5 Quadrilaterals

2464KQ

Let's study.

- Parallelogram
- Tests of parallelogram
- Rhombus
- Rectangles
- Square
- Trapezium
- Mid point theorem

Let's recall.

1. Write the following pairs considering □ABCD

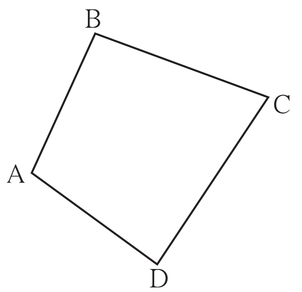


Fig. 5.1

Pairs of adjacent sides:
(1) ... , ... (2) ... , ...
(3) ... , ... (4) ... , ...

Pairs of adjacent angles :
(1) ... , ... (2) ... , ...
(3) ... , ... (4) ... , ...

Pairs of opposite sides (1) ..... , ... (2) ..... , ...
Pairs of opposite angles (1) ..... , ... (2) ..... , ...

Let's recall types of quadrilaterals and their properties .

I am a quadrilateral

My both pairs of opposite sides are parallel.

My properties
- Opposite sides congruent
- Opposite angels ......
- Diagonals .....

My all angles are right angles.

My properties
- Opposite sides .....
- Diagonals .....

My all sides are equal in length.

My properties
- Opposite sides .....
- Diagonals .....

My all angles are equal and all sides are equal.

My properties
- Diagonals .....

My only one pair of opposite sides is parallel.

You know different types of quadrilaterals and their properties. You have learned then through different activities like measuring sides and angles, by paper folding method etc. Now we will study these properties by giving their logical proofs.

A property proved logically is called a proof.

In this chapter you will learn that how a rectangle, a rhombus and a square are parallelograms. Let us start our study from parallelogram.

Let's learn.

Parallelogram

A quadrilateral having both pairs of opposite sides parallel is called a parallelogram.

For proving the theorems or for solving the problems we need to draw figure of a parallelogram frequently. Let us see how to draw a parallelogram.

Suppose we have to draw a parallelogram □ABCD.

Method I :

- Let us draw seg AB and seg BC of any length and making an angle of any measure with each other.
- Now we want seg AD and seg BC parallel to each other. So draw a line parallel to seg BC through the point A.
- Similarly we will draw line parallel to AB through the point C. These lines will intersect in point D.

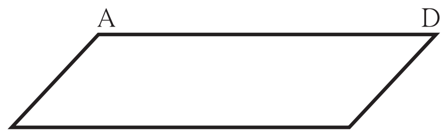


Fig. 5.2

So constructed quadrilateral ABCD will be a parallelogram.

Method II :

- Let us draw seg AB and seg BC of any length and making angle of any measure between them.
- Draw an arc with compass with centre A and radius BC.
- Similarly draw an arc with centre C and radius AB intersecting the arc previously drawn.
- Name the point of intersection of two arcs as D.

Draw seg AD and seg CD.

Quadrilateral so formed is a parallelogram ABCD

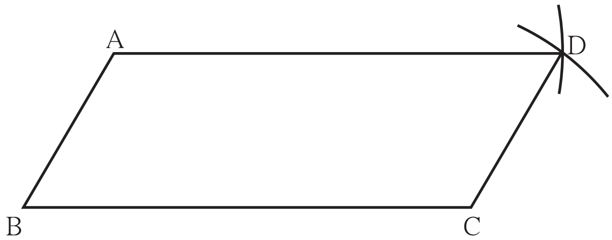


Fig. 5.3

In the second method we have actually drawn □ABCD in which opposite sides are equal. We will prove that a quadrilateral whose opposite sides are equal, is a parallelogram.

Activity I Draw five parallelograms by taking various measures of lengths and angles.

For the proving theorems on parallelogram, we use congruent triangles. To understand how they are used, let's do the following activity.

Activity II

- Draw a parallellogram ABCD on a card sheet. Draw diagonal AC. Write the names of vertices inside the triangle as shown in the figure. Then cut is out.

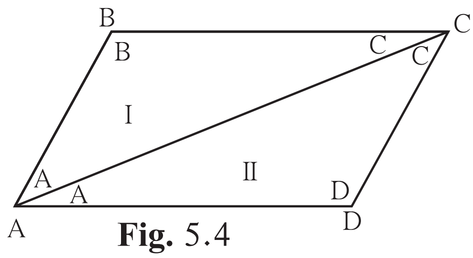


Fig. 5.4

- Fold the quadrilateral on the diagonal AC and see whether ΔADC and ΔCBA match with each other or not.

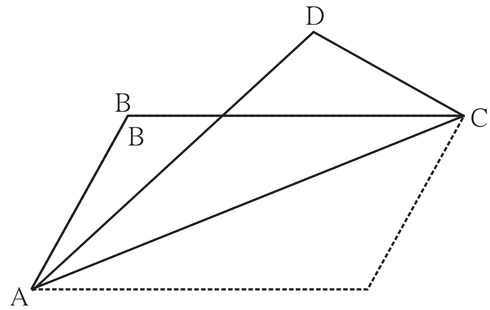


Fig. 5.5

- Cut □ABCD along diagonals AC and separate ΔADC and ΔCBA. By rotating and flipping ΔCBA, check whether it matchs exactly with ΔADC.

What did you find ? Which sides of ΔCBA match with which sides of ΔADC ? Which angles of ΔCBD match with which angles of ΔADC ?

Side DC matches with side AB and side AD matches with side CB. Similarly ∠B matches with ∠D.

