $a$ .

**Answer 13:** According to the question,

Let  $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$  and  $g(x) = x + 2a$

$$g(x) = 0$$

$$\Rightarrow x + 2a = 0$$

$$\Rightarrow x = -2a$$

Hence, zero of  $g(x) = -2a$

As per the factor theorem,

If  $g(x)$  is a factor of  $p(x)$ , then  $p(-2a) = 0$

So, substituting the value of  $x$  in  $p(x)$ , we get,

$$p(-2a) = (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3 = 0$$

$$\Rightarrow -32a^5 + 32a^5 - 2a + 3 = 0$$

$$\Rightarrow -2a = -3$$

$$\Rightarrow a = 3/2$$

**Question 14:** Find the value of  $x^3 + y^3 + z^3 - 3xyz$  if  $x^2 + y^2 + z^2 = 83$  and  $x + y + z = 1$

**Answer 14:** Consider the equation  $x + y + z = 15$

From algebraic identities, we know that  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

So,

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + xz)$$

From the question,  $x^2 + y^2 + z^2 = 83$  and  $x + y + z = 15$

So,

$$15^2 = 83 + 2(xy + yz + xz)$$

$$\Rightarrow 225 - 83 = 2(xy + yz + xz)$$

$$\text{Or, } xy + yz + xz = 142/2 = 71$$

Using algebraic identity  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ ,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - (xy + yz + xz))$$

Now,

$$x + y + z = 15, x^2 + y^2 + z^2 = 83 \text{ and } xy + yz + xz = 71$$

$$\text{So, } x^3 + y^3 + z^3 - 3xyz = 15(83 - 71)$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 15 \times 12$$

$$\text{Or, } x^3 + y^3 + z^3 - 3xyz = 180$$

**Question 15: Verify that:**

$$\text{(i) } x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$\text{(ii) } x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

**Answer 15:** (i)  $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$$\text{We know that } (x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$$

$$\Rightarrow x^3 + y^3 = (x + y)[(x + y)^2 - 3xy]$$

$$\text{Taking } (x + y) \text{ common } \Rightarrow x^3 + y^3 = (x + y)[(x^2 + y^2 + 2xy) - 3xy]$$

$$\Rightarrow x^3 + y^3 = (x+y)(x^2 + y^2 - xy)$$

$$(ii) x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

We know that  $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$$

$$\Rightarrow x^3 - y^3 = (x-y)[(x-y)^2 + 3xy]$$

Taking  $(x+y)$  common  $\Rightarrow x^3 - y^3 = (x-y)[(x^2 + y^2 - 2xy) + 3xy]$

$$\Rightarrow x^3 + y^3 = (x-y)(x^2 + y^2 + xy)$$

**Question 16:** For what value of  $m$  is  $x^3 - 2mx^2 + 16$  divisible by  $x + 2$ ?

**Answer 16:** According to the question,

Let  $p(x) = x^3 - 2mx^2 + 16$ , and  $g(x) = x + 2$

$$g(x) = 0$$

$$\Rightarrow x + 2 = 0$$

$$\Rightarrow x = -2$$

Hence, zero of  $g(x) = -2$

As per the factor theorem,

if  $p(x)$  is divisible by  $g(x)$ , then the remainder of  $p(-2)$  should be zero.

Thus, substituting the value of  $x$  in  $p(x)$ , we obtain,

$$p(-2) = 0$$

$$\Rightarrow (-2)^3 - 2m(-2)^2 + 16 = 0$$

$$\Rightarrow 0 - 8 - 8m + 16 = 0$$

$$\Rightarrow 8m = 8$$

$$\Rightarrow m = 1$$

**Question 17:** If  $a + b + c = 15$  and  $a^2 + b^2 + c^2 = 83$ , find the value of  $a^3 + b^3 + c^3 - 3abc$ .

**Answer 17:** We know that,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \dots(i)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \dots(ii)$$

