

Ch 10 Heron's Formula Class 9 Important Questions NCERT Maths

Q.1 Find the Area of a Triangle whose two sides are 18 cm and 10 cm respectively and the perimeter is 42cm.

Solution:

Let us consider the third side of the triangle to be "c".

Now, the three sides of the triangle are $a = 18$ cm, $b = 10$ cm, and "c" cm

It is given that the perimeter of the triangle = 42cm

So,

$$18 + 10 + c = 42$$

$$c = 42 - (18 + 10) \text{ cm} = 14 \text{ cm}$$

$$\therefore \text{The semi perimeter of triangle (s)} = 42/2 = 21 \text{ cm}$$

Using Heron's formula,

$$\begin{aligned} \text{Area of the triangle } A &= s(s-a)(s-b)(s-c) \text{-----} \sqrt{s(s-a)(s-b)(s-c)} \\ &= 21(21-18)(21-10)(21-14) \text{-----} \sqrt{21(21-18)(21-10)(21-14)} \\ &= 21 \times 3 \times 11 \times 7 \text{-----} \sqrt{21 \times 3 \times 11 \times 7} \\ &= 21 \times 21 \times 11 \text{-----} \sqrt{21 \times 21 \times 11} \\ &= 2111 \text{-----} \sqrt{2111} \text{cm}^2 \end{aligned}$$

Q2: Sides of a Triangle are in the ratio of 14 : 20: 25 and its perimeter is 590cm. Find its area.

Solution:

The ratio of the sides of the triangle is given as 14: 20: 25

Let us consider the common ratio between the sides of the triangle be "a"

$$\therefore \text{The sides are } 14a, 20a \text{ and } 25a$$

It is also given that the perimeter of the triangle = 590 cm

$$12a + 17a + 25a = 590$$

$$\Rightarrow 59a = 590$$

$$\text{So, } a = 10$$

Now, the sides of the triangle are 140 cm, 200 cm, 250 cm.

So, the semi perimeter of the triangle (s) = $590/2 = 295$ cm

Using Heron's formula for Area of the triangle

$$= s(s-a)(s-b)(s-c) \text{-----} \sqrt{s(s-a)(s-b)(s-c)}$$

$$= 295(295-140)(295-200)(295-250) \text{-----} \\ -\sqrt{295(295-140)(295-200)(295-250)}$$

$$= 295 \times 155 \times 95 \times 45 \text{-----} \sqrt{295 \times 155 \times 95 \times 45}$$

$$= 195,474,375 \text{-----} \sqrt{195,474,375}$$

$$= 13981.21 \text{cm}^2$$

Q3: A field is in the Shape of a Trapezium whose parallel sides are 22 m and 10 m. The non-parallel sides are given as 13 m and 14 m. Find the area of the field.

Solution:

Draw a line segment BE line AD. Then, draw a perpendicular on the line segment CD from point B

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Now, it can be seen that the quadrilateral ABED is a parallelogram. So,

$$AB = DE = 10 \text{ m}$$

$$AD = BE = 13 \text{ m}$$

$$EC = DC - ED$$

$$= 22 - 10 = 12 \text{ m}$$

Now, consider the triangle BEC,

$$\text{Its semi perimeter (s) } = (13 + 14 + 12)/2$$

$$= 39/2 \text{ m}$$

$$= 19.5 \text{m}$$

By using Heron's formula,

Area of $\Delta BEC =$

$$= s(s-a)(s-b)(s-c) - \sqrt{s(s-a)(s-b)(s-c)}$$

$$= 19.5(19.5-13)(19.5-14)(19.5-15) - \sqrt{19.5(19.5-13)(19.5-14)(19.5-15)}$$

$$= 19.5 \times 6.5 \times 5.5 \times 4.5 - \sqrt{19.5 \times 6.5 \times 5.5 \times 4.5}$$

$$= 3137.06 - \sqrt{3137.06}$$

$$= 56 \text{ m}^2$$

We also know that the area of $\Delta BEC = (1/2) \times EC \times BF$

$$56 \text{ cm}^2 = (1/2) \times 12 \times BF$$

$$BF = 56 \times 2 / 12 \text{ cm}$$

$$= 9.3 \text{ cm}$$

So, the total area of ABED will be $BF \times DE$ i.e. $9.3 \times 10 = 93 \text{ m}^2$

$$\therefore \text{Area of the field} = 93 + 56 = 149 \text{ m}^2$$

Q4: Find the Area of the Triangular field of sides 55 m, 60 m, and 65 m. Find the cost of laying the grass in the triangular field at the rate of Rs 8 per m^2 .

