4 Ratio and Proportion

ZPNAR2

Let's study.

- Ratio
- Operations on equal ratios
- Continued proportion
- Properties of ratios
- Theorem of equal ratios
- *k* method

Let's recall.

In earlier standards, we have learnt about ratio and proportion. We have also solved examples based on it. Let us discuss following example.

Ex. The rawa ladoo prepared by Vimal are tasty, for which she takes 1 bowl of ghee, 3 bowls of rawa and 2 bowls of sugar.

Here proportion of rawa and sugar is 3 : 2 or $\frac{3}{2}$.

If 12 units of rawa is used, how many units of sugar are required ?

Let the number of bowls of sugar required be x.

$\therefore$ from above information, $\frac{3}{2} = \frac{12}{x}$ $\quad \therefore\ 3x = 24 \quad \therefore\ x = 8$

That is for preparation of ladoo, with 12 units of rawa requires 8 units of sugar.
Alternatively we can solve the above example in the following way.

$3k$ bowls of rawa, $2k$ bowls of sugar is required because $\frac{3k}{2k} = \frac{3}{2}$

If $3k = 12$ then $k = 4$ $\quad \therefore\ 2k = 2 \times 4 = 8$ bowls of sugar is required.

Let's learn.

Ratio and proportion

The concept of ratio of two numbers can be extended to three or more numbers.

Let us see the above example of ladoos. The proportion of ghee, rawa and sugar is 1 : 3 : 2.

Here proportion of ghee and rawa is 1 : 3 and that of rawa and sugar is 3 : 2, This means the proportion of ghee, rawa and sugar is 1 : 3 : 2.

Let us take k bowls of ghee, $3k$ bowls of rawa and $2k$ bowls of sugar.

Hence for 12 bowls of rawa, how much quantity of ghee and sugar is required can be found as follows.

Now $3k = 12 \quad \therefore\ k = 4$ and $2k = 8$.

$\therefore$ 4 bowls of ghee and 8 bowls of sugar is required.

The same concept can be extended for proportion of 4 or more entities.

If a, b, c, d are in the ratio 2 : 3 : 7 : 4 then let us assume that the numbers are $2m$, $3m$, $7m$, $4m$. From the given information, value of m can be determined. For example if the sum of these four numbers is 48, we find these numbers.

$2m + 3m + 7m + 4m = 16\,m = 48$

$\therefore \quad m = 3$

$\therefore \quad 2m = 6,\ 3m = 9,\ 7m = 21,\ 4m = 12$

$\therefore$ required numbers are 6, 9, 21, 12

Ex (1) The proportion of compounds of nitrogen, phosphorous and potassium in certain fertilizer is 18 : 18 : 10. Here compound of nitrogen is 18%, compound of phosphorous is 18% and that of potassium is 10%. Remaining part is of other substances. Find the weight of each of the above compounds in 20 kg of fertilizer.

Solution : Let the weight of nitrogen compound in 20 kg of fertilizer be x kg.

$\therefore \quad \frac{18}{100} = \frac{x}{20} \qquad \therefore \quad x = \frac{18 \times 20}{100} = 3.6$

$\therefore$ weight of nitrogen compound is 3.6 kg.

The percentage of phosphorous compound is also 18%.

$\therefore$ Weight of compound of phosphorous is 3.6 kg.

If we assume the weight of potassium compound y kg then

$\frac{10}{100} = \frac{y}{20} \quad \therefore y = 2 \qquad \therefore$ weight of potassium compound is 2 kg.

Direct proportion

A car covers a distance of 10 km consuming 1 litre of petrol.

It will cover a distance of $20 \times 10 = 200$ km consuming 20 litre of petrol .

Consuming 40 litre of petrol, it will cover a distance of $40 \times 10 = 400$ km.

Let us write this information in tabular form.

Petrol : x litre	1	20	40	
Distance : y km	10	200	400	
$\frac{x}{y}$	$\frac{1}{10}$	$\frac{20}{200} = \frac{1}{10}$	$\frac{40}{400} = \frac{1}{10}$	$\frac{x}{y} = k$

The ratio of consumption of petrol (in litre) and distance covered by the car (in kilometres), is constant. In such case, it is said that the two quantities are in direct proportion or in direct variation.

Inverse proportion

A car takes two hours to cover a distance of 100 km at the speed of 50 km/hr. A bullock-cart travels 5 km in 1 hour. To cover a distance of 100 km at the speed of 5 km/hr, the bullock-cart takes 20 hours.

We know that, **Speed × time = distance**

By using the relation let us put the above information in a tabular form.

Vehicle	Speed/hr (x)	Time (y)	$x \times y$	$x \times y$ = k
Car	50	2	100	
Bullock-cart	5	20	100	

Hence, we see that, the product of speed of the vehicle and time is constant. In such a case it is said that the quantities are in inverse proportion or in inverse variation.

Let's recall.

Properties of ratio

(1) Ratio of numbers a and b is written as $a : b$ or $\frac{a}{b}$. a is called the predecessor (first term) and b is called successor (Second term).

(2) In the ratio of two numbers, if the second term is 100 then it is known as a percentage.

(3) The ratio remains unchanged, if its terms are multiplied or divided by non-zero number.
e.g.. 3 : 4 = 6:8 = 9:12, Similarly 2:3:5 = 8:12:20. If k is a non-zero number,
then $a : b = ak : bk$ $\quad$ $a : b : c = ak : bk : ck$

(4) The quantities taken in the ratio must be expressed in the same unit.

