Important Questions Class 10 Maths Chapter 7 Coordinate Geometry

Q. 1: Find the distance of the point P (2, 3) from the x-axis.

Solution:

We know that,

(x, y) = (2, 3) is a point on the Cartesian plane in the first quadrant.

x = Perpendicular distance from y-axis

y = Perpendicular distance from x-axis

Therefore, the perpendicular distance from x-axis = y coordinate = 3

Q. 2: Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).

Solution:

Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5).

Then, AP = BP

$AP^2 = BP^2$

Using distance formula,

$(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

$x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$

$x - y = 2$

Hence, the relation between x and y is $x - y = 2$.

Q. 3: Find the coordinates of the points of trisection (i.e., points dividing into three equal parts) of the line segment joining the points A(2, – 2) and B(– 7, 4).

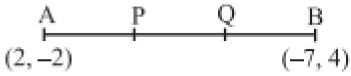
Solution:

Let P and Q be the points of trisection of AB, i.e., AP = PQ = QB.

Therefore, P divides AB internally in the ratio 1: 2.

Let $(x_1, y_1) = (2, -2)$

$(x_2, y_2) = (-7, 4)$



$m_1 : m_2 = 1 : 2$

Therefore, the coordinates of P, by applying the section formula,

$$\left(\frac{m_{1} x_{2}+m_{2} x_{1}}{m_{1}+m_{2}}, \frac{m_{1} y_{2}+m_{2} y_{1}}{m_{1}+m_{2}}\right)$$

$$=\left\lfloor \frac{1(-7)+2(2)}{1+2},\frac{1(4)+2(-2)}{1+2} \right\rfloor$$

= (-3/3, 0/3)

= (-1, 0)

Similarly, Q also divides AB internally in the ratio 2 : 1. and the coordinates of Q by applying the section formula,

$$=\left\lfloor \frac{2(-7)+1(2)}{2+1},\frac{2(4)+1(-2)}{2+1} \right\rfloor$$

= (-12/3, 6/3)

= (-4, 2)

Hence, the coordinates of the points of trisection of the line segment joining A and B are (−1, 0) and (– 4, 2).

Q. 4: Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Solution:

Let the ratio in which the line segment joining (-3, 10) and (6, -8) is divided by point (-1, 6) be k:1.

Therefore by section formula,

$$\left(\frac{k x_{2}+x_{1}}{k+1}, \frac{k y_{2}+y_{1}}{k+1}\right)$$

-1 = (6k-3)/(k+1)

−k – 1 = 6k -3

7k = 2

k = 2/7

Hence, the required ratio is 2 : 7.

Q. 5: Find the value of k if the points A(2, 3), B(4, k) and C(6, −3) are collinear.

Solution:

Given,

$A(2, 3) = (x_1, y_1)$

$B(4, k) = (x_2, y_2)$

$C(6, -3) = (x_3, y_3)$

If the given points are collinear, the area of the triangle formed by them will be 0.

$\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\frac{1}{2} [2(k + 3) + 4(-3 -3) + 6(3 - k)] = 0$

$\frac{1}{2} [2k + 6 - 24 + 18 - 6k] = 0$

$\frac{1}{2} (-4k) = 0$

$4k = 0$

$k = 0$

Q. 6: Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, −1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Solution:

Let the vertices of the triangle be A(0, -1), B(2, 1) and C(0, 3).

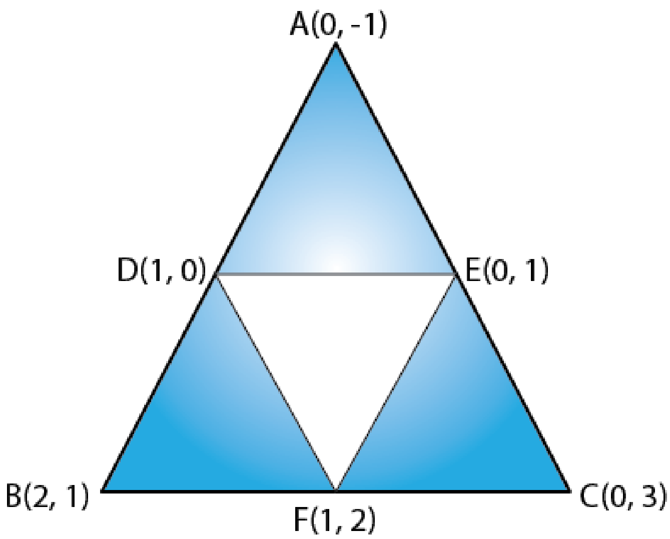
Let D, E, F be the mid-points of the sides of this triangle.

