6 Circle

Let's study.

- Circle
- Property of chord of the circle
- Incircle
- Circumcircle

Let's recall.

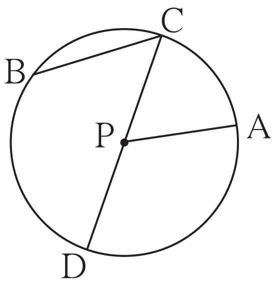


Fig. 6.1

In adjoining figure, observe the circle with center P. With reference to this figure, complete the following table.

---	seg PA	---	---	---	---	$\angle CPA$
chord	---	diameter	radius	centre	central angle	---

Let's learn.

Circle

Let us describe this circle in terms of a set of points.

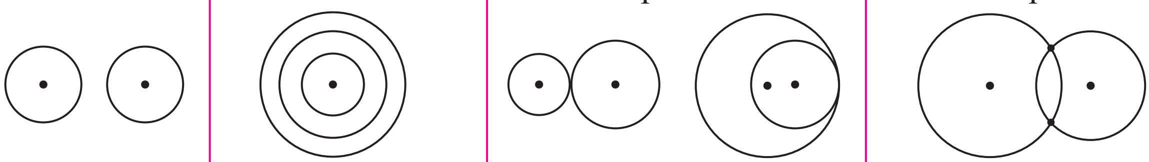
- The set of points in a plane which are equidistant from a fixed point in the plane is called a circle.

Some terms related with a circle.

- The fixed point is called the centre of the circle.
- The segment joining the centre of the circle and a point on the circle is called a radius of the circle.
- The distance of a point on the circle from the centre of the circle is also called the radius of the circle.
- The segment joining any two points of the circle is called a chord of the circle.
- A chord passing through the centre of a circle is called a diameter of the circle. A diameter is a largest chord of the circle.

Circles in a plane

Congruent circles	Concentric circles	Circles intersecting in a point	Circles intersecting in two points



• the same radii	• the same centre, different radii	• different centres, different radii, only one common point	• different centres, different radii, two common points

Fig. 6.2

Let's learn.

Properties of chord

Activity I : Every student in the group will do this activity. Draw a circle in your notebook. Draw any chord of that circle. Draw perpendicular to the chord through the centre of the circle. Measure the lengths of the two parts of the chord. Group leader will prepare a table and other students will write their observations in it.

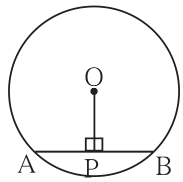


Fig. 6.3

Student / Length	1	2	3	4	5	6
l (AP)	...... cm					
l (PB)	...... cm					

Write theproperty which you have observed.
Let us write the proof of this property.

Theorem : A perpendicular drawn from the centre of a circle on its chord bisects the chord.

Given : seg AB is a chord of a circle with centre O.
seg OP $\perp$ chord AB

To prove : seg AP $\cong$ seg BP

Proof : Draw seg OA and seg OB

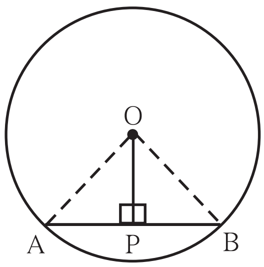


Fig. 6.4

In Δ OPA and Δ OPB

$\angle$OPA $\cong$ $\angle$OPB . . . . seg OP $\perp$ chord AB

seg OP $\cong$ seg OP . . . . common side

hypotenuse OA $\cong$ hypotenuse OB . . . . radii of the same circle

$\therefore$ Δ OPA $\cong$ Δ OPB . . . . hypotenuse side theorem

seg PA $\cong$ seg PB . . . . c.s.c.t.

Activity II : Every student from the group will do this activity. Draw a circle in your notebook. Draw a chord of the circle. Join the midpoint of the chord and centre of the circle. Measure the angles made by the segment with the chord. Discuss about the measures of the angles with your friends.

Which property do the observations suggest ?

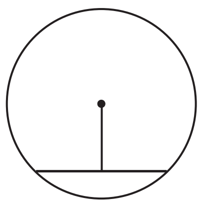


Fig. 6.5

Theorem : The segment joining the centre of a circle and the midpoint of its chord is perpendicular to the chord.

Given : seg AB is a chord of a circle with centre O and P is the midpoint of chord AB of the circle. That means seg AP $\cong$ seg PB.

To prove : seg OP $\perp$ chord AB

Proof : Draw seg OA and seg OB.