(5) The ratio of two quantities is unitless.
For example The ratio of 2 kg and 300 g is not 2 : 300 ,
but it is 2000 : 300 as (2 kg = 2000 gm) i.e. 20 : 3

Ex (1) The ratio of ages of Seema and Rajashree is 3 : 1. The ratio of ages of Rajashree and Atul is 2 : 3. Then find the ratio of ages of Seema, Rajashree and Atul.

Solution : Seema's age : Rajashree's age = 3 : 1 Rajashree's age : Atul's age = 2 : 3

Second term of first ratio should be the same as the first term of second ratio.

Hence to get the continuous ratio, multiplying each term of the first ratio by 2. We get

3:1 = 6:2 .

$$\frac{\text{Seema's age}}{\text{Rajashree's age}} = \frac{6}{2}, \quad \frac{\text{Rajashree's age}}{\text{Atul's age}} = \frac{2}{3}$$

$\therefore$ Seema's age : Rajashree's age : Atul's age 6 : 2 : 3.

Ex (2) The length of a rectangular field is 1.2 km and its breadth is 400 metre. Find the ratio of length to breadth.

Solution : Here the length is in kilometer and breadth is in meter. In order to find the ratio of length to breadth, they must be expressed in same unit. Hence we convert kilometre to meter.

1.2 km= 1.2 ×1000 = 1200 m

$\therefore$ ratio of 1200 m , to 400 m is $\frac{1200}{400} = \frac{3}{1}$, that is 3 : 1

Ex (3) The ratio of expenditure and income of Mahesh is 3 : 5. Find the percentage of expenses to his income.

Solution : The ratio of expenditure to income is 3 : 5. To convert it into percentage, convert second term into 100.

$\frac{3}{5} = \frac{3\times 20}{5\times 20} = \frac{60}{100}$ $\therefore$ $\frac{\text{Expenditure}}{\text{Income}} = \frac{60}{100} = 60\%$ $\therefore$ Mahesh spends 60% of his income.

Ex (4) The ratio of number of mango trees to chikoo trees in an orchard is 2 : 3. If 5 more trees of each type are planted the ratio of trees would be 5 : 7. Then find the number of mango and chickoo trees in the orchard.

Solution : The ratio of trees is 2 : 3.

Let the number of mango trees = $2x$ and chikoo trees = $3x$

From given condition, $\frac{2x+5}{3x+5} = \frac{5}{7}$

$14x + 35 = 15x + 25$

$\therefore x = 10$

$\therefore$ number of mango trees in the orchard = $2x$ = 2 ×10= 20

and number of chikoo trees = $3x$ = 3 ×10= 30

Ex (5) The ratio of two numbers is 5 : 7. If 40 is added in each number, then the ratio becomes 25 : 31, Find the numbers.

Solution : Let the first number be $5x$ and and second number be $7x$.

From the given condition, $\frac{5x+40}{7x+40}=\frac{25}{31}$

$$31(5x+40)=25(7x+40)$$

$$155x+1240=175x+1000$$

$$1240-1000 = 175x-155x$$

$$240=20x$$

$$x=12$$

$\therefore$ first number = 5 × 12 = 60 and second number = 7 × 12 = 84

$\therefore$ given numbers are 60 and 84.

Practice set 4.1

(1) From the following pairs of numbers, find the reduced form of ratio of first number to second number.

(i) 72, 60 (ii) 38,57 (iii) 52,78

(2) Find the reduced form of the ratio of the first quantity to second quantity.

(i) 700 ₹, 308 ₹ (ii) 14 ₹, 12 ₹. 40 paise.

(iii) 5 litre, 2500 ml (iv) 3 years 4 months, 5 years 8 months

(v) 3.8 kg, 1900 gm (vi) 7 minutes 20 seconds, 5 minutes 6 seconds.

(3) Express the following percentages as ratios in the reduced form.

(i) 75 : 100 (ii) 44 : 100 (iii) 6.25% (iv) 52 : 100 (v) 0.64%

(4) Three persons can build a small house in 8 days. To build the same house in 6 days, how many persons are required?

(5) Convert the following ratios into percentage.

(i) 15 : 25 (ii) 47 : 50 (iii) $\frac{7}{10}$ (iv) $\frac{546}{600}$ (v) $\frac{7}{16}$

(6) The ratio of ages of Abha and her mother is 2 : 5. At the time of Abha's birth her mothers age was 27 year. Find the present ages of Abha and her mother.

(7) Present ages of Vatsala and Sara are 14 years and 10 years respectively. After how many years the ratio of their ages will become 5 : 4?

(8) The ratio of present ages of Rehana and her mother is 2 : 7. After 2 years, the ratio of their ages will be 1 : 3. What is Rehana's present age ?

Let's learn.

Comparison of ratios

The numbers a, b, c, d being positive, comparison of ratios $\frac{a}{b}$, $\frac{c}{d}$ can be done using following rules :

(i) If $ad > bc$ then $\frac{a}{b} > \frac{c}{d}$ (ii) If $ad < bc$ then $\frac{a}{b} < \frac{c}{d}$

(iii) If $ad = bc$ then $\frac{a}{b} = \frac{c}{d}$

Compare the following pairs of ratios.

Ex (1) $\frac{4}{9}, \frac{7}{8}$

Solution : 4×8 [?] 7×9

$32 < 63$

$\therefore \frac{4}{9} < \frac{7}{8}$

Ex (2) $\frac{\sqrt{13}}{\sqrt{8}}, \frac{\sqrt{7}}{\sqrt{5}}$

$\sqrt{13} \times \sqrt{5},$ [?] $\sqrt{8} \times \sqrt{7}$

$\sqrt{65}$ [?] $\sqrt{56}$

$\sqrt{65} > \sqrt{56}$

$\therefore \frac{\sqrt{13}}{\sqrt{8}} > \frac{\sqrt{7}}{\sqrt{5}}$

Ex (3) If a and b are integers and $a < b$, $b > 1$ then compare $\frac{a-1}{b-1}, \frac{a+1}{b+1}$.