Solution:

Given that

Sides of the triangular field are 50 m, 60 m and 65 m.

Cost of laying grass in a triangular field = Rs 8 per m^2

$$\text{Let } a = 55, b = 60, c = 65$$

$$\text{Semi- Perimeter } s = (a + b + c)/2$$

$$\Rightarrow s = (55 + 60 + 65)/2$$

$$= 180/2$$

$$= 90.$$

By using Heron's formula,

Area of $\Delta BEC =$

$$= s(s-a)(s-b)(s-c) - \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \frac{90(90-55)(90-60)(90-65)}{(90-65)} = \sqrt{90(90-55)(90-60)}$$

$$= \frac{90 \times 35 \times 30 \times 25}{25} = \sqrt{90 \times 35 \times 30 \times 25}$$

$$= \frac{2362500}{25} = \sqrt{2362500}$$

$$= 1537 \text{m}^2$$

Cost of laying grass = Area of triangle \times Cost of laying grass per m^2

$$= 1537 \times 8$$

$$= \text{Rs.}12296$$

Q5: The Perimeter of an Isosceles triangle is 42 cm. The ratio of the equal side to its base is 3: 4. Find the area of the triangle.

Solution:

Given that,

The perimeter of the isosceles triangle = 42 cm

It is also given that,

Ratio of equal side to base = 3 : 4

Let the equal side = 3x

So, base = 4x

Perimeter of the triangle = 42

$$\Rightarrow 3x + 3x + 4x = 42$$

$$\Rightarrow 10x = 42$$

$$\Rightarrow x = 4.2$$

Equal side = 3x = 3 \times 4.2 = 12.6

Base = 4x = 4 \times 4.2 = 16.8

The sides of the triangle = 12.6cm, 12.6cm and 16.8cm.

Let a = 12.6, b = 12.6, c = 16.8

$$s = (a + b + c)/2$$

$$\Rightarrow s = (12.6 + 12.6 + 16.8)/2$$

$$= 42/2$$

$$= 21.$$

By using Heron's formula,

Area of $\Delta BEC =$

$$= s(s-a)(s-b)(s-c) \text{-----} \sqrt{s(s-a)(s-b)(s-c)}$$

$$= 21(21-12.6)(21-12.6)(21-16.8) \text{-----} \sqrt{21(21-12.6)(21-12.6)(21-16.8)}$$

$$= 21 \times 8.4 \times 8.4 \times 4.2 \text{-----} \sqrt{21 \times 8.4 \times 8.4 \times 4.2}$$

$$= 6223.39 \text{-----} \sqrt{6223.39}$$

$$= 78.88 \text{cm}^2$$

Q.6 Find the area of a triangle whose two sides are 18 cm and 10 cm and the perimeter is 42cm.

Solution:

Assume that the third side of the triangle to be "x".

Now, the three sides of the triangle are 18 cm, 10 cm, and "x" cm

It is given that the perimeter of the triangle = 42cm

$$\text{So, } x = 42 - (18 + 10) \text{ cm} = 14 \text{ cm}$$

$$\therefore \text{The semi perimeter of triangle} = 42/2 = 21 \text{ cm}$$

Using Heron's formula,

Area of the triangle,

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-18)(21-10)(21-14)} \text{ cm}^2$$

$$= \sqrt{21 \times 3 \times 11 \times 7} \text{ m}^2$$

$$= 21\sqrt{11} \text{ cm}^2$$

Q.7: The sides of a triangle are in the ratio of 12: 17: 25 and its perimeter is 540cm. Find its area.

Solution:

The ratio of the sides of the triangle is given as 12: 17: 25

Now, let the common ratio between the sides of the triangle be “x”

∴ The sides are 12x, 17x and 25x

It is also given that the perimeter of the triangle = 540 cm

$$12x + 17x + 25x = 540 \text{ cm}$$

$$\Rightarrow 54x = 540 \text{ cm}$$

$$\text{So, } x = 10$$

Now, the sides of the triangle are 120 cm, 170 cm, 250 cm.

