5 Linear Equations in Two Variables

ZPK6SP

Let's study.

- **Introduction**
  Linear equation in two variables
- **Solving simultaneous equations**
- **Simultaneous equations**
- **Word problems based on simultaneous equations**

Let's recall.

Ex. Solve the following equations.

(1) $m + 3 = 5$ (2) $3y + 8 = 22$ (3) $\frac{x}{3} = 2$ (4) $2p = p + \frac{4}{9}$

$m = \square$ $y = \square$ $x = \square$ $p = \square$

(5) Which number should be added to 5 to obtain 14 ?

$\square + 5 = 14$

$x + 5 = 14$

$x = \square$

(6) Which number should be subtracted from 8 to obtain 2 ?

$8 - \square = 2$

$8 - y = 2$

$y = \square$

In all above equations, degree of the variable is 1. These are called as Linear equations.

Let's learn.

Linear equations in two variables

Find two numbers whose sum is 14.

Using variables x and y for the two numbers, we can form the equation $x + y = 14$.

This is an equation in two variables.

We can find many values of x and y satisfying the condition.

e.g. $9 + 5 = 14$ $7 + 7 = 14$ $8 + 6 = 14$ $4 + 10 = 14$

$(-1) + 15 = 14$ $15 + (-1) = 14$ $2.6 + 11.4 = 14$ $0 + 14 = 14$

$100 + (-86) = 14$ $(-100) + (114) = 14$ $\square + \square = 14$ $\square + \square = 14$

Hence, above equation has many solutions like ($x = 9, y = 5$); ($x = 7, y = 7$); ($x = 8, y = 6$) etc.

Conventionally, the solution $x = 9, y = 5$ is written as an ordered pair (9, 5) where 9 is the value of x and 5 is the value of y. To satisfy the equation $x + y = 14$, we can get infinite ordered pairs like (9,5), (7,7), (8,6), (4,10), (10,4), (–1,15), (2.6, 11.4), … etc. All of these are the solutions of $x + y = 14$.

Consider second example.

Find two numbers such that their difference is 2.

Let the greater number be x and the smaller number be y.

Then we get the equation $x - y = 2$

For the values of x and y, we can get following equations.

$10 - 8 = 2$	$9 - 7 = 2$	$8 - 6 = 2$	$(-3) - (-5) = 2$	$5.3 - 3.3 = 2$
$15 - 13 = 2$	$100 - 98 = 2$	$\square - \square = 2$	$\square - \square = 2$	

Here if we take values $x = 10$ and $y = 8$, then the ordered pair (10, 8) satisfies the above equation. Here we cannot write as (8, 10) because (8, 10) will imply $x = 8$ and $y = 10$ and it does not satisfy the equation $x - y = 2$. Therefore, note that, the order of numbers in the pair indicating solution is very important.

Now let us write the solutions of $x - y = 2$ in the form of ordered pairs.

(7, 5), (–2, –4), (0, –2), (5.2, 3.2), (8, 6) etc. There are infinite solutions.

Find the solution of $4m - 3n = 2$.

Construct 3 different equations and find their solutions.

Now, observe the first two equations.

$x + y = 14$ I

$x - y = 2$ II

Solution of equation I : (9, 5), (7, 7), (8, 6)...

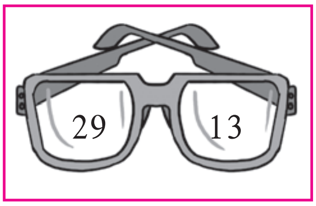
Solutions of Equation II : (7, 5), (–2, –4), (0, –2), (5.2, 3.2), (8, 6)...

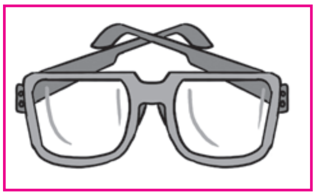
(8, 6) is the only common solution of both the equations. This solution satisfies both the equations. Hence it is the unique common solution of both the equations.