Using the mid-point formula, coordinates of D, E, and F are:

$D = [(0+2)/2, (-1+1)/2] = (1, 0)$

$E = [(0+0)/2, (-1+3)/2] = (0, 1)$

$F = [(0+2)/2, (3+1)/2] = (1, 2)$



We know that,

Area of triangle = ½ $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of triangle DEF = ½ $\{(1(2 - 1) + 1(1 - 0) + 0(0 - 2)\}$

= ½ $(1 + 1)$

= 1

Area of triangle DEF = 1 sq.unit

Area of triangle ABC = ½ $\{0(1 - 3) + 2(3 - (-1)) + 0(-1 - 1)\}$

= ½ (8)

= 4

Area of triangle ABC = 4 sq.units

Hence, the ratio of the area of triangle DEF and ABC = 1 : 4.

Q. 7: Name the type of triangle formed by the points A (–5, 6), B (–4, –2) and C (7, 5).

Solution:

The points are A (–5, 6), B (–4, –2) and C (7, 5).

Using distance formula,

$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

$AB = \sqrt{((-4+5)^2 + (-2-6)^2)}$

$= \sqrt{(1+64)}$

$=\sqrt{65}$

$BC=\sqrt{((7+4)^2 + (5+2)^2)}$

$=\sqrt{(121 + 49)}$

$=\sqrt{170}$

$AC=\sqrt{((7+5)^2 + (5-6)^2)}$

$=\sqrt{144} + 1$

$=\sqrt{145}$

Since all sides are of different lengths, ABC is a scalene triangle.

Q.8: Find the area of triangle PQR formed by the points P(-5, 7), Q(-4, -5) and R(4, 5).

Solution:

Given,

P(-5, 7), Q(-4, -5) and R(4, 5)

Let P(-5, 7) = (x_1, y_1)

Q(-4, -5) = (x_2, y_2)

R(4, 5) = (x_3, y_3)

Area of the triangle PQR = $(\frac{1}{2})|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= (\frac{1}{2}) |-5(-5 - 5) + (-4)(5 - 7) + 4(7 + 5)|$

$= (\frac{1}{2}) |-5(-10) -4(-2) + 4(12)|$

$= (\frac{1}{2}) |50 + 8 + 48|$

$= (\frac{1}{2}) \times 106$

$= 53$

Therefore, the area of triangle PQR is 53 sq. units.

Q.9: If the point C(-1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B.

Solution:

Given,

C(-1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4.

Here,

A(2, 5) = (x_1, y_1)

B(x, y) = (x_2, y_2)

m : n = 3 : 4

Using section formula,

C(-1, 2) = $[(mx_2 + nx_1)/(m + n), (my_2 + ny_1)/(m + n)]$

= [(3x + 8)/(3 + 4), (3y + 20)/(3 + 7)]

By equating the corresponding coordinates,

(3x + 8)/7 = -1

3x + 8 = -7

3x = -7 – 8

3x = -15

x = -5

And

(3y + 20)/7 = 2

3y + 20 = 14

3y = 14 – 20

3y = -6

y = -2

Therefore, the coordinates of B(x, y) = (-5, -2).

Q.10: Find the ratio in which the line x – 3y = 0 divides the line segment joining the points (-2, -5) and (6, 3). Find the coordinates of the point of intersection.

Solution:

Let the given points be:

$A(-2, -5) = (x_1, y_1)$

$B(6, 3) = (x_2, y_2)$

The line $x - 3y = 0$ divides the line segment joining the points A and B in the ratio k : 1.

Using section formula,

Point of division $P(x, y) = [(kx_2 + x_1)/(k + 1), (ky_2 + y_1)/(k + 1)]$

$x = (6k - 2)/(k + 1)$ and $y = (3k - 5)/(k + 1)$

Here, the point of division lies on the line $x - 3y = 0$.

Thus,

$[(6k - 2)/(k + 1)] - 3[(3k - 5)/(k + 1)] = 0$

$6k - 2 - 3(3k - 5) = 0$

$6k - 2 - 9k + 15 = 0$

$-3k + 13 = 0$

$-3k = -13$

$k = 13/3$

Thus, the ratio in which the line $x - 3y = 0$ divides the line segment AB is 13 : 3.

Therefore, $x = [6(13/3) - 2]/ [(13/3) + 1]$

$= (78 - 6)/(13 + 3)$

$= 72/16$

$= 9/2$

And

$y = [3(13/3) - 5]/ [(13/3) + 1]$

$= (39 - 15)/(13 + 3)$

$= 24/16$

$= 3/2$

Therefore, the coordinates of the point of intersection = (9/2, 3/2).