So, the semi perimeter of the triangle (s) = $540/2 = 270$ cm

Using Heron’s formula,

$$\begin{aligned} \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \left[\sqrt{270(270-120)(270-170)(270-250)} \right] \text{cm}^2 \\ &= \left[\sqrt{270 \times 150 \times 100 \times 20} \right] \text{cm}^2 \\ &= 9000 \text{ cm}^2 \end{aligned}$$

Q.8: A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Solution:

First, draw a line segment BE parallel to the line AD. Then, from B, draw a perpendicular on the line segment CD.

Now, it can be seen that the quadrilateral ABED is a parallelogram. So,

$$AB = ED = 10 \text{ m}$$

$$AD = BE = 13 \text{ m}$$

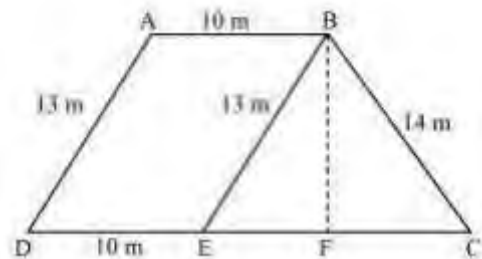
$$EC = 25 - ED = 25 - 10 = 15 \text{ m}$$

Now, consider the triangle BEC,

$$\text{Its semi perimeter (s)} = (13 + 14 + 15)/2 = 21 \text{ m}$$

By using Heron’s formula,

Area of $\triangle BEC =$



$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\left(\sqrt{21 \times (21-13) \times (21-14) \times (21-15)}\right) m^2$$

$$\left(\sqrt{21 \times 8 \times 7 \times 6}\right) m^2$$

$$= 84 m^2$$

We also know that the area of $\triangle BEC = (1/2) \times CE \times BF$

$$84 m^2 = (1/2) \times 15 \times BF$$

$$\Rightarrow BF = (168/15) cm = 11.2 cm$$

So, the total area of ABED will be $BF \times DE$, i.e. $11.2 \times 10 = 112 m^2$

$$\therefore \text{Area of the field} = 84 + 112 = 196 m^2$$

Q.9: A rhombus-shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Solution:

Draw a rhombus-shaped field first with the vertices as ABCD. The diagonal AC divides the rhombus into two congruent triangles which are having equal areas. The diagram is as follows.

Consider the triangle BCD,

$$\text{Its semi-perimeter} = (48 + 30 + 30)/2 m = 54 m$$

Using Heron's formula,

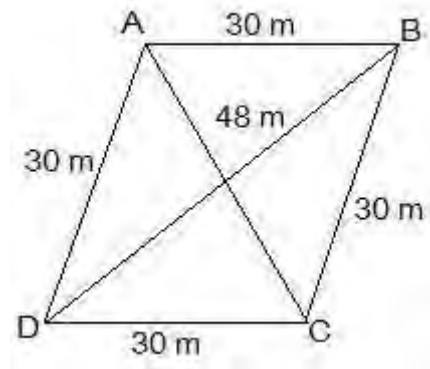
$$\text{Area of the } \triangle BCD =$$

$$= 432 m^2$$

$$\therefore \text{Area of field} = 2 \times \text{area of the } \triangle BCD = (2 \times 432) m^2 = 864 m^2$$

Thus, the area of the grass field that each cow will be getting = $(864/18) m^2 = 48 m^2$

Q.10: Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m^2 .



$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\left(\sqrt{54(54-48)(54-30)(54-30)}\right) m^2$$

$$\left(\sqrt{54 \times 6 \times 24 \times 24}\right) m^2$$

Solution:

According to the question,

Sides of the triangular field are 50 m, 65 m and 65 m.

Cost of laying grass in a triangular field = Rs 7 per m²

Let a = 50, b = 65, c = 65

$$s = (a + b + c)/2$$

$$\Rightarrow s = (50 + 65 + 65)/2$$

$$= 180/2$$

$$= 90.$$

$$\text{Area of triangle} = \sqrt{(s(s-a)(s-b)(s-c))}$$

$$= \sqrt{(90(90-50)(90-65)(90-65))}$$

$$= \sqrt{(90 \times 40 \times 25 \times 25)}$$

$$= 1500\text{m}^2$$

$$\text{Cost of laying grass} = \text{Area of triangle} \times \text{Cost per m}^2$$

$$= 1500 \times 7$$

$$= \text{Rs.}10500$$