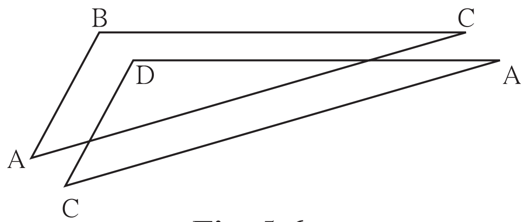


Fig. 5.6

So we can see that opposite sides and angles of a parallelogram are congruent.

We will prove these properties of a parallelogram.

Theorem 1. Opposite sides and opposite angles of a parallelogram are congruent.

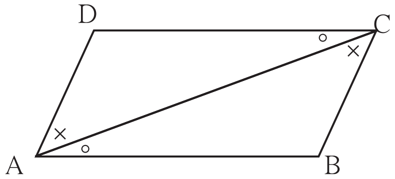


Fig. 5.7

Given : □ABCD is a parallelogram.
It means side AB || side DC,
side AD || side BC.

To prove : seg AD ≅ seg BC ; seg DC ≅ seg AB
∠ADC ≅ ∠CBA, and ∠DAB ≅ ∠BCD.

Construction : Draw diagonal AC.

Proof : seg DC || seg AB and diagonal AC is a transversal.

∴ ∠DCA ≅ ∠BAC ....(1)
and ∠DAC ≅ ∠BCA ....(2) }.....alternate angles

Now , in ΔADC and ΔCBA,

∠DAC ≅ ∠BCA .... from (2)
∠DCA ≅ ∠BAC .... from (1)
seg AC ≅ seg CA .... common side
∴ ΔADC ≅ ΔCBA .... ASA test
∴ side AD ≅ side CB .... c.s.c.t.
and side DC ≅ side AB .... c.s.c.t.,
Also, ∠ADC ≅ ∠CBA .... c.a.c.t.

Similarly we can prove ∠DAB ≅ ∠BCD.

Use your brain power!

In the above theorem, to prove ∠DAB ≅ ∠BCD, is any change in the construction needed ? If so, how will you write the proof making the change ?

To know one more property of a parallelogram let us do the following activity.

Activity : Draw a parallelogram PQRS. Draw diagonals PR and QS. Denote the intersection of diagonals by letter O. Compare the two parts of each diagonal with a divider. What do you find ?

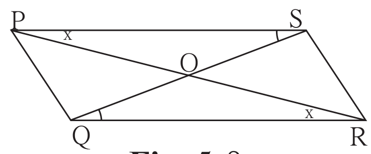


Fig. 5.8

Theorem : Diagonals of a parallelogram bisect each other.

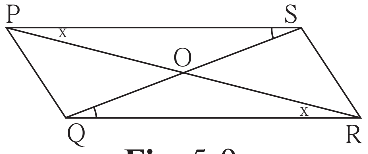


Fig. 5.9

Given : $\square$PQRS is a parallelogram. Diagonals PR and QS intersect in point O.

To prove : seg PO $\cong$ seg RO,
seg SO $\cong$ seg QO.

Proof : In ΔPOS and ΔROQ

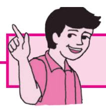
$\angle$OPS $\cong$ $\angle$ORQ .... alternate angles

side PS $\cong$ side RQ .... opposite sides of parallelogram

$\angle$PSO $\cong$ $\angle$RQO .... alternate angles

$\therefore \Delta$POS $\cong$ ΔROQ .... ASA test

$\therefore$ seg PO $\cong$ seg RO ....
and seg SO $\cong$ seg QO .... } ..... corresponding sides of congruent triangles



Remember this !

- Adjacent angles of a parallelogram are supplementary.
- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Diagonals of a parallelogram bisect each other.

Solved Examples

Ex (1) $\square$PQRS is a parallelogram. PQ = 3.5, PS = 5.3 $\angle Q = 50^\circ$ then find the lengths of remaining sides and measures of remaining angles.

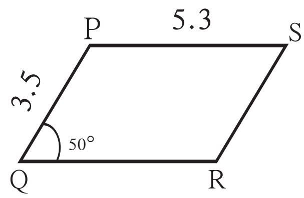


Fig. 5.10

Solution : $\square$PQRS is a parallelogram.

$\therefore \angle Q + \angle P = 180^\circ$ .... interior angles are supplementary.

$\therefore\ 50^\circ + \angle P = 180^\circ$

$\therefore\ \angle P = 180^\circ - 50^\circ = 130^\circ$

Now , $\angle P = \angle R$ and $\angle Q = \angle S$ ....opposite angles of a parallelogram.

$\therefore\ \angle R = 130^\circ$ and $\angle S = 50^\circ$

Similarly, PS = QR and PQ = SR ....opposite sides of a parallelogram.

$\therefore$ QR = 5.3 and SR = 3.5

Ex (2) □ABCD is a parallelogram. If $\angle A = (4x + 13)^\circ$ and $\angle D = (5x - 22)^\circ$ then find the measures of $\angle B$ and $\angle C$.

Solution : Adjacent angles of a parallelogram are supplementary.

$\angle A$ and $\angle D$ are adjacent angles.