Solution : $a < b$ $\therefore a - 1 < b - 1$

Now consider the subtraction $\frac{a-1}{b-1} - \frac{a+1}{b+1}$

$$\frac{a-1}{b-1} - \frac{a+1}{b+1} = \frac{(a-1)(b+1)-(a+1)(b-1)}{(b-1)(b+1)}$$

$$= \frac{(ab-b+a-1)-(ab+b-a-1)}{b^2-1}$$

$$= \frac{ab-b+a-1-ab-b+a+1}{b^2-1}$$

$$= \frac{2a-2b}{b^2-1}$$

$$= \frac{2(a-b)}{b^2-1} \quad \dots\dots\dots\dots (1)$$

Now $a < b$ $\therefore a - b < 0$

also $b^2 - 1 > 0$ because $b > 1$

$\frac{2(a-b)}{b^2-1} < 0 \dots\dots\dots\dots (2)$

$\frac{a-1}{b-1} - \frac{a+1}{b+1} < 0$.....from (1) & (2)

$\frac{a-1}{b-1} < \frac{a+1}{b+1}$

Ex (4) If $a : b = 2 : 1$ and $b : c = 4 : 1$ then find the value of $\left(\frac{a^4}{32b^2c^2}\right)^3$.

Solution : $\frac{a}{b} = \frac{2}{1}$ $\therefore a = 2b$ $\frac{b}{c} = \frac{4}{1}$ $\therefore b = 4c$

$a = 2b = 2 \times 4c = 8c$ $\therefore a = 8c$

Now substituting the values $a = 8c$, $b = 4c$

$$\left(\frac{a^4}{32b^2c^2}\right)^3 = \left(\frac{(8c)^4}{32 \times 4^2 \times c^2 \times c^2}\right)^3$$

$$= \left[\frac{8 \times 8 \times 8 \times 8 \times c^4}{32 \times 16 \times c^2 \times c^2}\right]^3$$

$$= (8)^3$$

$$\therefore \left(\frac{a^4}{32b^2c^2}\right)^3 = 512$$

Practice set 4.2

(1) Using the property $\frac{a}{b} = \frac{ak}{bk}$, fill in the blanks substituting proper numbers in the following.

(i) $\frac{5}{7} = \frac{....}{28} = \frac{35}{....} = \frac{....}{3.5}$ (ii) $\frac{9}{14} = \frac{4.5}{....} = \frac{....}{42} = \frac{....}{3.5}$

(2) Find the following ratios.

(i) The ratio of radius to circumference of the circle.

(ii) The ratio of circumference of circle with radius r to its area.

(iii) The ratio of diagonal of a square to its side, if the length of side is 7 cm.

(iv) The lengths of sides of a rectangle are 5 cm and 3.5 cm. Find the ratio of its perimeter to area.

(3) Compare the following pairs of ratios.

(i) $\frac{\sqrt{5}}{3}, \frac{3}{\sqrt{7}}$ (ii) $\frac{3\sqrt{5}}{5\sqrt{7}}, \frac{\sqrt{63}}{\sqrt{125}}$ (iii) $\frac{5}{18}, \frac{17}{121}$

(iv) $\frac{\sqrt{80}}{\sqrt{48}}, \frac{\sqrt{45}}{\sqrt{27}}$ (v) $\frac{9.2}{5.1}, \frac{3.4}{7.1}$

(4) (i) □ABCD is a parallelogram. The ratio of ∠A and ∠B of this parallelogram is 5 : 4. Find the measure of ∠ B.

(ii) The ratio of present ages of Albert and Salim is 5 : 9. Five years hence ratio of their ages will be 3 : 5. Find their present ages.

(iii) The ratio of length and breadth of a rectangle is 3 : 1, and its perimeter is 36 cm. Find the length and breadth of the rectangle.

(iv) The ratio of two numbers is 31 : 23 and their sum is 216. Find these numbers.

(v) If the product of two numbers is 360 and their ratio is 10 : 9, then find the numbers.

(5*) If $a : b = 3 : 1$ and $b : c = 5 : 1$ then find the value of (i) $\left(\frac{a^3}{15b^2c}\right)^3$ (ii) $\frac{a^2}{7bc}$

(6*) If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$ then find the ratio $\frac{a}{b}$.

(7) $(x + 3) : (x + 11) = (x - 2) : (x + 1)$ then find the value of x.

Let's learn.

Operations on equal ratios

Using the properties of equality, we can perform some operations on ratios. Let's study them.

Let us learn some properties of the equal ratios, if *a, b, c, d,* are positive integers.