Given,  $a + b + c = 15$  and  $a^2 + b^2 + c^2 = 83$

From (ii), we have

$$15^2 = 83 + 2(ab + bc + ca)$$

$$\Rightarrow 225 - 83 = 2(ab + bc + ca)$$

$$\Rightarrow 142/2 = ab + bc + ca$$

$$\Rightarrow ab + bc + ca = 71$$

Now, (i) can be written as

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$$

$$a^3 + b^3 + c^3 - 3abc = 15 \times [83 - 71] = 15 \times 12 = 180.$$

**Question 18: Factorise:  $27x^3 + y^3 + z^3 - 9xyz$**

**Answer 18:** The expression  $27x^3 + y^3 + z^3 - 9xyz$  can be written as  $(3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$

$$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$$

We know that  $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$$

$$= (3x + y + z)[(3x)^2 + y^2 + z^2 - 3xy - yz - 3xz]$$

$$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$$

**Question 19: If  $(x - 1/x) = 4$ , then evaluate  $(x^2 + 1/x^2)$  and  $(x^4 + 1/x^4)$ .**

**Answer 19:** Given,  $(x - 1/x) = 4$

Squaring both sides, we get,

$$(x - 1/x)^2 = 16$$

$$\Rightarrow x^2 - 2 \cdot x \cdot 1/x + 1/x^2 = 16$$

$$\Rightarrow x^2 - 2 + 1/x^2 = 16$$

$$\Rightarrow x^2 + 1/x^2 = 16 + 2 = 18$$

$$\therefore (x^2 + 1/x^2) = 18 \dots(i)$$

Again, squaring both sides of (i), we get

$$(x^2 + 1/x^2)^2 = 324$$

$$\Rightarrow x^4 + 2 \cdot x^2 \cdot 1/x^2 + 1/x^4 = 324$$

$$\Rightarrow x^4 + 2 + 1/x^4 = 324$$

$$\Rightarrow x^4 + 1/x^4 = 324 - 2 = 322$$

$$\therefore (x^4 + 1/x^4) = 322.$$

### Question 20: Factorise

$$64m^3 - 343n^3$$

**Answer 20:** The expression  $64m^3 - 343n^3$  can be written as  $(4m)^3 - (7n)^3$

$$64m^3 - 343n^3 = (4m)^3 - (7n)^3$$

We know that  $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$$64m^3 - 343n^3 = (4m)^3 - (7n)^3$$

$$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$$

$$= (4m - 7n)(16m^2 + 28mn + 49n^2)$$

**Question 21:** Find out the values of a and b so that  $(2x^3 + ax^2 + x + b)$  has  $(x + 2)$  and  $(2x - 1)$  as factors.

**Answer 21:** Let  $p(x) = 2x^3 + ax^2 + x + b$ . Then,  $p(-2) = 0$  and  $p(1/2) = 0$ .

$$p(2) = 2(2)^3 + a(2)^2 + 2 + b = 0$$

$$\Rightarrow -16 + 4a - 2 + b = 0 \Rightarrow 4a + b = 18 \dots(i)$$

$$p(1/2) = 2(1/2)^3 + a(1/2)^2 + (1/2) + b = 0$$

$$\Rightarrow a + 4b = -3 \dots(ii)$$

On solving (i) and (ii), we get  $a = 5$  and  $b = -2$ .

Hence,  $a = 5$  and  $b = -2$ .

**Question 22: Explain that  $p - 1$  is a factor of  $p^{10} - 1$  and  $p^{11} - 1$ .**

**Answer 22:** According to the question,

Let  $h(p) = p^{10} - 1$ , and  $g(p) = p - 1$

zero of  $g(p) \Rightarrow g(p) = 0$

$$p - 1 = 0$$

$$p = 1$$

Therefore, zero of  $g(x) = 1$

We know that,

According to the factor theorem, if  $g(p)$  is a factor of  $h(p)$ , then  $h(1)$  should be zero

So,

$$h(1) = (1)^{10} - 1 = 1 - 1 = 0$$

$\Rightarrow g(p)$  is a factor of  $h(p)$ .