$\therefore (4x + 13)^\circ + (5x - 22)^\circ = 180$

$\therefore 9x - 9 = 180$

$\therefore 9x = 189$

$\therefore x = 21$

$\therefore \angle A = 4x + 13 = 4 \times 21 + 13 = 84 + 13 = 97^\circ$ $\quad \therefore \angle C = 97^\circ$

$\angle D = 5x - 22 = 5 \times 21 - 22 = 105 - 22 = 83^\circ$ $\quad \therefore \angle B = 83^\circ$

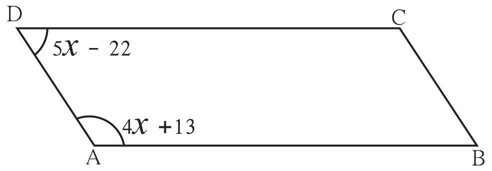


Fig. 5.11

Practice set 5.1

1. Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135^\circ$ then what is the measure of $\angle XWZ$ and $\angle YZW$?
   If $l(OY) = 5$ cm then $l(WY) = $?

2. In a parallelogram ABCD, If $\angle A = (3x + 12)^\circ$, $\angle B = (2x - 32)^\circ$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.

3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.

4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.

5*. Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13 then show that □ABCD is a rhombus.

6. In the figure 5.12, □PQRS and □ABCR are two parallelograms.
   If $\angle P = 110^\circ$ then find the measures of all angles of □ABCR.

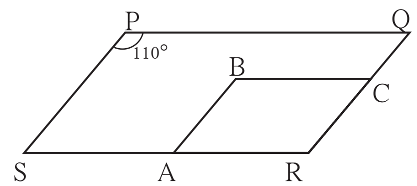


Fig. 5.12

7. In figure 5.13 □ABCD is a parallelogram. Point E is on the ray AB such that BE = AB then prove that line ED bisects seg BC at point F.

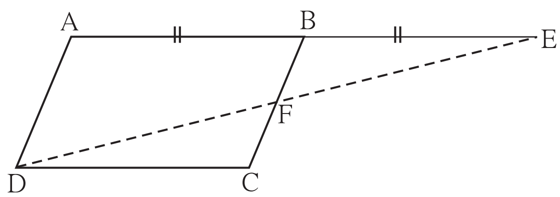


Fig. 5.13

Let's recall.

Tests for parallel lines

1. If a transversal interesects two lines and a pair of corresponding angles is congruent then those lines are parallel.
2. If a transversal intersects two lines and a pair of alternate angles is corgruent then those two lines are parallel.
3. If a transversal intersects two lines and a pair of interior angles is supplementary then those two lines are parallel.

Let's learn.

Tests for parallelogram

Suppose, in □PQRS, PS = QR and PQ = SR and we have to prove that □PQRS is a parallelogram. To prove it, which pairs of sides of □PQRS should be shown parallel ?

Which test can we use to show the sides parallel ? Which line will be convenient as a transversal to obtain the angles necessary to apply the test ?

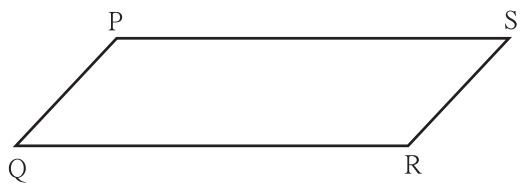


Fig. 5.14

Theorem : If pairs of opposite sides of a quadrilateral are congruent then that quadrilateral is a parallelogram.

Given : In □PQRS

side PS ≅ side QR

side PQ ≅ side SR

To prove : □PQRS is a parallelogram.

Construction : Draw diagonal PR.

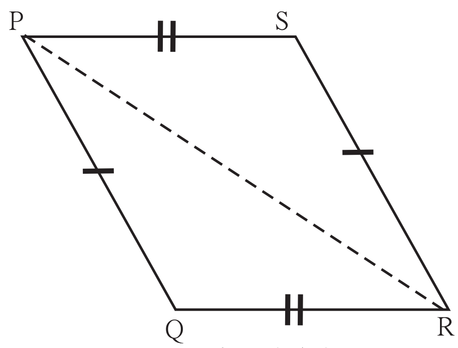


Fig. 5.15

Proof : In Δ SPR and Δ QRP

side PS ≅ side QR ....given

side SR ≅ side QP .... given

side PR ≅ side RP .... common side

∴ Δ SPR ≅ Δ QRP ...... sss test

∴ ∠SPR ≅ ∠QRP .... c.a.c.t.

Similarly, ∠PRS ≅ ∠RPQ ..... c.a.c.t.

∠SPR and ∠QRP are alternate angles formed by the transversal PR of seg PS and seg QR.

$\therefore$ side PS || side QR ......(I) alternate angles test for parallel lines.

Similarly $\angle$PRS and $\angle$RPQ are the alternate angles formed by transversal PR of seg PQ and seg SR.

$\therefore$ side PQ || side SR ......(II) .....alternate angle test

$\therefore$ from (I) and (II) □PQRS is a parallelogram.

On page 56, two methods to draw a parallelogram are given. In the second method actually we have drawn a quadrilateral of which opposite sides are equal. Did you now understand why such a quadrilateral is a parallelogram ?

Theorem : If both the pairs of opposite angles of a quadrilateral are congruent then it is a parallelogram.

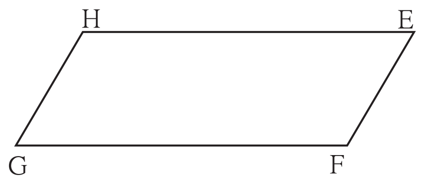


Fig. 5.16

Given : In □EFGH $\angle E \cong \angle G$
and $\angle$.......... $\cong$ $\angle$..........

To prove : □EFGH is a ....

Proof : Let $\angle E = \angle G = x$ and $\angle H = \angle F = y$

Sum of all angles of a quadrilateral is ....

$\therefore$ $\angle E + \angle G + \angle H + \angle F =$ ....