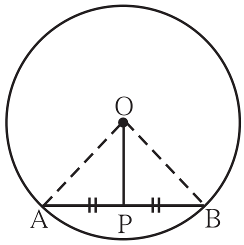


Fig. 6.6

In Δ AOP and Δ BOP

seg OA $\cong$ seg OB . . . . radii of the same circle

seg OP $\cong$ seg OP. . . . . common sides

seg AP $\cong$ seg BP . . . . given

$\therefore$ Δ AOP $\cong$ Δ BOP . . . . SSS test

$\therefore$ $\angle$OPA $\cong$ $\angle$OPB . . . . c.a.c.t. . . .(I)

Now $\angle OPA + \angle OPB = 180°$ . . . angles in linear pair

$\therefore$ $\angle OPB + \angle OPB = 180°$ . . . . . . from (I)

$\therefore$ $2 \angle OPB = 180°$

$\therefore$ $\angle OPB = 90°$

$\therefore$ seg OP $\perp$ chord AB

Solved examples

Ex (1) Radius of a circle is 5 cm. The length of a chord of the circle is 8 cm. Find the distance of the chord from the centre.

Solution :

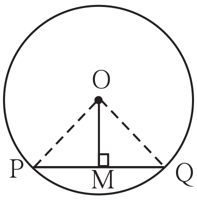


Fig. 6.7

Let us draw a figure from the given information.
O is the centre of the circle.
Length of the chord is 8 cm.
seg OM $\perp$ chord PQ.

We know that a perpendicular drawn from the centre of a circle on its chord bisects the chord.

$\therefore$ PM = MQ = 4 cm

Radius of the circle is 5 cm, means OQ = 5 cm .... given

In the right angled Δ OMQ using Pythagoras' theorem,

$OM^2 + MQ^2 = OQ^2$

$\therefore$ $OM^2 + 4^2 = 5^2$

$\therefore$ $OM^2 = 5^2 - 4^2 = 25 - 16 = 9 = 3^2$

$\therefore$ OM = 3

Hence distance of the chord from the centre of the circle is 3 cm.

Ex (2) Radius of a circle is 20 cm. Distance of a chord from the centre of the circle is 12 cm. Find the length of the chord.

Solution : Let the centre of the circle be O. Radius = OD = 20 cm.

Distance of the chord CD from O is12 cm. seg OP $\perp$ seg CD

$\therefore$ OP = 12 cm

Now CP = PD ...... perpendicular drawn from the centre bisects the chord

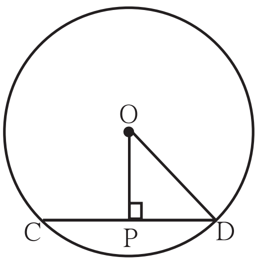


Fig. 6.8

In the right angled Δ OPD, using Pythagoras' theorem

$OP^2 + PD^2 = OD^2$

$(12)^2 + PD^2 = 20^2$

$PD^2 = 20^2 - 12^2$

$PD^2 = (20+12)(20-12)$

$= 32 \times 8 = 256$

$\therefore$ PD = 16 $\quad\quad$ $\therefore$ CP = 16

CD = CP + PD = 16 + 16 = 32

$\therefore$ the length of the chord is 32 cm.

Practice set 6.1

1. Distance of chord AB from the centre of a circle is 8 cm. Length of the chord AB is 12 cm. Find the diameter of the circle.
2. Diameter of a circle is 26 cm and length of a chord of the circle is 24 cm. Find the distance of the chord from the centre.
3. Radius of a circle is 34 cm and the distance of the chord from the centre is 30 cm, find the length of the chord.
4. Radius of a circle with centre O is 41 units. Length of a chord PQ is 80 units, find the distance of the chord from the centre of the circle.
5. In figure 6.9, centre of two circles is O. Chord AB of bigger circle intersects the smaller circle in points P and Q. Show that AP = BQ
6. Prove that, if a diameter of a circle bisects two chords of the circle then those two chords are parallel to each other.

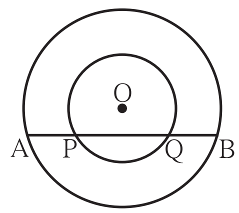


Fig. 6.9

Activity I

(1) Draw circles of convenient radii.

(2) Draw two equal chords in each circle.

(3) Draw perpendicular to each chord from the centre.

(4) Measure the distance of each chord from the centre.