(I) Invertendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$

$$\frac{a}{b} = \frac{c}{d}$$

$$\therefore \quad a \times d = b \times c$$

$$\therefore \quad b \times c = a \times d$$

$$\therefore \quad \frac{b \times c}{a \times c} = \frac{a \times d}{a \times c} \quad \text{...(dividing both sides by } a \times c \text{)}$$

$$\frac{b}{a} = \frac{d}{c}$$

$\therefore$ If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$. **This property is known as Invertendo.**

(II) Alternando : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$

$$\frac{a}{b} = \frac{c}{d}$$

$$\therefore \quad a \times d = b \times c$$

$$\frac{a \times d}{c \times d} = \frac{b \times c}{c \times d} \quad \text{...(dividing both sides by } c \times d\text{)}$$

$$\frac{a}{c} = \frac{b}{d}$$

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$. **This property is known as Alternando.**

(III) Componendo : If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$

$$\frac{a}{b}=\frac{c}{d}$$

$$\frac{a}{b}+1=\frac{c}{d}+1 \quad \text{...(adding 1 to both sides)}$$

$$\frac{a+b}{b}=\frac{c+d}{d}$$

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$. **This property is known as Componendo.**

(IV) Dividendo : If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-b}{b}=\frac{c-d}{d}$

$$\therefore \quad \frac{a}{b}=\frac{c}{d}$$

$$\therefore \quad \frac{a}{b}-1=\frac{c}{d}-1 \quad \text{...(subtracting 1 from both sides)}$$

$$\therefore \quad \frac{a-b}{b}=\frac{c-d}{d}$$

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-b}{b}=\frac{c-d}{d}$. **This property is known as Dividendo.**

(V) Componendo-Dividendo : If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$, $a \neq b, c \neq d$

If $\frac{a}{b}=\frac{c}{d}$ $\quad \therefore \frac{a+b}{b}=\frac{c+d}{d}$ ...(using componendo) ....(1)

$$\therefore \quad \frac{a-b}{b}=\frac{c-d}{d} \quad \text{...(using dividendo) ....(2)}$$

$$\therefore \quad \frac{a+b}{a-b}=\frac{c+d}{c-d} \quad \text{...from (1) and (2)}$$

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ **. This property is known as Componendo-dividendo.**

General form of Componendo and Dividendo

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$ ...(performing componendo once)

$$\frac{a+2b}{b}=\frac{c+2d}{d} \quad \text{...(performing componendo twice)}$$

Generally $\frac{a+mb}{b}=\frac{c+md}{d}$ ... (performing componendo *m* times) ...(I)

Similarly if $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-mb}{b}=\frac{c-md}{d}$ ...(performing dividendo *m* time) ...(II)

and if $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+mb}{a-mb}=\frac{c+md}{c-md}$ ...[dividing (I) by (II)]

Remember this !

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{b}{a}=\frac{d}{c}$ (Invertendo)

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a}{c}=\frac{b}{d}$ (Alternando)

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$ (Componendo)

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-b}{b}=\frac{c-d}{d}$ (Dividendo)

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (Componendo-Dividendo)

Solved Examples :

Ex (1) If $\frac{a}{b}=\frac{5}{3}$ then find the ratio $\frac{a+7b}{7b}$ = .

Method I

Solution : If $\frac{a}{b}=\frac{5}{3}$ then $\frac{a}{5}=\frac{b}{3}=k$,

...(using alternando)

$\therefore\ a = 5k,\ b = 3k$

$\therefore\ \frac{a+7b}{7b}=\frac{5k+7\times 3k}{7\times 3k}$

$=\frac{5k+21k}{21k}$

$=\frac{26k}{21k}\quad=\frac{26}{21}$

Method II

$\frac{a}{b}=\frac{5}{3}$

$\therefore\ \frac{a}{7b}=\frac{5}{21}$

$\therefore\ \frac{a+7b}{7b}=\frac{5+21}{21}$ ...(using componendo)

$\therefore\ \frac{a+7b}{7b}=\frac{26}{21}$

Ex. (2) If $\frac{a}{b}=\frac{7}{4}$ then find the ratio $\frac{5a-b}{b}$.

Method I

Solution : $\frac{a}{b}=\frac{7}{4}$

$\therefore\ \frac{a}{7}=\frac{b}{4}$ ...(using alternando)

Let $\frac{a}{7}=\frac{b}{4}=m$

$\therefore\ a = 7m,\quad b = 4m$

$\therefore\ \frac{5a-b}{b}=\frac{5(7m)-4m}{4m}$

$=\frac{35m-4m}{4m}$

$=\frac{31}{4}$

Method II

$\frac{a}{b}=\frac{7}{4}$

$\frac{5a}{b}=\frac{5\times 7}{4}$

$=\frac{35}{4}$

$\frac{5a-b}{b}=\frac{35-4}{4}$ ...(using dividendo)

$\frac{5a-b}{b}=\frac{31}{4}$

Ex. (3) If $\frac{a}{b} = \frac{7}{3}$ then find the value of the ratio $\frac{a+2b}{a-2b}$.

Solution : Method I :

Let $a = 7m$, $b = 3m$

$$\therefore \frac{a+2b}{a-2b} = \frac{7m+2\times 3m}{7m-2\times 3m}$$

$$= \frac{7m+6m}{7m-6m}$$

$$= \frac{13m}{m} = \frac{13}{1}$$

Method II : $\therefore \frac{a}{b} = \frac{7}{3}$

$\therefore \frac{a}{2b} = \frac{7}{6}$ ...(multiplying both sides by $\frac{1}{2}$)

$\therefore \frac{a+2b}{a-2b} = \frac{7+6}{7-6}$ (using componendo - dividendo)

$\therefore \frac{a+2b}{a-2b} = \frac{13}{1}$

Ex (4) If $\frac{a}{3} = \frac{b}{2}$ then find the value of the ratio $\frac{5a+3b}{7a-2b}$.