$\therefore$ $x + y +$ .... $+$ .... $=$ ....

$\therefore$ $\square x + \square y =$ .....

$\therefore$ $x + y = 180°$

$\therefore$ $\angle G + \angle H =$ ....

$\angle$G and $\angle$H are interior angles formed by transversal HG of seg HE and seg GF.

$\therefore$ side HE || side GF .... (I) interior angle test for parallel lines.

Similarly, $\angle G + \angle F =$ ....

$\therefore$ side .... || side .... (II) interior angle test for parallel lines.

$\therefore$ From (I) and (II), □EFGH is a ....

Theorem : If the diagonals of a quadrilateral bisect each other then it is a parallelogram.

Given : Diagonals of □ABCD bisect each other in the point E.
It means seg AE ≅ seg CE
and seg BE ≅ seg DE

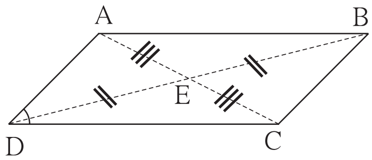


Fig. 5.17

To prove : □ABCD is a parallelogram.

Proof : Find the answers for the following questions and write the proof of your own.

1. Which pair of alternate angles should be shown congruent for proving seg AB || seg DC ? Which transversal will form a pair of alternate angles ?
2. Which triangles will contain the alternate angles formed by the transversal?
3. Which test will enable us to say that the two triangles congruent ?
4. Similarly, can you prove that seg AD || seg BC?

The three theorems above are useful to prove that a given quadrilateral is a parallelogram. Hence they are called as tests of a parallelogram.

One more theorem which is useful as a test for parallelogram is given below.

Theorem : A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

Given : In □ABCD
seg CB ≅ seg DA and seg CB || seg DA

To prove : □ABCD is a parallelogram.

Construction : Draw diagonal BD.

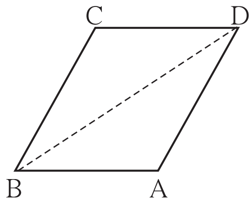


Fig. 5.18

Write the complete proof which is given in short.

Δ CBD ≅ Δ ADB ....SAS test

∴ ∠CDB ≅ ∠ABD ..... c.a.c.t.

∴ seg CD || seg BA ..... alternate angle test for parallel lines

Remember this !

- A quadrilateral is a parallelogram if its pairs of opposite angles are congruent.
- A quadrilateral is a parallelogram if its pairs of opposite sides are congruent.
- A quadrilateral is a parallelogram if its diagonals bisect each other.
- A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

These theorems are called tests for parallelogram.

Let's recall.

Lines in a note book are parallel. Using these lines how can we draw a parallelogram ?

Solved examples -

Ex (1) □PQRS is parallelogram. M is the midpoint of side PQ and N is the mid point of side RS. Prove that □PMNS and □MQRN are parallelograms.

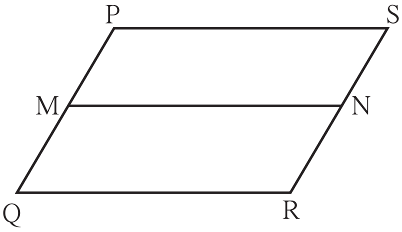


Fig. 5.19

Given : □ PQRS is a parallelogram. M and N are the midpoints of side PQ and side RS respectively.

To prove : □PMNS is a parallelogram.

□MQRN is a parallelogram.

Proof : side PQ || side SR

∴ side PM || side SN ...... (∵ P-M-Q; S-N-R) ......(I)

side PQ ≅ side SR.

∴ $\frac{1}{2}$ side PQ = $\frac{1}{2}$ side SR

∴ side PM ≅ side SN ..... (∵ M and N are midpoints.)......(II)

∴ From (I) and (II), □PMNQ is a parallelogram,

Similarly, we can prove that □MQRN is parallelogram.

Ex (2) Points D and E are the midpoints of side AB and side AC of Δ ABC respectively. Point F is on ray ED such that ED = DF. Prove that □AFBE is a parallelogram. For this example write 'given' and 'to prove' and complete the proof given below.

Given : -------------------

To prove : ----------------------

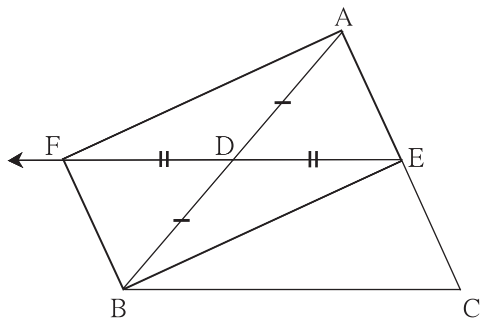


Fig. 5.20

Proof : seg AB and seg EF are [ ] of □AFBE.

seg AD ≅ seg DB.......[ ]

seg [ ] ≅ seg [ ].......construction.

∴ Diagonals of □AFBE [ ] each other

∴ □AFBE is a parallelogram ...by [ ] test.

Ex (3) Prove that every rhombus is a parallelogram.

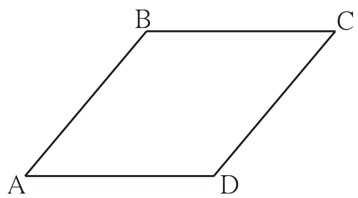


Fig. 5.21

Given : □ABCD is a rhombus.

To prove : □ABCD is parallelogram.