Remember this !

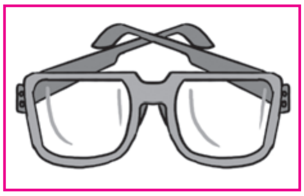
When we consider at the same time two linear equations in two variables those equations are called **Simultaneous equations**.

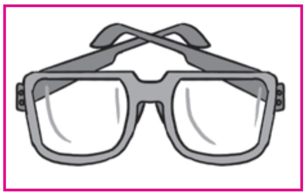
Activity : On the glasses of following spectacles, write numbers such that





(i) Their sum is 42 and difference is 16 (ii) Their sum is 37 and difference is 11





(iii) Their sum is 54 and difference is 20 (iv) Their sum is ... and difference is ...

Let's recall.

$x+y = 5$ and $2x + 2y = 10$ are two equations in two variables. Find five different solutions of $x+y = 5$, verify whether same solutions satisfy the equation $2x + 2y = 10$ also.

Observe both equations.

Find the condition where two equations in two variables have all solutions in common.

Let's learn.

Elimination method of solving simultaneous equations

By taking different values of variables we have solved the equations $x + y = 14$ and $x - y = 2$. But every time, it is not easy to solve by this method, e.g. , $2x + 3y = -4$ and $x - 5y = 11$. Try to solve these equations by taking different values of x and y. By this method observe that it is not easy to obtain the solution.

Therefore to solve simultaneous equations we use different method. In this method, we eliminate one of the variables to obtain equations in one variable. We can solve and find the value of one of the two variables and then substituting this value in one of the given equations we can find the value of the other variable.

Study the following example to understand this method.

Ex (1) Solve $x + y = 14$ and $x - y = 2$.

Solution : By adding both the equations we get an equation in one variable

$$\begin{array}{rrcll} & x+y & = & 14 & \text{.........I} \\ + & x-y & = & 2 & \text{.........II} \\ \hline & 2x+0 & = & 16 & \\ & 2x & = & 16 & \\ & x & = & 8 & \end{array}$$

Substituting $x = 8$ in the equation (I)

$$x + y = 14$$

$$\therefore \; 8 + y = 14$$

$$\therefore \; y = 6$$

Here (8, 6) is the solution of first equation. Let us check, whether it satisfies the second equation also.

$$x - y = 8 - 6 = 2 \text{ is true.}$$

$\therefore$ (8,6) is the solution for both the equations.

Hence (8, 6) is the solution of simultaneous equations $x + y = 14$ and $x - y = 2$.

Ex (2) Sum of the ages of mother and son is 45 years. If son's age is subtracted from twice of mother's age then we get answer 54. Find the ages of mother and son.

It becomes easy to solve a problem if we make use of variables.

Solution : Let the mother's today's age be x years and son's today's age be y years.

From the first condition $x+y = 45$ I

From the second condition $2x-y = 54$ II

Adding equations (I) and (II)

$$\begin{aligned} 3x+0 &= 99 \\ 3x &= 99 \\ x &= 33 \end{aligned}$$

Substituting $x = 33$ in equation (I),

$$\begin{aligned} 33+y &= 45 \\ y &= 45-33 \\ y &= 12 \end{aligned}$$

Verify that $x=33$ and $y = 12$ is the solution of second equation.

Today's age of mother = 33 and today's age of son = 12.

General form of linear equation in two variables

The general form of a linear equation in two variables is $ax + by + c = 0$ where a, b, c are real numbers and a and b are non-zero at the same time.

In this equation the index of both the variables is 1. Hence it is a linear equation.

Ex (1) Solve the following equations

$$3x + y = 5 \text{........... (I)}$$

$$2x + 3y = 1 \text{........(II)}$$

Solution : To eliminate one of the variables, we observe that in both equations, not a single coefficient is equal or opposite number. Hence we will make one of them equal.