Solution : Method I

$$\frac{a}{3} = \frac{b}{2}$$

$\therefore \frac{a}{b} = \frac{3}{2}$........ (using Alternando)

Now dividing each term of $\frac{5a+3b}{7a-2b}$ by b.

$$\frac{\frac{5a}{b}+\frac{3b}{b}}{\frac{7a}{b}-\frac{2b}{b}} = \frac{5\left(\frac{a}{b}\right)+3}{7\left(\frac{a}{b}\right)-2}$$

$$= \frac{5\left(\frac{3}{2}\right)+3}{7\left(\frac{3}{2}\right)-2}$$

$$= \frac{\frac{15}{2}+3}{\frac{21}{2}-2}$$

$$= \frac{15+6}{21-4}$$

$$= \frac{21}{17}$$

Method II

$$\frac{a}{3} = \frac{b}{2}$$

Let $\frac{a}{3} = \frac{b}{2} = t$.

$\therefore$ by substituting $a = 3t$ and $b = 2t$,

$$\frac{5a+3b}{7a-2b} = \frac{5(3t)+3(2t)}{7(3t)-2(2t)} \quad (t \neq 0)$$

$$= \frac{15t+6t}{21t-4t}$$

$$= \frac{21t}{17t}$$

$$= \frac{21}{17}$$

Ex (5) If $\frac{x}{y} = \frac{4}{5}$ then find the value of the ratio $\frac{4x-y}{4x+y}$.

Solution :

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{4x}{y} = \frac{16}{5} \qquad \text{...(multiplying both sides by 4)}$$

$$\therefore \frac{4x+y}{4x-y} = \frac{16+5}{16-5} \qquad \text{...(using componendo-dividendo)}$$

$$\therefore \frac{4x+y}{4x-y} = \frac{21}{11}$$

$$\therefore \frac{4x-y}{4x+y} = \frac{11}{21}$$

$\therefore$

Ex (6) If $5x = 4y$ then find the value of the ratio $\frac{3x^2+y^2}{3x^2-y^2}$.

Solution :

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{x^2}{y^2} = \frac{16}{25}$$

$$\therefore \frac{3x^2}{y^2} = \frac{48}{25} \qquad \text{...(multiplying both sides by 3)}$$

$$\therefore \frac{3x^2+y^2}{3x^2-y^2} = \frac{48+25}{48-25} \qquad \text{...(using componendo-dividendo)}$$

$$\therefore \frac{3x^2+y^2}{3x^2-y^2} = \frac{73}{23}$$

$\therefore$

Let's learn.

Application of properties of equal ratios

To solve some types of equations, it is convenient to use properties of equal ratios rather than using other methods.

Ex (1) Solve the equation. $\frac{3x^2+5x+7}{10x+14} = \frac{3x^2+4x+3}{8x+6}$

Solution :

$$\frac{3x^2+5x+7}{10x+14} = \frac{3x^2+4x+3}{8x+6}$$

$$\frac{(6x^2+10x+14)}{10x+14} = \frac{(6x^2+8x+6)}{8x+6} \qquad \text{....(multiplying both sides by 2)}$$

$$\frac{\left(6x^2+10x+14\right)-\left(10x+14\right)}{10x+14}=\frac{\left(6x^2+8x+6\right)-\left(8x+6\right)}{8x+6}$$ ...(using dividendo)

$\therefore \quad \frac{6x^2}{10x+14}=\frac{6x^2}{8x+6}$

This equation is true for $x = 0$ $\quad\therefore\quad x = 0$ is a solution of the given equation.

If $x \neq 0$ then $x^2 \neq 0$, $\therefore$ dividing by $6x^2$, $\quad \frac{1}{10x+14}=\frac{1}{8x+6}$

$\therefore \quad 8x+6=10x+14$

$\therefore \quad 6-14=10x-8x$

$\therefore \quad -8=2x$

$\therefore \quad x=-4$

$\therefore \quad x=-4$ or $x=0$ are the solutions of the given equation.

Ex (2) Solve. $\quad \frac{\sqrt{x+7}+\sqrt{x-2}}{\sqrt{x+7}-\sqrt{x-2}}=\frac{5}{1}$

Solution : $\quad \frac{(\sqrt{x+7}+\sqrt{x-2})+(\sqrt{x+7}-\sqrt{x-2})}{(\sqrt{x+7}+\sqrt{x-2})-(\sqrt{x+7}-\sqrt{x-2})}=\frac{5+1}{5-1}$ ...(using componendo-dividendo)

$\therefore \quad \frac{2\sqrt{x+7}}{2\sqrt{x-2}}=\frac{6}{4}$

$\therefore \quad \frac{\sqrt{x+7}}{\sqrt{x-2}}=\frac{3}{2}$

$\therefore \quad \frac{x+7}{x-2}=\frac{9}{4}$ ...(squaring both sides of the equation)

$\therefore \quad 4x+28=9x-18$

$\therefore \quad 28+18=9x-4x$

$\therefore \quad 46=5x$

$\therefore \quad \frac{46}{5}=x$

$\therefore \quad x=\frac{46}{5}$ is the solution of the given equation.

Activity :

Take 5 pieces of card paper. Write the following statements, one on each paper.

(i) $\frac{a+b}{b}=\frac{c+d}{d}$ (ii) $\frac{a}{c}=\frac{b}{d}$ (iii) $\frac{a}{b}=\frac{ac}{bd}$ (iv) $\frac{c}{d}=\frac{c-a}{d-b}$ (v) $\frac{a}{b}=\frac{rc}{rd}$

a, b, c, d are positive numbers and $\frac{a}{b}=\frac{c}{d}$ is given. Which of the above statements are true or false, write at the back of each card, if false explain why.

Practice set 4.3

(1) If $\frac{a}{b}=\frac{7}{3}$ then find the values of the following ratios.