Let's learn.

Relation between congruent chords of a circle and their distances from the centre

Activity II : Measure the lengths of the perpendiculars on chords in the following figures.

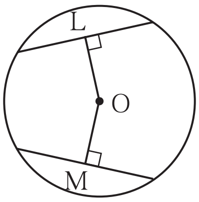


Figure (i)

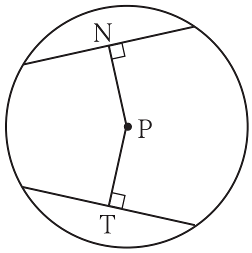


Figure (ii)

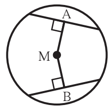


Figure (iii)

Did you find OL = OM in fig (i), PN = PT in fig (ii) and MA = MB in fig (iii) ?
Write the property which you have noticed from this activity.

Let's learn.

Properties of congruent chords

Theorem : Congruent chords of a circle are equidistant from the centre of the circle.

Given : In a circle with centre O

chord AB $\cong$ chord CD

OP $\perp$ AB, OQ $\perp$ CD

To prove : OP = OQ

Construction : Join seg OA and seg OD.

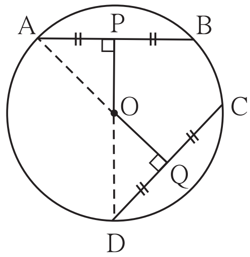


Fig. 6.10

Proof : AP = $\frac{1}{2}$ AB, DQ = $\frac{1}{2}$ CD . . . perpendicular drawn from the centre of a circle to its chord bisects the chord.

AB = CD . . . . given

$\therefore$ AP = DQ

$\therefore$ seg AP $\cong$ seg DQ . . . . (I) . . . segments of equal lengths

In right angled Δ APO and right angled Δ DQO

seg AP $\cong$ seg DQ . . . . from (I)

hypotenuse OA $\cong$ hypotenuse OD . . . . radii of the same circle

$\therefore$ Δ APO $\cong$ Δ DQO . . . . hypotenuse side theorem

seg OP $\cong$ seg OQ . . . . c.s.c.t.

$\therefore$ OP = OQ . . . . Length of congruent segments.

Congruent chords in a circle are equidistant from the centre of the circle.

Theorem : The chords of a circle equidistant from the centre of a circle are congruent.

Given : In a circle with centre O

seg OP $\perp$ chord AB

seg OQ $\perp$ chord CD

and OP = OQ

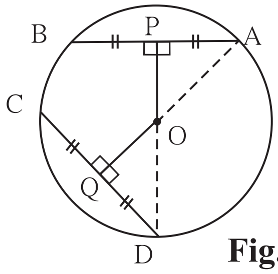


Fig. 6.11

To prove : chord AB $\cong$ chord CD

Construction : Draw seg OA and seg OD.

Proof : (Complete the proof by filling in the gaps.)

In right angled Δ OPA and right Δ OQD

hypotenuse OA $\cong$ hypotenuse OD . . . .

seg OP $\cong$ seg OQ . . . . given

$\therefore$ Δ OPA $\cong$ Δ OQD . . . .

$\therefore$ seg AP $\cong$ seg QD . . . . c.s.c.t.

$\therefore$ AP = QD . . . . (I)

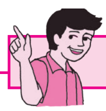
But AP = $\frac{1}{2}$ AB, and DQ = $\frac{1}{2}$ CD . . . . .

and AP = QD . . . . from (I)

$\therefore$ AB = CD

$\therefore$ seg AB $\cong$ seg CD

Note that both the theorems are converses of each other.



Remember this !

Congruent chords of a circle are equidistant from the centre of the circle.
The chords equidistant from the centre of a circle are congruent.

Activity : The above two theorems can be proved for two congruent circles also.

1. Congruent chords in congruent circles are equidistant from their respective centres.
2. Chords of congruent circles which are equidistant from their respective centres are congruent.

   Write 'Given', 'To prove' and the proofs of these theorems.

Solved example

Ex. In the figure 6.12, O is the centre of the circle and AB = CD. If OP = 4 cm, find the length of OQ.

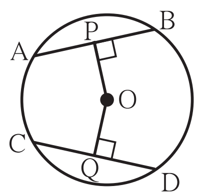


Fig. 6.12

Solution : O is the centre of the circle,

chord AB $\cong$ chord CD .....given

OP $\perp$ AB, OQ $\perp$ CD

OP = 4 cm, means distance of AB from the centre O is 4 cm.