Proof : seg AB ≅ seg BC ≅ seg CD ≅ seg DA (given)

∴ side AB ≅ side CD and side BC ≅ side AD

∴ □ABCD is a parallelogram..... opposite side test for parallelogram

Practice set 5.2

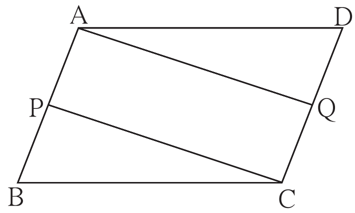


Fig. 5.22

1. In figure 5.22, □ABCD is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove □APCQ is a parallelogram.

2. Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.

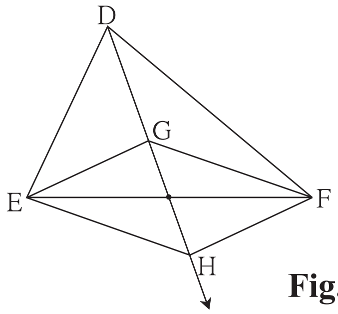


Fig. 5.23

3. In figure 5.23, G is the point of concurrence of medians of Δ DEF. Take point H on ray DG such that D-G-H and DG = GH, then prove that □GEHF is a parallelogram.

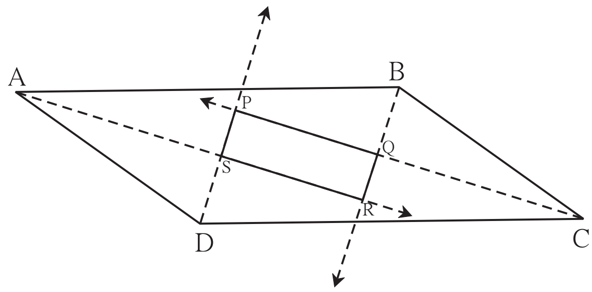


Fig. 5.24

4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. (Figure 5.24)

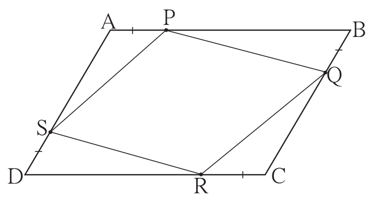
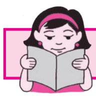


Fig. 5.25

5. In figure 5.25, if points P, Q, R, S are on the sides of parallelogram such that AP = BQ = CR = DS then prove that □PQRS is a parallelogram.


Let's learn.

Properties of rectangle, rhombus and square

Rectangle, rhombus and square are also parallelograms. So the properties that opposite sides are equal, opposite angles are equal and diagonals bisect each other hold good in these types of quadrilaterals also. But there are some more properties of these quadrilaterals.

Proofs of these properties are given in brief. Considering the steps in the given proofs, write the proofs in detail.

Theorem : Diagonals of a rectangle are congruent.

Given : □ABCD is a rectangle.

To prove : Diagonal AC ≅ diagonal BD

Proof : Complete the proof by giving suitable reasons.

Δ ADC ≅ Δ DAB ...... SAS test

$\therefore$ diagonal AC ≅ diagonal BD..... c.s.c.t.

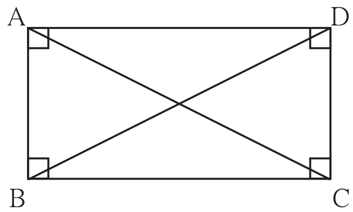


Fig. 5.26

Theorem : Diagonals of a square are congruent.

Write 'Given', 'To prove' and 'proof' of the theorem.

Theorem : Diagonals of a rhombus are perpendicular bisectors of each other.

Given : □EFGH is a rhombus

To prove : (i) Diagonal EG is the perpendicular bisector of diagonal HF.

(ii) Diagonal HF is the perpendicular bisector of diagonal EG.

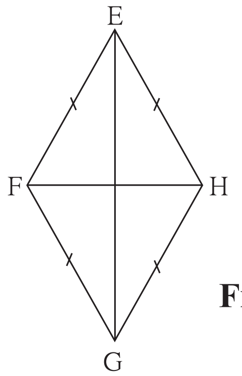


Fig. 5.27

Proof : (i) seg EF ≅ seg EH, seg GF ≅ seg GH } given

Every point which is equidistant from end points of a segment is on the perpendicular bisector of the segment.

$\therefore$ point E and point G are on the perpendicular bisector of seg HF.

One and only one line passes through two distinct points.

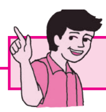
$\therefore$ line EG is the perpendicular bisector of diagonal HF.

$\therefore$ diagonal EG is the perpendicular bisector of diagonal HF.

(ii) Similarly, we can prove that diagonal HF is the perpendicular bisector of EG.

Write the proofs of the following statements.

- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect its opposite angles.
- Diagonals of a square bisect its opposite angles.



Remember this !

- Diagonals of a rectangle are congruent.
- Diagonals of a square are congruent.
- Diagonals of a rhombus are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect the pairs of opposite angles.
- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a square bisect opposite angles.

Practice set 5.3

1. Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm then find the length of BO and if $\angle CAD = 35°$ then find the measure of $\angle ACB$.

2. In a rhombus PQRS if PQ = 7.5 then find the length of QR. If $\angle QPS = 75°$ then find the measure of $\angle PQR$ and $\angle SRQ$.

3. Diagonals of a square IJKL intersects at point M, Find the measures of $\angle IMJ$, $\angle JIK$ and $\angle LJK$.

4. Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.

5. State with reasons whether the following statements are 'true' or 'false'.

(i) Every parallelogram is a rhombus.

(ii) Every rhombus is a rectangle.