(i) $\frac{5a+3b}{5a-3b}$ (ii) $\frac{2a^2+3b^2}{2a^2-3b^2}$ (iii) $\frac{a^3-b^3}{b^3}$ (iv) $\frac{7a+9b}{7a-9b}$

(2) If $\frac{15a^2+4b^2}{15a^2-4b^2}=\frac{47}{7}$ then find the values of the following ratios.

(i) $\frac{a}{b}$ (ii) $\frac{7a-3b}{7a+3b}$ (iii) $\frac{b^2-2a^2}{b^2+2a^2}$ (iv) $\frac{b^3-2a^3}{b^3+2a^3}$

(3) If $\frac{3a+7b}{3a-7b}=\frac{4}{3}$ then find the value of the ratio $\frac{3a^2-7b^2}{3a^2+7b^2}$.

(4) Solve the following equations.

(i) $\frac{x^2+12x-20}{3x-5}=\frac{x^2+8x+12}{2x+3}$ (ii) $\frac{10x^2+15x+63}{5x^2-25x+12}=\frac{2x+3}{x-5}$

(iii) $\frac{(2x+1)^2+(2x-1)^2}{(2x+1)^2-(2x-1)^2}=\frac{17}{8}$ (iv*) $\frac{\sqrt{4x+1}+\sqrt{x+3}}{\sqrt{4x+1}-\sqrt{x+3}}=\frac{4}{1}$

(v) $\frac{(4x+1)^2+(2x+3)^2}{4x^2+12x+9}=\frac{61}{36}$ (vi) $\frac{(3x-4)^3-(x+1)^3}{(3x-4)^3+(x+1)^3}=\frac{61}{189}$

Activity : In the following activity, the values of a and b can be changed. That is by changing $a : b$ we can create many examples. Teachers should give lot of practice to the students and encourage them to construct their own examples.

$\frac{a}{b}=\frac{3}{4}$

$\frac{5a^2+2b^2}{5a^2-2b^2}=$ ------

$\frac{3a}{4b}=$ ------

$\frac{2a-b}{2a+b}=$ ------

$\frac{a^2+b^2}{b^2}=$ ---

$\frac{a}{2b}=$ ------

Let's learn.

Theorem on equal ratios

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a}{b}=\frac{a+c}{b+d}=\frac{c}{d}$ This property is called the theorem of equal ratios.

Prrof : Let $\frac{a}{b}=\frac{c}{d}=k$. $\therefore$ $a = bk$ and $c = dk$

$$\therefore \frac{a+c}{b+d}=\frac{bk+dk}{b+d}=\frac{k(b+d)}{b+d}=k$$

$$\therefore \frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}$$

We know that, $\frac{a}{b}=\frac{al}{bl}$

$\therefore$ If $\frac{a}{b}=\frac{c}{d}=k$, then $\frac{al}{bl}=\frac{cm}{dm}=\frac{al+cm}{bl+dm}=k$

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=$........ (finite terms) and if l, m, n are non-zero numbers

then each ratio = $\frac{al+cm+en+\ldots}{bl+dm+fn+\ldots}$ (finite terms) is the general form of the above theorem.

Use your brain power !

In a certain gymnasium, there are 35 girls and 42 boys in the kid's section, 30 girls and 36 boys in the children's section and 20 girls and 24 boys in the teens' section. What is the ratio of the number of boys to the number of girls in every section ?

For physical exercises, all three groups gathered on the ground. Now what is the ratio of number of boys to the number of girls ?

From the answers of the above questions, did you verify the theorem of equal ratios ?

Ex (1) Fill in the blanks in the following statements.

(i) $\frac{a}{3}=\frac{b}{7}=\frac{4a+9b}{\ldots\ldots\ldots}$ (ii) $\frac{x}{3}=\frac{y}{5}=\frac{z}{4}=\frac{5x-3y+4z}{\ldots\ldots\ldots\ldots\ldots}$

Solution : (i) $\frac{a}{3}=\frac{b}{7}=\frac{4a+9b}{4\times3+9\times7}=\frac{4a+9b}{12+63}=\frac{4a+9b}{75}$

(ii) $$\frac{x}{3}=\frac{y}{5}=\frac{z}{4}=\frac{5\times x}{5\times3}=\frac{-3\times y}{-3\times5}=\frac{4\times z}{4\times4}$$

$$\therefore \quad =\frac{5x}{15}=\frac{-3y}{-15}=\frac{4z}{16}$$

$$=\frac{5x-3y+4z}{15-15+16} \quad \text{----(by the theorem of equal ratio)}$$

$$=\frac{5x-3y+4z}{16}$$

Ex (2) If $\dfrac{a}{(x-2y+3z)}=\dfrac{b}{(y-2z+3x)}=\dfrac{c}{(z-2x+3y)}$ and $x+y+z\neq 0$

then prove that each ratio = $\dfrac{a+b+c}{2(x+y+z)}$

Solution : Let $\dfrac{a}{(x-2y+3z)}=\dfrac{b}{(y-2z+3x)}=\dfrac{c}{(z-2x+3y)}=$ k.

$\therefore$ by theorem of equal ratios

$$k=\frac{a+b+c}{(x-2y+3z)+(y-2z+3x)+(z-2x+3y)}$$

$$=\frac{a+b+c}{2x+2y+2z}$$

$$=\frac{a+b+c}{2(x+y+z)}$$

$$\therefore \quad \frac{a}{x-2y+3z}=\frac{b}{y-2z+3x}=\frac{c}{z-2x+3y}=\frac{a+b+c}{2(x+y+z)}$$

Ex (3) If $\dfrac{y}{b+c-a}=\dfrac{z}{c+a-b}=\dfrac{x}{a+b-c}$ then prove that $\dfrac{a}{z+x}=\dfrac{b}{x+y}=\dfrac{c}{y+z}$.