The congruent chords of a circle are equidistant from the centre of the circle.

$\therefore$ OQ = 4 cm

Practice set 6.2

1. Radius of circle is 10 cm. There are two chords of length 16 cm each. What will be the distance of these chords from the centre of the circle ?
2. In a circle with radius 13 cm, two equal chords are at a distance of 5 cm from the centre. Find the lengths of the chords.
3. Seg PM and seg PN are congruent chords of a circle with centre C. Show that the ray PC is the bisector of $\angle$NPM.

Let's recall.

In previous standard we have verified the property that the angle bisectors of a triangle are concurrent. We denote the point of concurrence by letter I.

Let's learn.

Incircle of a triangle

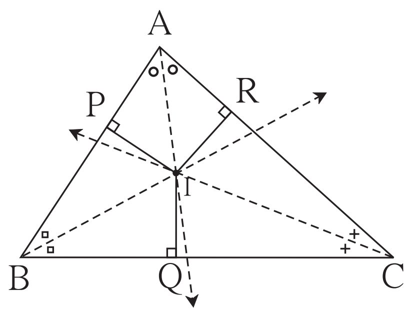


Fig. 6.13

In fig. 6.13, bisectors of all angles of a ΔABC meet in the point I. Perpendiculars on three sides are drawn from the point of concurrence.

$$IP \perp AB, \quad IQ \perp BC, \quad IR \perp AC$$

We know that, every point on the angle bisector is equidistant from the sides of the angle.

Point I is on the bisector of $\angle$B. $\therefore$ IP = IQ.

Point I is on the bisector of $\angle$C $\therefore$ IQ = IR

$$\therefore IP = IQ = IR$$

That is point I is equidistant from all the sides of ΔABC.

$\therefore$ if we draw a circle with centre I and radius IP, it will touch the sides AB, AC, BC of ΔABC internally.

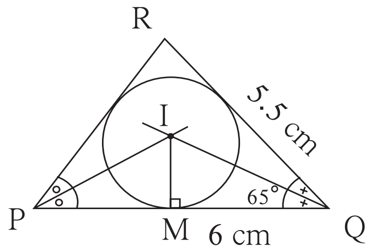
This circle is called the Incircle of the triangle ABC.

Let's learn.

To construct the incircle of a triangle

Ex. Construct Δ PQR such that PQ = 6 cm, $\angle Q = 35^\circ$, QR = 5.5 cm. Draw incircle of Δ PQR.

Draw a rough figure and show all measures in it.



Rough fig. 6.14

(1) Construct Δ PQR of given measures.

(2) Draw bisectors of any two angles of the triangle.

(3) Denote the point of intersection of angle bisectors as I.

(4) Draw perpendicular IM from the point I to the side PQ.

(5) Draw a circle with centre I and radius IM.

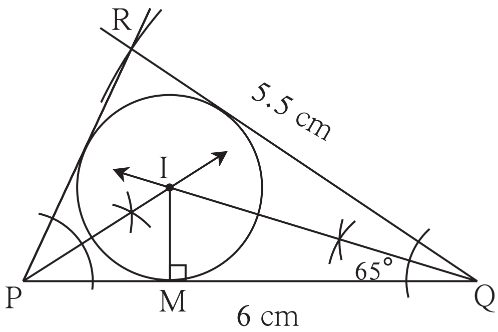


Fig. 6.15

Remember this !

The circle which touches all the sides of a triangle is called incircle of the triangle and the centre of the circle is called the incentre of the triangle.

Let's recall.

In previous standards we have verified the property that perpendicular bisectors of sides of a triangle are concurrent. That point of concurrence is denoted by the letter C.

Let's learn.

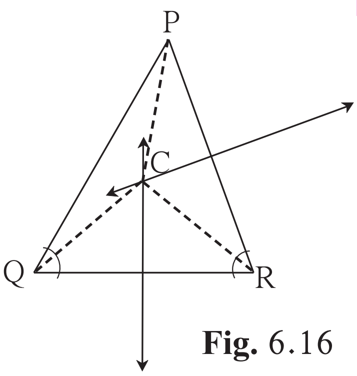


Fig. 6.16

In fig. 6.16, the perpendicular bisectors of sides of Δ PQR are intersecting at point C. So C is the point of concurrence of perpendicular bisectors.