Multiply both sides of the equation (I) by 3.

$$\therefore \quad 3x \times 3 + 3 \times y = 5 \times 3$$

$$\therefore \quad 9x + 3y = 15 \quad \text{.......(III)}$$

$$2x + 3y = 1 \quad \text{.......(II)}$$

Now subtracting eqn (II) from eqn (III)

$$\begin{array}{rcl} 9x + 3y & = & 15 \\ \underset{-}{+} 2x \underset{-}{+} 3y & \underset{-}{=} & 1 \\ \hline 7x & = & 14 \\ x & = & 2 \end{array}$$

Substituting $x = 2$ in one of the equations.

$$\begin{array}{rcl} 2x + 3y & = & 1 \\ \therefore \; 2 \times 2 + 3y & = & 1 \\ \therefore \; 4 + 3y & = & 1 \\ \therefore \; 3y & = & -3 \\ \therefore \; y & = & -1 \end{array}$$

Verify that $(2, -1)$ satisfies the second equation.

Ex (2) Solve the following simultaneous equations.

$$3x - 4y - 15 = 0 \text{ .... (I)}$$

$$y + x + 2 = 0 \text{ ....(II)}$$

Solution : Let us write the equations by shifting constant terms to RHS

$$3x - 4y = 15 \text{........(I)}$$

$$x + y = -2 \text{ .... (II)}$$

To eliminate y, multiply second equation by 4 and add to equation (I).

$$\begin{array}{rcl} 3x - 4y & = & 15 \\ +\quad 4x + 4y & = & -8 \\ \hline 7x & = & 7 \\ x & = & 1 \end{array}$$

Substituting $x = 1$ in the equation (II).

$$\begin{array}{rcl} x + y & = & -2 \\ \therefore 1 + y & = & -2 \\ \therefore y & = & -2 - 1 \\ \therefore y & = & -3 \end{array}$$

$(1, -3)$ is the solution of the above equations.

Verify that it satisfies equation (I) also.

Use your brain power!

$3x - 4y - 15 = 0$ and $y + x + 2 = 0$. Can these equations be solved by eliminating x ? Is the solution same ?

Let's learn.

Substitution method of solving simultaneous equations

There is one more method to eliminate a variable. We can express one variable in terms of other from one of the equations. Then substituting it in the other equation we can eliminate the variable. Let us discuss this method from following examples.

Ex (1) Solve $8x + 3y = 11$; $3x - y = 2$

Solution : $8x + 3y = 11$.................. (I)

$3x - y = 2$.......................(II)

In Equation (II), it is easy to express y in terms of x.

$3x - y = 2$

$3x - 2 = y$

Substituting $y = 3x - 2$ in equation (I).

$8x + 3y = 11$

$\therefore$ $8x + 3(3x - 2) = 11$

$\therefore$ $8x + 9x - 6 = 11$

$\therefore$ $17x - 6 = 11$

$\therefore$ $17x = 11 + 6 = 17$

$\therefore$ $x = 1$

Now, substituting this value of x in the equation $y = 3x - 2$.

$y = 3 \times 1 - 2$

$\therefore$ $y = 1$

$\therefore$ (1, 1) is the solution of the given equations

Ex (2) Solve. $3x - 4y = 16$; $2x - 3y = 10$

Solution : $3x - 4y = 16$..........(I)

$2x - 3y = 10$.............(II)

Writing x in terms of y from equation (I).

$3x - 4y = 16$

$3x = 16 + 4y$

$x = \frac{16 + 4y}{3}$

Substituting this value of x in equation (II)

$2x - 3y = 10$

$2\left(\frac{16 + 4y}{3}\right) - 3y = 10$

$\frac{32 + 8y}{3} - 3y = 10$

$\frac{32 + 8y - 9y}{3} = 10$

$32 + 8y - 9y = 30$

$32 - y = 30$ $\therefore$ $y = 2$

Now, substituting $y = 2$ in equation (I)

$3x - 4y = 16$

$\therefore$ $3x - 4 \times 2 = 16$

$\therefore$ $3x - 8 = 16$

$\therefore$ $3x = 16 + 8$

$\therefore$ $3x = 24$

$\therefore$ $x = 8$

$\therefore$ $x = 8$ and $y = 2$

$\therefore$ (8, 2) is the solution of the given equations.