Solution : By invertendo, we get

$$\frac{b+c-a}{y}=\frac{c+a-b}{z}=\frac{a+b-c}{x}$$

Now let $\dfrac{b+c-a}{y}=\dfrac{c+a-b}{z}=\dfrac{a+b-c}{x}=k.$

$\therefore$ by theorem of equal ratios

$$k=\frac{(c+a-b)+(a+b-c)}{z+x}$$

$$=\frac{2a}{z+x} \quad .....(I)$$

$$k=\frac{(a+b-c)+(b+c-a)}{x+y}$$

$$=\frac{2b}{x+y} \quad ....(II)$$

$$k=\frac{(b+c-a)+(c+a-b)}{y+z}$$

$$=\frac{2c}{y+z} \quad .....(III)$$

$$\therefore \quad \frac{2a}{z+x}=\frac{2b}{x+y}=\frac{2c}{y+z}$$

$$\therefore \quad \frac{a}{z+x}=\frac{b}{x+y}=\frac{c}{y+z}$$

Ex (4) Solve : $\dfrac{14x^2-6x+8}{10x^2+4x+7}=\dfrac{7x-3}{5x+2}$

Solution : By observation, we see that multiplying by $2x$ the predecessor and the successor of right hand side, we get two terms of the predecessor and the successor of the left hand side.

But before multiplying, we must ensure that $x\neq 0$.

If $x = 0$ then $\frac{14x^2-6x+8}{10x^2+4x+7} = \frac{8}{7}$ and $\frac{7x-3}{5x+2}=\frac{-3}{2}$

$\therefore \frac{8}{7}=\frac{-3}{2}$ Which is a contradiction.

$\therefore \; x \neq 0$

$\therefore$ multiplying predecessor and successor of RHS by $2x$.

$$\frac{14x^2-6x+8}{10x^2+4x+7}=\frac{2x(7x-3)}{2x(5x+2)}=k$$

$$\therefore \frac{14x^2-6x+8}{10x^2+4x+7}=\frac{14x^2-6x}{10x^2+4x}=k$$

$$\therefore \frac{14x^2-6x+8-14x^2+6x}{10x^2+4x+7-10x^2-4x}=\frac{8}{7}=k$$

$$\therefore \quad k=\frac{8}{7}$$

$$\therefore \quad \frac{7x-3}{5x+2}=\frac{8}{7}$$

$$\therefore \quad 49x-21=40x+16$$

$$\therefore \quad 49x-40x=16+21$$

$$\therefore \quad 9x=37 \qquad \therefore \; x=\frac{37}{9}$$

Practice set 4.4

(1) Fill in the blanks of the following

(i) $\frac{x}{7}=\frac{y}{3}=\frac{3x+5y}{\ldots\ldots\ldots}=\frac{7x-9y}{\ldots\ldots}$ (ii) $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=\frac{a-2b+3c}{\ldots\ldots}=\frac{\ldots\ldots}{6-8+14}$

(2) $5m-n=3m+4n$ then find the values of the following expressions.

(i) $\frac{m^2+n^2}{m^2-n^2}$ (ii) $\frac{3m+4n}{3m-4n}$

(3) (i) If $a(y+z) = b(z+x) = c(x+y)$ and out of a, b, c no two of them are equal then show that, $\frac{y-z}{a(b-c)}=\frac{z-x}{b(c-a)}=\frac{x-y}{c(a-b)}$.

(ii) If $\frac{x}{3x-y-z}=\frac{y}{3y-z-x}=\frac{z}{3z-x-y}$ and $x+y+z \neq 0$ then show that the value of each ratio is equal to 1.

(iii) If $\frac{ax+by}{x+y}=\frac{bx+az}{x+z}=\frac{ay+bz}{y+z}$ and $x+y+z \neq 0$ then show that $\frac{a+b}{2}$.

(iv) If $\frac{y+z}{a}=\frac{z+x}{b}=\frac{x+y}{c}$ then show that $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$.

(v) If $\frac{3x-5y}{5z+3y}=\frac{x+5z}{y-5x}=\frac{y-z}{x-z}$ then show that every ratio = $\frac{x}{y}$.

(4) Solve. (i) $\frac{16x^2-20x+9}{8x^2+12x+21}=\frac{4x-5}{2x+3}$ (ii) $\frac{5y^2+40y-12}{5y+10y^2-4}=\frac{y+8}{1+2y}$

Let's learn.

Continued Proportion

Let us consider the ratios 4 : 12 and 12 : 36. They are equal ratios. In the two ratios, the successor (second term) of the first ratio is equal to the predecessor (first term) of the second ratio. Hence 4, 12, 36 are said to be in continued proportion.

If $\frac{a}{b}=\frac{b}{c}$ **then *a, b, c* are in continued proportion.**

If $ac = b^2$, then dividing both sides by bc we get $\frac{a}{b}=\frac{b}{c}$.

$\therefore$ if $ac = b^2$, then a, b, c are in continued proportion.

When a, b, c are in continued proportion then b is known as **Geometric mean** of a and c or **Mean proportional** of a and c.

Hence all the following statements convey the same meaning.

$\therefore$ (1) $\frac{a}{b}=\frac{b}{c}$ (2) $b^2 = a\,c$ (3) a, b, c are in continued proportion.

(4) b is the geometric mean of a and c.

(5) b is the mean proportional of a and c.

We can generalise the concept of continued proportion.