Q.11: Write the coordinates of a point on the x-axis which is equidistant from points A(-2, 0) and B(6, 0).

Solution:

Let P(x, 0) be a point on the x-axis.

Given that point, P is equidistant from points A(-2, 0) and B(6, 0).

AP = BP

Squaring on both sides,

$(AP)^2 = (BP)^2$

Using distance formula,

$(x + 2)^2 + (0 – 0)^2 = (x – 6)^2 + (0 – 0)^2$

$x^2 + 4x + 4 = x^2 – 12x + 36$

$4x + 12x = 36 – 4$

$16x = 32$

$x = 2$

Therefore, the coordinates of a point on the x-axis = (2, 0).

Q.12: If A(-2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence, find the lengths of its sides.

Solution:

Given vertices of a parallelogram ABCD are:

A(-2, 1), B(a, 0), C(4, b) and D(1, 2)

We know that the diagonals of a parallelogram bisect each other.

So, midpoint of AC = midpoint of BD

$[(-2 + 4)/2, (1 + b)/2] = [(a + 1)/2, (0 + 2)/2]$

By equating the corresponding coordinates,

$2/2 = (a + 1)/2$ and $(1 + b)/2 = 2/2$

$a + 1 = 2$ and $b + 1 = 2$

$a = 1$ and $b = 1$

Therefore, a = 1 and b = 1.

Let us find the lengths of sides of a parallelogram, i.e. AB, BC, CD and DA

Using the distance formula,

$AB = \sqrt{[(1 + 2)^2 + (0 - 1)^2]} = \sqrt{(9 + 1)} = \sqrt{10}$ units

$BC = \sqrt{[(4 - 1)^2 + (1 - 0)^2]} = \sqrt{(9 + 1)} = \sqrt{10}$ units

And $CD = \sqrt{10}$ and $DA = \sqrt{10}$ {the opposite sides of a parallelogram are parallel and equal}

Hence, the length of each side of the parallelogram ABCD = $\sqrt{10}$ units.

Q.13: If A(-5, 7), B(-4, -5), C(-1, -6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

Solution:

Given vertices of a quadrilateral are:

A(-5, 7), B(-4, -5), C(-1, -6) and D(4, 5)

The quadrilateral ABCD can be divided into two triangles ABD and BCD.

Area of the triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) = $(1/2)\ |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Area of triangle ABD = $(1/2)\ |-5(-5 - 5) + (-4)(5 - 7) + 4(7 + 5)|$

$= (1/2)\ |-5(-10) -4(-2) + 4(12)|$

$= (1/2)\ |50 + 8 + 48|$

$= (1/2) \times 106$

$= 53$

Area of triangle BCD = $(1/2)\ |-4(-6 - 5) + (-1)(5 + 5) + 4(-5 + 6)|$

$= (1/2)\ |-4(-11) -1(10) + 4(1)|$

$= (1/2)\ |44 - 10 + 4|$

$= (1/2) \times 38$

$= 19$

Therefore, the area of quadrilateral ABCD = Area of triangle ABD + Area of triangle BCD

$= 53 + 19$

$= 72$ sq.units

Q.14: Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, -3). Hence, find m.

Solution:

Let P(4, m) divides the line segment joining the points A(2, 3) and B(6, -3) in the ratio k : 1.

Here,

P(4, m) = (x, y)

A(2, 3) = (x_1, y_1)

B(6, -3) = (x_2, y_2)

Using section formula,

$p(x, y) = [(kx_2 + x_1)/(k + 1), (ky_2 + y_1)/(k + 1)]$

(4, m) = [(6k + 2)/(k + 1), (-3k + 3)/(k + 1)]

By equating the x-coodinate,

(6k + 2)/(k + 1) = 4

6k + 2 = 4k + 4

6k – 4k = 4 – 2

2k = 2

k = 1

Thus, the point P divides the line segment joining A and B in the ratio 1 : 1.

Now by equating the y-coodinate,

(-3k + 3)/(k + 1) = m

Substituting k = 1,

[-3(1) + 3]/(1 + 1) = m

m = (3 – 3)/2

m = 0

Q.15: Find the distance of a point P(x, y) from the origin.

Solution:

Given,

$P(x, y)$

Coordinates of origin = $O(0, 0)$

Let $P(x, y) = (x_1, y_1)$

$O(0, 0) = (x_2, y_2)$

Using distance formula,

$OP = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$= \sqrt{[(x - 0)^2 + (y - 0)^2]}$

$= \sqrt{(x^2 + y^2)}$

Hence, the distance of the point $P(x, y)$ from the origin is $\sqrt{(x^2 + y^2)}$ units.