Here, we have  $h(p) = p^{11} - 1$ ,  $g(p) = p - 1$

Putting  $g(p) = 0 \Rightarrow p - 1 = 0 \Rightarrow p = 1$

As per the factor theorem, if  $g(p)$  is a factor of  $h(p)$ ,

Then  $h(1) = 0$

$$\Rightarrow (1)^{11} - 1 = 0$$

Hence,  $g(p) = p - 1$  is the factor of  $h(p) = p^{10} - 1$

**Question 23: Examine whether  $(7 + 3x)$  is a factor of  $(3x^3 + 7x)$ .**

**Answer 23:** Let  $p(x) = 3x^3 + 7x$  and  $g(x) = 7 + 3x$ . Now  $g(x) = 0 \Rightarrow x = -7/3$ .

By the remainder theorem,  $p(x)$  is divided by  $g(x)$ , and then the remainder is  $p(-7/3)$ .

$$\text{Now, } p(-7/3) = 3(-7/3)^3 + 7(-7/3) = -490/9 \neq 0.$$

$\therefore g(x)$  is not a factor of  $p(x)$ .

**Question 24: Prove that:**

$$x^3+y^3+z^3-3xyz = (1/2) (x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$$

**Answer 24:** We know that,

$$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz)$$

$$\Rightarrow x^3+y^3+z^3-3xyz = (1/2)(x+y+z)[2(x^2+y^2+z^2-xy-yz-xz)]$$

$$= (1/2)(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2xz)$$

$$= (1/2)(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2xz)]$$

$$= (1/2)(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$$

**Question 25: Find out which of the following polynomials has  $x - 2$  a factor:**

(i)  $3x^2 + 6x - 24$ .

(ii)  $4x^2 + x - 2$ .

**Answer 25:** (i) According to the question,

Let  $p(x) = 3x^2 + 6x - 24$  and  $g(x) = x - 2$

$$g(x) = x - 2$$

$$\text{zero of } g(x) \Rightarrow g(x) = 0$$

$$x - 2 = 0$$

$$x = 2$$

Hence, zero of  $g(x) = 2$

Thus, substituting the value of  $x$  in  $p(x)$ , we get,

$$p(2) = 3(2)^2 + 6(2) - 24$$

$$= 12 + 12 - 24$$

$$= 0$$

the remainder = zero,

We can derive that,

$g(x) = x - 2$  is factor of  $p(x) = 4x^2 + 2x - 2$

(ii) According to the question,

Let  $p(x) = 4x^2 + 2x - 2$  and  $g(x) = x - 2$

$g(x) = x - 2$

zero of  $g(x) \Rightarrow g(x) = 0$

$x - 2 = 0$

$x = 2$

Hence, zero of  $g(x) = 2$

Thus, substituting the value of  $x$  in  $p(x)$ , we get,

$p(2) = 4(2)^2 + 2 - 2$

$= 16 \neq 0$

Since the remainder  $\neq$  zero,

We can say that,

$g(x) = x - 2$  is not a factor of  $p(x) = 4x^2 + 2x - 2$

**Question 26: Factorise  $x^2 + 1/x^2 + 2 - 2x - 2/x$ .**

**Answer 26:**  $x^2 + 1/x^2 + 2 - 2x - 2/x = (x^2 + 1/x^2 + 2) - 2(x + 1/x)$

$= (x + 1/x)^2 - 2(x + 1/x)$

$= (x + 1/x)(x + 1/x - 2)$ .

**Question 27: Factorise**

**$8a^3 + b^3 + 12a^2b + 6ab^2$**

**Answer 27:** The expression,  $8a^3 + b^3 + 12a^2b + 6ab^2$  can be written as  $(2a)^3 + b^3 + 3(2a)^2b + 3(2a)(b)^2$

$8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + b^3 + 3(2a)^2b + 3(2a)(b)^2$

$= (2a + b)^3$

$$= (2a+b)(2a+b)(2a+b)$$

Here, the identity,  $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$  is used.