(iii) Every rectangle is a parallelogram.

(iv) Every squre is a rectangle.

(v) Every square is a rhombus.

(vi) Every parallelogram is a rectangle.

Let's learn.

Trapezium

When only one pair of opposite sides of a quadrilateral is parallel then the quadrilateral is called a trapezium.

In the adjacent figure only side AB and side DC of □ABCD are parallel to each other. So this is a trapezium. $\angle A$ and $\angle D$ is a pair of adjacent angles and so is the pair of $\angle B$ and $\angle C$. Therefore by property of parallel lines both the pairs are supplementary.

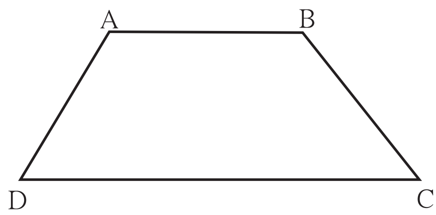


Fig. 5.28

If non-parallel sides of a trapezium are congruent then that quadrilateral is called as an **Isoceles trapezium.**

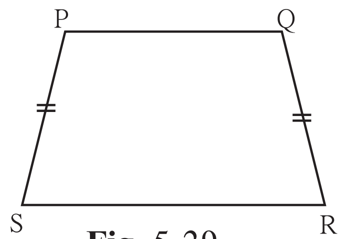


Fig. 5.29

The segment joining the midpoints of non parallel sides of a trapezium is called the median of the trapezium.

Solved examples

Ex (1) Measures of angles of □ABCD are in the ratio 4 : 5 : 7 : 8. Show that □ABCD is a trapezium.

Solution : Let measures of ∠A, ∠B, ∠C and ∠D are $(4x)^\circ$, $(5x)^\circ$, $(7x)^\circ$, and $(8x)^\circ$ respectively.

Sum of all angles of a quadrialteral is 360°.

∴ $4x + 5x + 7x + 8x = 360$

∴ $24x = 360$ ∴ $x = 15$

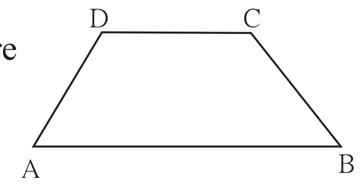


Fig. 5.30

∠A = 4 × 15 = 60°, ∠B = 5 × 15 = 75°, ∠C = 7 × 15 = 105°,

and ∠D = 8 × 15 = 120°

Now , ∠B + ∠C = 75° + 105°= 180°

∴ side CD || side BA...... (I)

But ∠B + ∠A = 75°+ 60°= 135° ≠ 180°

∴ side BC and side AD are not parallel ....(II)

∴ □ABCD is a trapezium. ....[from (I) and (II)]

Ex (2) In □PQRS, side PS || side QR and side PQ ≅ side SR, side QR > side PS then prove that ∠PQR ≅ ∠SRQ

Given : In □PQRS, side PS || side QR, side PQ ≅ side SR and side QR > side PS.

To prove : ∠PQR ≅ ∠SRQ

Construction : Draw the segment parallel to side PQ through the point S which intersects side QR in T.

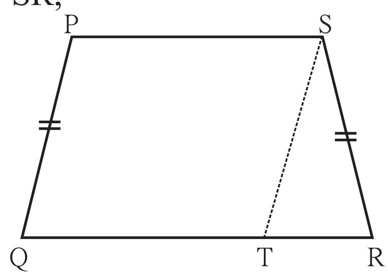


Fig. 5.31

Proof : In □PQRS,

seg PS || seg QT ....given

seg PQ || seg ST ....construction

∴ □PQTS is a parallelogram

∴ ∠PQT ≅ ∠STR ..... corresponding angles (I)

and seg PQ ≅ seg ST ......opposite sides of parallelogram

But seg PQ ≅ seg SR ......given

∴ seg ST ≅ seg SR

∴ ∠STR ≅ ∠SRT.....isosceles triangle theorem (II)

∴ ∠PQT ≅ ∠SRT ....[from (I) and (II)]

∴ ∠PQR ≅ ∠SRQ

Hence, it is proved that base angles of an isosceles trapezium are congruent.

Practice set 5.4

1. In □IJKL, side IJ || side KL $\angle I = 108^\circ$ $\angle K = 53^\circ$ then find the measures of $\angle J$ and $\angle L$.
2. In □ABCD, side BC || side AD, side AB ≅ side DC If $\angle A = 72^\circ$ then find the measures of $\angle B$, and $\angle D$.
3. In □ABCD, side BC < side AD (Figure 5.32)
   side BC || side AD and if
   side BA ≅ side CD
   then prove that ∠ABC ≅ ∠DCB.

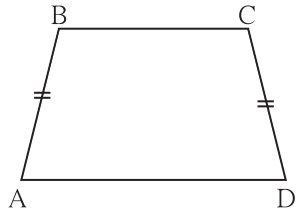
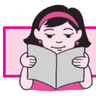


Fig. 5.32


Let's learn.

Theorem of midpoints of two sides of a triangle

Statement : The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.

Given : In Δ ABC, point P is the midpoint of seg AB and point Q is the midpoint of seg AC.

To prove : seg PQ || seg BC

and $PQ = \frac{1}{2} BC$

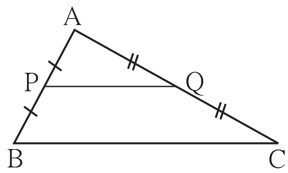


Fig. 5.33

Construction : Produce seg PQ upto R such that PQ = QR
Draw seg RC.