Circumcircle

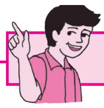
Point C is on the perpendicular bisectors of the sides of triangle PQR. Join PC, QC and RC. We know that, every point on the perpendicular bisector is equidistant from the end points of the segment.

Point C is on the perpendicular bisector of seg PQ. $\therefore$ PC = QC I

Point C is on the perpendicular bisector of seg QR. $\therefore$ QC = RC II

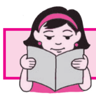
$\therefore$ PC = QC = RC . . . . . From I and II

$\therefore$ the circle with centre C and radius PC will pass through all the vertices of Δ PQR. This circle is called as the circumcircle.



Remember this !

Circle passing through all the vertices of a triangle is called circumcircle of the triangle and the centre of the circle is called the circumcentre of the triangle.



Let's learn.

To draw the circumcircle of a triangle

Ex. Construct Δ DEF such that DE = 4.2 cm, $\angle$D = 60°, $\angle$E = 70° and draw circumcircle of it. Draw rough figure. Write the given measures.

Rough figure

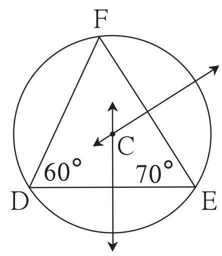


Fig. 6.17

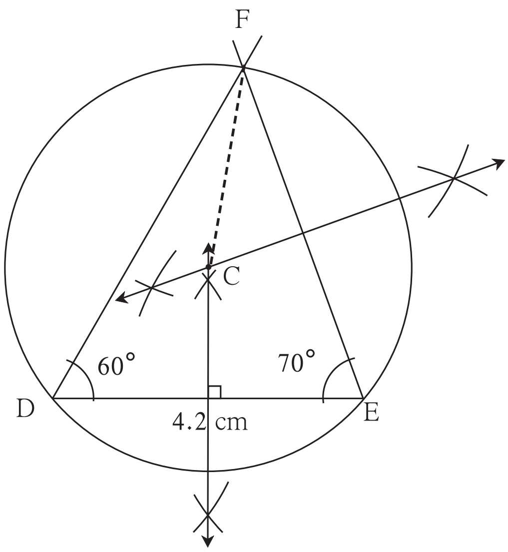


Fig. 6.18

Steps of construction **:**

(1) Draw Δ DEF of given measures.

(2) Draw perpendicular bisectors of any two sides of the triangle.

(3) Name the point of intersection of perpendicular bisectors as C.

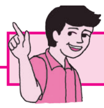
(4) Join seg CF.

(5) Draw circle with centre C and radius CF.

Activity :
Draw different triangles of different measures and draw incircles and circumcircles of them. Complete the table of observations and discuss.

Type of triangle	Equilateral triangle	Isosceles triangles	Scalene triangle
Position of incenter	Inside the triangle	Inside the triangle	Inside the triangle
Position of circumcentre	Inside the triangle	Inside, outside on the triangle	

Type of triangle	Acute angled triangle	Right angled triangle	Obtuse angled triangle
Position of incentre			
Position of circumcircle		Midpoint of hypotenuse	



Remember this !

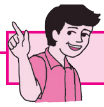
- Incircle of a triangle touches all sides of the triangle from inside.
- For construction of incircle of a triangle we have to draw bisectors of any two angles of the triangle.
- Circumcircle of a triangle passes through all the vertices of a triangle.
- For construciton of a circumcircle of a triangle we have to draw perpendicular bisectors of any two sides of the triangle.
- Circumcentre of an acute angled triangle lies inside the triangle.
- Circumcentre of a right angled triangle is the midpoint of its hypotenuse.
- Circumcentre of an obtuse angled triangle lies in the exterior of the triangle.
- Incentre of any triangle lies in the interior of the triangle.

Activity : Draw any equilateral triangle. Draw incircle and circumcircle of it. What did you observe while doing this activity ?

(1) While drawing incircle and circumcircle, do the angle bisectors and perpendicular bisectors coincide with each other ?

(2) Do the incentre and circumcenter coincide with each other ? If so, what can be the reason of it ?

(3) Measure the radii of incircle and circumcircle and write their ratio.



Remember this !