**Question 28: By Remainder Theorem, find out the remainder when  $p(x)$  is divided by  $g(x)$ , where**

(i)  $p(x) = x^3 - 2x^2 - 4x - 1$ ,  $g(x) = x + 1$

(ii)  $p(x) = x^3 - 3x^2 + 4x + 50$ ,  $g(x) = x - 3$

(iii)  $p(x) = 4x^3 - 12x^2 + 14x - 3$ ,  $g(x) = 2x - 1$

(iv)  $p(x) = x^3 - 6x^2 + 2x - 4$ ,  $g(x) = 1 - \frac{3}{2}x$

**Answer 28:** (i) Given  $p(x) = x^3 - 2x^2 - 4x - 1$  and  $g(x) = x + 1$

Here zero of  $g(x) = -1$

By applying the remainder theorem

$$P(x) \text{ divided by } g(x) = p(-1)$$

$$P(-1) = (-1)^3 - 2(-1)^2 - 4(-1) - 1 = 0$$

Therefore, the remainder = 0

(ii) given  $p(x) = x^3 - 3x^2 + 4x + 50$ ,  $g(x) = x - 3$

Here zero of  $g(x) = 3$

By applying the remainder theorem  $p(x)$  divided by  $g(x) = p(3)$

$$p(3) = 3^3 - 3 \times (3)^2 + 4 \times 3 + 50 = 62$$

Therefore, the remainder = 62

(iii)  $p(x) = 4x^3 - 12x^2 + 14x - 3$ ,  $g(x) = 2x - 1$

Here zero of  $g(x) = \frac{1}{2}$

By applying the remainder theorem  $p(x)$  divided by  $g(x) = p(\frac{1}{2})$

$$P(\frac{1}{2}) = 4(\frac{1}{2})^3 - 12(\frac{1}{2})^2 + 14(\frac{1}{2}) - 3$$

$$= \frac{4}{8} - \frac{12}{4} + \frac{14}{2} - 3$$

$$= \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

Hence, the remainder =  $\frac{3}{2}$

$$(iv) p(x) = x^3 - 6x^2 + 2x - 4, g(x) = 1 - \frac{3}{2}x$$

so, zero of  $g(x) = \frac{2}{3}$

By applying the remainder theorem  $p(x)$  divided by  $g(x) = p(\frac{2}{3})$

$$p(\frac{2}{3}) = (\frac{2}{3})^3 - 6(\frac{2}{3})^2 + 2(\frac{2}{3}) - 4$$

$$= -\frac{136}{27}$$

Therefore, the remainder =  $-\frac{136}{27}$

**Question 29: Factorise  $x^2 - 1 - 2a - a^2$ .**

$$\text{Answer 29: } x^2 - 1 - 2a - a^2 = x^2 - (1 + 2a + a^2)$$

$$= x^2 - (1 + a)^2$$

$$= [x - (1 + a)][x + 1 + a]$$

$$= (x - 1 - a)(x + 1 + a)$$

$$\therefore x^2 - 1 - 2a - a^2 = (x - 1 - a)(x + 1 + a).$$

**Question 30: Evaluate the following using suitable identity**

$$(998)^3$$

**Answer 30:** We can write 99 as  $1000 - 2$

$$\text{Using identity, } (x - y)^3 = x^3 - y^3 - 3xy(x - y)$$

$$(998)^3 = (1000 - 2)^3$$

$$= (1000)^3 - 2^3 - (3 \times 1000 \times 2)(1000 - 2)$$

$$= 1000000000 - 8 - 6000(1000 - 2)$$

$$= 1000000000 - 8 - 6000000 + 12000$$

$$= 994011992$$

**Question 31: Find the zeroes of the polynomial:**

$$p(x) = (x - 2)^2 - (x + 2)^2$$

**Answer 31:**  $p(x) = (x - 2)^2 - (x + 2)^2$

We know that,

Zero of the polynomial  $p(x) = 0$

Hence, we get,

$$\Rightarrow (x - 2)^2 - (x + 2)^2 = 0$$

Expanding using the identity,  $a^2 - b^2 = (a - b)(a + b)$

$$\Rightarrow (x - 2 + x + 2)(x - 2 - x - 2) = 0$$

$$\Rightarrow 2x(-4) = 0$$

$$\Rightarrow -8x = 0$$

Therefore, the zero of the polynomial = 0