Practice set 5.1

(1) By using variables x and y form any five linear equations in two variables.

(2) Write five solutions of the equation $x + y = 7$.

(3) Solve the following sets of simultaneous equations.

(i) $x + y = 4$; $2x - 5y = 1$ (ii) $2x + y = 5$; $3x - y = 5$

(iii) $3x - 5y = 16$; $x - 3y = 8$ (iv) $2y - x = 0$; $10x + 15y = 105$

(v) $2x + 3y + 4 = 0$; $x - 5y = 11$ (vi) $2x - 7y = 7$; $3x + y = 22$

Let's learn.

Word problems based on simultaneous equations

While solving word problems, converting the given information into mathematical form is an important step in this process. In the following flow-chart, the procedure for finding solutions of word problems is given.

Steps	Example
Read the given word problem carefully and try to understand it.	Sum of two numbers is 36. If 9 is subtracted from 8 times the first number, we get second number. Find the numbers.
↓	↓
From given information, use proper variables for given quantities	Let first number = x and second number = y
↓	↓
Form the mathematical statements from above variables	Sum of the numbers 36 $\therefore x + y = 36$ 8 times first number = $8x$ 8 times first number $- 9 = 8x - 9$ $\therefore$ Second number $= y = 8x - 9$
↓	↓
Use suitable method to solve the equations	$x + y = 36$ $+ \; 8x - y = 9$ $\therefore 9x = 45$ $\therefore x = 5$ $x + y = 36$ $\therefore 5 + y = 36$ $\therefore y = 36 - 5$ $\therefore y = 31$
↓	↓
Find the solution	$x = 5, \; y = 31$
↓	↓
Substituting the values of the variables in both the equations, verify your result.	$31 + 5 = 36$ ....(I) $31 = 8 \times 5 - 9$ ....(II)
↓	↓
Write the answer.	$\therefore$ the numbers are 5 and 31.

Word Problems

Now we will see various types of word problems.

(1) Problems regarding age.

(2) Problems regarding numbers.

(3) Problems based on fractions.

(4) Problems based on money transactions.

(5) Problems based on geometrical properties

(6) Problems based on speed, distance, time.

Ex (1) Sum of two numbers is 103. If greater number is divided by smaller number then the quotient is 2 and the remainder is 19. Then find the numbers.

Solution : Step 1 : To understand the given problem.

Step 2 : Use proper variables for given quantities. Also note the rule

dividend = divisor × quotient + remainder.

Let the greater number be x and the smaller number be y

Step 3 : Given information : Sum of the numbers = 103

$x + y = 103$ is the first equation.

By dividing greater number by smaller numbers quotient is 2 and remainder is 19.

$x = 2 \times y + 19$ ...(dividend = divisor × quotient + remainder)

$x - 2y = 19$ is the second equation.

Step 4 : Let us find the solution of the equations.

$$x + y = 103 \quad \text{................(I)}$$
$$x - 2y = 19 \quad \text{................(II)}$$

Subtracting eqn. (II) from eqn. (I)

$$\begin{array}{rcl} x + y & = & 103 \\ x - 2y & = & 19 \\ - \quad + & & - \\ \hline 0 + 3y & = & 84 \\ \therefore \quad y & = & 28 \end{array}$$

Step 5 : Substituting value of y in equation $x + y = 103$.

$$\therefore \quad x + 28 = 103$$
$$\therefore \quad x = 103 - 28$$
$$\therefore \quad x = 75$$

Step 6 : Given numbers are 75 and 28.