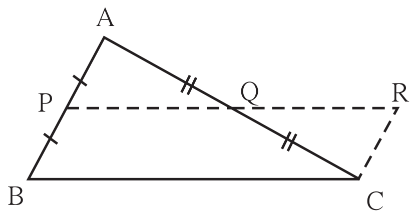


Fig. 5.34

Proof : In Δ AQP and Δ CQR

seg PQ ≅ seg QR ...... construction

seg AQ ≅ seg QC ...... given

∠AQP ≅ ∠CQR ..... vertically opposite angles.

∴ Δ AQP ≅ Δ CQR .... SAS test

∠PAQ ≅ ∠RCQ ..... (1) c.a.c.t.

∴ seg AP ≅ seg CR ......(2) c.s.c.t.

From (1) line AB || line CR.........alternate angle test

from (2) seg AP ≅ seg CR

Now, seg AP ≅ seg PB ≅ seg CR and seg PB || seg CR

∴ □PBCR is a parallelogram.

∴ seg PQ || seg BC and PR = BC ..... opposite sides are congruent

$PQ = \frac{1}{2} PR$ ...... (construction)

$\therefore \ PQ = \frac{1}{2} BC \qquad \because \ PR = BC$

Converse of midpoint theroem

Theorem : If a line drawn through the midpoint of one side of a triangle and parallel to the other side then it bisects the third side.

For this theorem 'Given', To prove', 'construction' is given below. Try to write the proof.

Given : Point D is the midpoint of side AB of Δ ABC. Line l passing through the point D and parallel to side BC intersects side AC in point E.

To prove : AE = EC

Construction : Draw a line parallel to seg AB passing through the point C. Name the point of intersection where this line and line l will intersect as F.

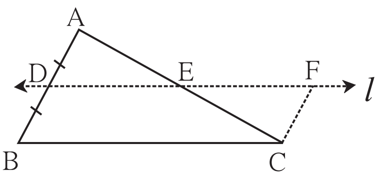


Fig. 5.35

Proof : Use the construction and line $l \parallel$ seg BC which is given. Prove Δ ADE $\cong$ Δ CFE and complete the proof.

Ex (1) Points E and F are mid points of seg AB and seg AC of Δ ABC respectively. If EF = 5.6 then find the length of BC.

Solution : In Δ ABC, point E and F are midpoints of side AB and side AC respectively.

$EF = \frac{1}{2} BC$ ....midpoint theorem

$5.6 = \frac{1}{2} BC \qquad \therefore \ BC = 5.6 \times 2 = 11.2$

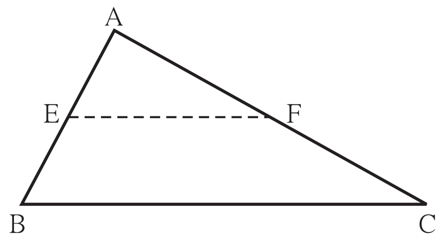


Fig. 5.36

Ex (2) Prove that the quadrilateral formed by joining the midpoints of sides of a quadrilateral in order is a parallelogram.

Given : □ABCD is a quadrilateral. P, Q, R, S are midpoints of the sides AB, BC, CD and AD respectively.

To prove : □PQRS is a parallelogram.

Construction : Draw diagonal BD

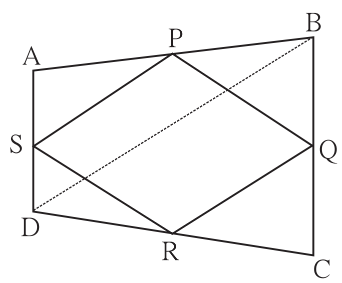


Fig. 5.37

Proof : In Δ ABD, the midpoint of side AD is S and the midpoint of side AB is P.

$\therefore$ by midpoint theorem, PS || DB and PS = $\frac{1}{2}$ BD .... (1)

In Δ DBC point Q and R are midpoints of side BC and side DC respectively.

$\therefore$ QR || BD and QR = $\frac{1}{2}$ BD ....by midpoint theorem (2)

$\therefore$ PS || QR and PS = QR .... from (1) and (2)

$\therefore$ □PQRS is a parallelogram.

Practice set 5.5

1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of Δ ABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the length of XY, YZ, XZ.

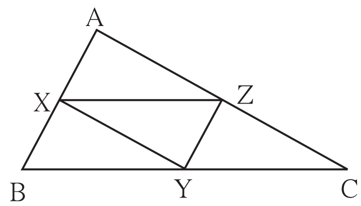


Fig. 5.38

2. In figure 5.39, □PQRS and □MNRL are rectangles. If point M is the midpoint of side PR then prove that, (i) SL = LR, (ii) LN = $\frac{1}{2}$SQ.

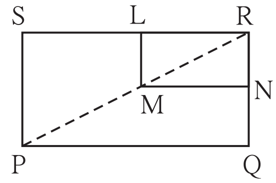


Fig. 5.39

3. In figure 5.40, ΔABC is an equilateral traingle. Points F,D and E are midpoints of side AB, side BC, side AC respectively. Show that Δ FED is an equilateral traingle.

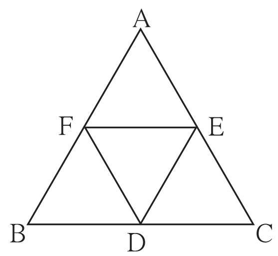


Fig. 5.40

4. In figure 5.41, seg PD is a median of Δ PQR. Point T is the mid point of seg PD. Produced QT intersects PR at M. Show that $\frac{PM}{PR} = \frac{1}{3}$.