- The perpendicular bisectors and angle bisectors of an equilateral triangle are coincedent.
- The incentre and the circumcentre of an equilateral triangle are coincedent.
- Ratio of radius of circumcircle to the radius of incircle of an equilateral triangle is 2 : 1

Practice set 6.3

1. Construct Δ ABC such that $\angle B = 100^\circ$, BC = 6.4 cm, $\angle C = 50^\circ$ and construct its incircle.

2. Construct Δ PQR such that $\angle P = 70^\circ$, $\angle R = 50^\circ$, QR = 7.3 cm. and construct its circumcircle.

3. Construct Δ XYZ such that XY = 6.7 cm, YZ = 5.8 cm, XZ = 6.9 cm. Construct its incircle.

4. In Δ LMN, LM = 7.2 cm, $\angle M = 105^\circ$, MN = 6.4 cm, then draw Δ LMN and construct its circumcircle.

5. Construct Δ DEF such that DE = EF = 6 cm, $\angle F = 45^\circ$ and construct its circumcircle.

Problem set 6

1. Choose correct alternative answer and fill in the blanks.

(i) Radius of a circle is 10 cm and distance of a chord from the centre is 6 cm. Hence the length of the chord is ....

(A) 16 cm (B) 8 cm (C) 12 cm (D) 32 cm

(ii) The point of concurrence of all angle bisectors of a triangle is called the ......

(A) centroid (B) circumcentre (C) incentre (D) orthocentre

(iii) The circle which passes through all the vertices of a triangle is called .....

(A) circumcircle (B) incircle (C) congruent circle (D) concentric circle

(iv) Length of a chord of a circle is 24 cm. If distance of the chord from the centre is 5 cm, then the radius of that circle is ....

(A) 12 cm (B) 13 cm (C) 14 cm (D) 15 cm

(v) The length of the longest chord of the circle with radius 2.9 cm is .....

(A) 3.5 cm (B) 7 cm (C) 10 cm (D) 5.8 cm

(vi) Radius of a circle with centre O is 4 cm. If l(OP) = 4.2 cm, say where point P will lie.

(A) on the centre (B) Inside the circle (C) outside the circle(D) on the circle

(vii) The lengths of parallel chords which are on opposite sides of the centre of a circle are 6 cm and 8 cm. If radius of the circle is 5 cm, then the distance between these chords is .....

(A) 2 cm (B) 1 cm (C) 8 cm (D) 7 cm

2. Construct incircle and circumcircle of an equilateral Δ DSP with side 7.5 cm. Measure the radii of both the circles and find the ratio of radius of circumcircle to the radius of incircle.

3. Construct Δ NTS where NT = 5.7 cm, TS = 7.5 cm and $\angle$NTS = 110° and draw incircle and circumcircle of it.

4. In the figure 6.19, C is the centre of the circle.
seg QT is a diameter
CT = 13, CP = 5, find the length of chord RS.

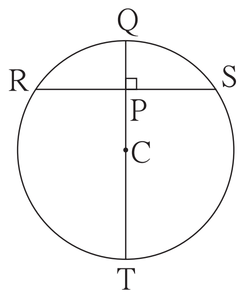


Fig. 6.19

5. In the figure 6.20, P is the centre of the circle.
Chord AB and chord CD intersect
on the diameter at the point E.
If $\angle$AEP $\cong$ $\angle$DEP
then prove that AB = CD.

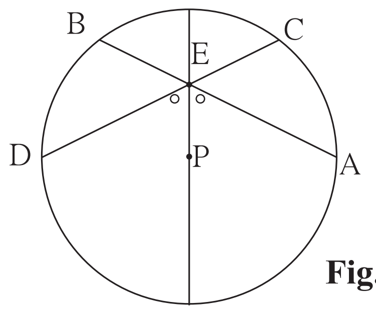


Fig. 6.20

6. In the figure 6.21, CD is a diameter of the circle with centre O. Diameter CD is perpendicular to chord AB at point E. Show that Δ ABC is an isosceles triangle.

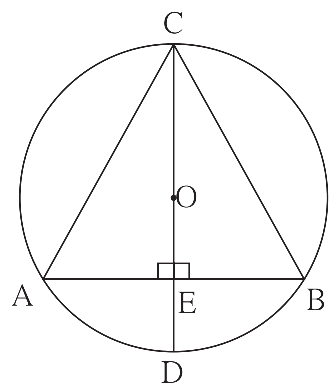


Fig. 6.21

ICT Tools or Links

Draw different circles with Geogebra software. Verify and experience the properties of chords. Draw circumcircle and incircle of different triangles. Using 'Move Option' experience how the incentre and circumcentre changes if the size of a triangle is changed.