Ex (2) Salil's age is 23 years more than half of the Sangram's age. Five years ago, the sum of their ages was 55 years. Find their present ages.

Solution : Let Salil's present age be x and Sangram's present age be y .

Salil's age is 23 years more than half of the Sangram's age $\therefore\ x = \frac{y}{2} + \square$

Five years ago Salil's age = $x - 5$. Five years ago Sangram's age = $y - 5$

The sum of their ages five years ago = 55

$\square + \square = 55$

Finding the solution by solving equations

$2x = y + 46$ $\qquad 2x - y = 46$ ....(I)

$(x - 5) + (y - 5) = 55$

$x + y = 65$ ....(II)

Adding equation (I) and (II)

$$\begin{aligned} 2x - y &= 46 \\ + \quad x + y &= 65 \\ \hline \therefore\ 3x &= 111 \\ \therefore\ x &= 37 \end{aligned}$$

Substituting $x = 37$ in equation (II)

$$\begin{aligned} x + y &= 65 \\ \therefore\ 37 + y &= 65 \\ \therefore\ y &= 65 - 37 \\ \therefore\ y &= 28 \end{aligned}$$

Salil's present age is 37 years and Sangram's present age is 28 years.

Ex (3) A two digit number is 4 times the sum of its digits. If we interchange the digits, the number obtained is 9 less than 4 times the original number. Then find the number.

Solution : Let the units place digit in original number be x, and tens place be y.

	Digit in tens place	Digit in units place	Number	Sum of the digits
For original number	y	x	$10y + x$	$y + x$
Number obtained by interchanging the digits	x	y	$10x + y$	$x + y$

From first condition, $10y + x = 4(y + x)$

$\therefore 10y + x = 4y + 4x$

$\therefore\ x - 4x + 10y - 4y = 0$

$\therefore\ -3x + 6y = 0 \quad \therefore\ -3x = -6y \qquad \therefore x = 2y$ .....(I)

From second condition, $10x + y = 2(10y+x)-9$

$10x+y = 20y + 2x-9$

$10x-2x+y-20y = -9$

$8x - 19y = -9$ ....(II)

$x = 2y$ ....(I)

Substituting $x = 2y$ in equation (II).

$16y - 19y = -9$ ....(I)

$\therefore\ -3y = -9$

$\therefore\ y = 3$

Substituting $y = 3$ in equation (I). $x - 2y = 0$

$x - 2 \text{ x } 3 = 0$ $\quad\therefore\ x - 6 = 0$ $\quad\therefore\ x = 6$

Original two digit number : $10y + x = 10 \times 3 + 6$

$= 36$

Ex (4) The population of a certain town was 50,000. In a year, male population was increased by 5% and female population was increased by 3%. Now the population became 52020. Then what was the number of males and females in the previous year?

Solution : Let the number of males in previous year be x, number of females be y.

By first condition $\square + \square = 50000$ ....(I)

Male population increased by 5% $\quad\therefore$ number of males $= \frac{\square}{\square}x$

Female population increased by 3% $\quad\therefore$ number of females $= \frac{\square}{\square}y$.

From second condition $\frac{\square}{\square}x + \frac{\square}{\square}y = 52020$

$\square x + \square y = 5202000$ ....(II)

Multiplying equation (I) by 103

$\square x + \square y = 5150000$ ....(III)