[Hint : draw DN || QM.]

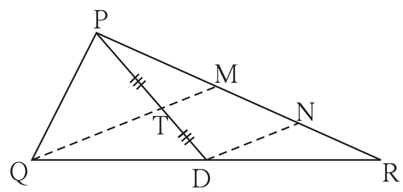


Fig. 5.41

Problem set 5

1. Choose the correct alternative answer and fill in the blanks.

(i) If all pairs of adjacent sides of a quadrilateral are congruent then it is called ....

(A) rectangle (B) parallelogram (C) trapezium, (D) rhombus

(ii) If the diagonal of a square is $12\sqrt{2}$ cm then the perimeter of square is ......

(A) 24 cm (B) $24\sqrt{2}$ cm (C) 48 cm (D) $48\sqrt{2}$ cm

(iii) If opposite angles of a rhombus are $(2x)^\circ$ and $(3x - 40)^\circ$ then value of x is ...

(A) 100° (B) 80° (C) 160° (D) 40°

2. Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.

3. If diagonal of a square is 13 cm then find its side.

4. Ratio of two adjacent sides of a parallelogram is 3 : 4, and its perimeter is 112 cm. Find the length of its each side.

5. Diagonals PR and QS of a rhombus PQRS are 20 cm and 48 cm respectively. Find the length of side PQ.

6. Diagonals of a rectangle PQRS are intersecting in point M. If $\angle QMR = 50^\circ$ then find the measure of $\angle MPS$.

7. In the adjacent Figure 5.42, if
seg AB $\parallel$ seg PQ , seg AB $\cong$ seg PQ,
seg AC $\parallel$ seg PR, seg AC $\cong$ seg PR
then prove that,
seg BC $\parallel$ seg QR and seg BC $\cong$ seg QR.

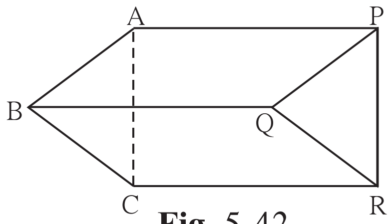


Fig. 5.42

8*. In the Figure 5.43, □ABCD is a trapezium. AB $\parallel$ DC. Points P and Q are midpoints of seg AD and seg BC respectively. Then prove that, PQ $\parallel$ AB and $PQ = \frac{1}{2}(AB + DC)$.

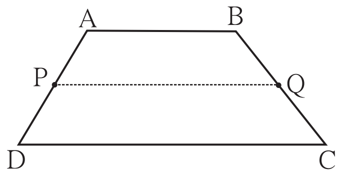


Fig. 5.43

9. In the adjacent figure 5.44, □ABCD is a trapezium. AB $\parallel$ DC. Points M and N are midpoints of diagonal AC and DB respectively then prove that MN $\parallel$ AB.

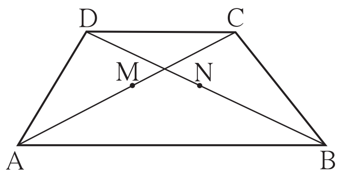


Fig. 5.44

Activity

To verify the different properties of quadrilaterals

Material : A piece of plywood measuring about 15 cm× 10 cm, 15 thin screws, twine, scissors.

Note : On the plywood sheet, fix five screws in a horizontal row keeping a distance of 2cm between any two adjacent screws. Similarly make two more rows of screws exactly below the first one. Take care that the vertical distance between any two adjacent screws is also 2cm.

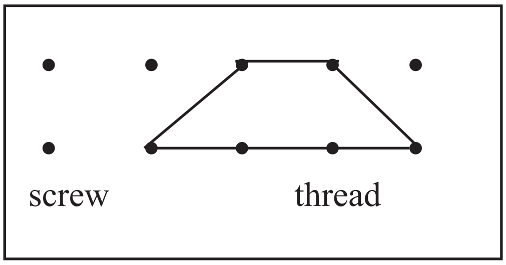


Fig. 5.45

With the help of the screws, make different types of quadrilaterals of twine. Verify the properties of sides and angles of the quadrilaterals.

Additional information

You know the property that the point of concurrence of medians of a triangle divides the medians in the ratio 2 : 1. Proof of this property is given below.

Given : seg AD and seg BE are the medians of Δ ABC which intersect at point G.

To prove: AG : GD = 2 : 1

Construction : Take point F on ray AD such that G-D-F and GD = DF

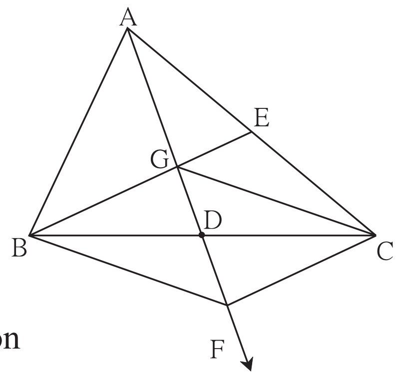


Fig. 5.46

Proof : Diagonals of □BGCF bisect each other

.... given and construction

∴ □BGCF is a parallelogram.

∴ seg BE || set FC

Now point E is the midpoint of side AC of Δ AFC . ... given

seg EB || seg FC

Line passing through midpoint of one side and parallel to the other side bisects the third side.

∴ point G is the midpoint of side AF.

∴ AG = GF

But GF = 2GD .... construction

∴ AG = 2 GD

∴ $\frac{AG}{GD} = \frac{2}{1}$ i.e. AG : GD = 2 : 1