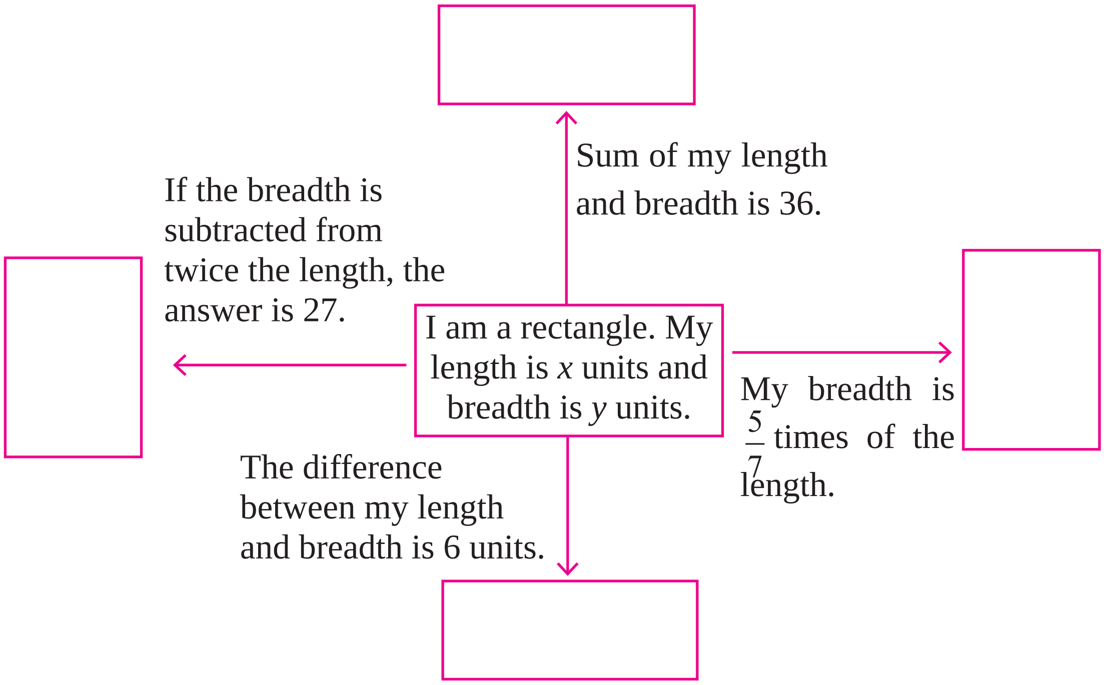
Subtracting equation (III) from equation (II) .

$2x = 5202000 - 5150000$

$2x = 52000$

$\therefore$ number of males $= x = \square$ $\quad\therefore$ number of females $= y = \square$

Activity I : There are instructions written near the arrows in the following diagram. From this information form suitable equations and write in the boxes indicated by arrows. Select any two equations from these boxes and find their solutions. Also verify the solutions. By taking one pair of equations at a time, how many pairs can be formed ? Discuss the solutions for these pairs.



Practice set 5.2

(1) In an envelope there are some 5 rupee notes and some 10 rupee notes. Total amount of these notes together is 350 rupees. Number of 5 rupee notes are less by 10 than number of 10 rupee notes. Then find the number of 5 rupee and 10 rupee notes.

(2) The denominator of a fraction is 1 less than twice its numerator. If 1 is added to numerator and denominator respectively, the ratio of numerator to denominator is 3 : 5. Find the fraction.

(3) The sum of ages of Priyanka and Deepika is 34 years. Priyanka is elder to Deepika by 6 years. Then find their today's ages.

(4) The total number of lions and peacocks in a certain zoo is 50. The total number of their legs is140. Then find the number of lions and peacocks in the zoo.

(5) Sanjay gets fixed monthly income. Every year there is a certain increment in his salary. After 4 years, his monthly salary was Rs. 4500 and after 10 years his monthly salary became 5400 rupees, then find his original salary and yearly increment.

(6) The price of 3 chairs and 2 tables is 4500 rupees and price of 5 chairs and 3 tables is 7000 rupees, then find the total price of 2 chairs and 2 tables.

(7) The sum of the digits in a two-digits number is 9. The number obtained by interchanging the digits exceeds the original number by 27. Find the two-digit number.

(8*) In Δ ABC, the measure of angle A is equal to the sum of the measures of $\angle$ B and $\angle$ C. Also the ratio of measures of $\angle$ B and $\angle$ C is 4 : 5. Then find the measures of angles of the triangle.

(9*) Divide a rope of length 560 cm into 2 parts such that twice the length of the smaller part is equal to $\frac{1}{3}$ of the larger part. Then find the length of the larger part.

(10) In a competitive examination, there were 60 questions. The correct answer would carry 2 marks, and for incorrect answer 1 mark would be subtracted. Yashwant had attempted all the questions and he got total 90 marks. Then how many questions he got wrong ?

Problem set 5

(1) Choose the correct alternative answers for the following questions.

(i) If $3x + 5y = 9$ and $5x + 3y = 7$ then What is the value of $x + y$?

(A) 2 (B) 16 (C) 9 (D) 7

(ii) 'When 5 is subtracted from length and breadth of the rectangle, the perimeter becomes 26.' What is the mathematical form of the statement ?

(A) $x - y = 8$ (B) $x + y = 8$ (C) $x + y = 23$ (D) $2x + y = 21$

(iii) Ajay is younger than Vijay by 5 years. Sum of their ages is 25 years. What is Ajay's age ?

(A) 20 (B) 15 (C) 10 (D) 5

(2) Solve the following simultaneous equations.

(i) $2x + y = 5$; $3x - y = 5$ (ii) $x - 2y = -1$; $2x - y = 7$

(iii) $x + y = 11$; $2x - 3y = 7$ (iv) $2x + y = -2$; $3x - y = 7$

(v) $2x - y = 5$; $3x + 2y = 11$ (vi) $x - 2y = -2$; $x + 2y = 10$

(3) By equating coefficients of variables, solve the following equations.

(i) $3x - 4y = 7$; $5x + 2y = 3$ (ii) $5x + 7y = 17$; $3x - 2y = 4$

(iii) $x - 2y = -10$; $3x - 5y = -12$ (iv) $4x + y = 34$; $x + 4y = 16$

(4) Solve the following simultaneous equations.

(i) $\frac{x}{3} + \frac{y}{4} = 4$; $\frac{x}{2} - \frac{y}{4} = 1$ (ii) $\frac{x}{3} + 5y = 13$; $2x + \frac{y}{2} = 19$

(iii) $\frac{2}{x} + \frac{3}{y} = 13$; $\frac{5}{x} - \frac{4}{y} = -2$

(5*) A two digit number is 3 more than 4 times the sum of its digits. If 18 is added to this number, the sum is equal to the number obtained by interchanging the digits. Find the number.

(6) The total cost of 8 books and 5 pens is 420 rupees and the total cost of 5 books and 8 pens is 321 rupees. Find the cost of 1 book and 2 pens.

(7*) The ratio of incomes of two persons is 9 : 7. The ratio of their expenses is 4 : 3. Every person saves rupees 200, find the income of each.

(8*) If the length of a rectangle is reduced by 5 units and its breadth is increased by 3 units, then the area of the rectangle is reduced by 8 square units. If length is reduced by 3 units and breadth is increased by 2 units, then the area of rectangle will increase by 67 square units. Then find the length and breadth of the rectangle.

(9*) The distance between two places A and B on road is 70 kilometers. A car starts from A and the other from B. If they travel in the same direction, they will meet after 7 hours. If they travel towards each other they will meet after 1 hour, then find their speeds.

(10*) The sum of a two digit number and the number obtained by interchanging its digits is 99. Find the number.

Activity : Find the fraction.

$$\frac{\text{Numerator } x}{\text{Denominator } y}$$

If numerator is multiplied by 3 and 3 is subtracted from the denominator then the fraction obtained is $\frac{18}{11}$.

Equation I

$11x - 6y + 18 = 0$

If numerator is increased by 8 and denominator is doubled then the resulting fraction is $\frac{1}{2}$.

Equation II

$x - y + 8 = 0$

$\therefore$ Given fraction = $\frac{\square}{\square}$

Verify the answer obtained.