If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\frac{d}{e}=\frac{e}{f}$ then a, b, c, d, e and f are said to be in continued proportion.

Ex (1) If x is the geometric mean of 25 and 4, then find the value of x.

Solution : x is the geometric mean of 25 and 4.

$$\therefore x^2 = 25 \times 4$$
$$\therefore x^2 = 100$$
$$\therefore x = 10$$

Ex (2) If $4\,a^2b,\ 8\,ab^2,\ p$ are in continued proportion then find the value of p.

Solution : From given information, $4\,a^2b,\ 8\,ab^2,\ p$ are in continued proportion.

$$\therefore \quad \frac{4a^2b}{8ab^2} = \frac{8ab^2}{p}$$

$$p = \frac{8ab^2 \times 8ab^2}{4a^2b} = 16b^3$$

Ex (3) Which number should be subtracted from 7, 12 and 18 such that the resultant numbers are in continued proportion?

Solution : Let x be subtracted from 7, 12 and 18 such that resultant numbers are in continued proportion.

$(7-x), (12-x), (18-x)$ are in continued proportion.

$\therefore \quad (12-x)^2 = (7-x)(18-x)$

$\therefore \quad 144-24x + x^2 = 126 - 25x + x^2$

$\therefore \quad -24x + 25x = 126 - 144$

$\therefore \quad x = -18$

Tally

$(7-x) = 7-(-18) = 25$

$(12-x) = 12-(-18) = 30$

$(18-x) = 18-(-18) = 36$

$30^2 = 900$ and $25 \times 36 = 900$

25, 30, 36 are in continued proportion

$\therefore$ If -18 is subtracted from 7, 12, 18 the resultant numbers are in continued proportion.

k - method

The k-method is used to solve examples based on equal ratios, i.e. equal proportions. In this simple method every equal ratio is assumed to be equal to k.

Ex (1) If $\frac{a}{b} = \frac{c}{d}$ then show that $\frac{5a-3c}{5b-3d} = \frac{7a-2c}{7b-2d}$

Solution : Let $\frac{a}{b} = \frac{c}{d} = k \quad \therefore\ a = bk,\ c = dk$

Substituting values of a and c in both sides,

$$\text{LHS} = \frac{5a-3c}{5b-3d} = \frac{5(bk)-3(dk)}{5b-3d} = \frac{k(5b-3d)}{(5b-3d)} = k$$

$$\text{RHS} = \frac{7a-2c}{7b-2d} = \frac{7(bk)-2(dk)}{7b-2d} = \frac{k(7b-2d)}{7b-2d} = k$$

$\therefore$ LHS = RHS.

$$\therefore \frac{5a-3c}{5b-3d} = \frac{7a-2c}{7b-2d}$$

Ex (2) If a, b, c are in continued proportion then show that, $\frac{(a+b)^2}{ab} = \frac{(b+c)^2}{bc}$.

Solution : a, b, c are in continued proportion. Let $\frac{a}{b} = \frac{b}{c} = k$.

$\therefore b = ck, \quad a = bk = ck \times k = ck^2$

Substituting values of a and b.

$$\text{LHS} = \frac{(a+b)^2}{ab} = \frac{(ck^2+ck)^2}{(ck^2)(ck)} = \frac{c^2k^2(k+1)^2}{c^2k^3} = \frac{(k+1)^2}{k}$$

$$\text{RHS} = \frac{(b+c)^2}{bc} = \frac{(ck+c)^2}{(ck)c} = \frac{c^2(k+1)^2}{c^2k} = \frac{(k+1)^2}{k}$$

$\therefore$ LHS = RHS. $\qquad \therefore \frac{(a+b)^2}{ab} = \frac{(b+c)^2}{bc}$

Ex (3) If a, b, c are in continued proportion then show that $\frac{a}{c} = \frac{a^2+ab+b^2}{b^2+bc+c^2}$

Solution : a, b, c are in continued proportion.

$\therefore \frac{a}{b} = \frac{b}{c}$

Let, $\frac{a}{b} = \frac{b}{c} = k \quad \therefore b = ck$ and $a = ck^2$

$$\text{LHS} = \frac{a}{c} = \frac{ck^2}{c} = k^2$$

$$\text{RHS} = \frac{a^2+ab+b^2}{b^2+bc+c^2}$$

$$= \frac{(k^2c)^2 + k^2c(ck) + (ck)^2}{(ck)^2 + (ck)(c) + c^2}$$

$$= \frac{k^4c^2 + k^3c^2 + c^2k^2}{c^2k^2 + c^2k + c^2}$$

$$= \frac{c^2k^2(k^2+k+1)}{c^2(k^2+k+1)}$$

$$= k^2$$

$\therefore$ LHS = RHS

$\therefore \frac{a}{c} = \frac{a^2+ab+b^2}{b^2+bc+c^2}$

Ex (4) Five numbers are in continued proportion. The first term is 5 and the last term is 80. Find these numbers.

Solution : Let the numbers in continued proportion be a, ak, ak^2, ak^3, ak^4.

Here $a = 5$ and $ak^4 = 80$

$\therefore 5 \times k^4 = 80$

$\therefore k^4 = 16$

$\therefore k = 2 \quad \because 2^4 = 16$

$ak = 5 \times 2 = 10 \qquad ak^2 = 5 \times 4 = 20$

$ak^3 = 5 \times 8 = 40 \qquad ak^4 = 5 \times 16 = 80$

$\therefore$ the numbers are 5, 10, 20, 40, 80.

Practice set 4.5

(1) Which number should be subtracted from 12, 16 and 21 so that resultant numbers are in continued proportion?

(2) If $(28-x)$ is the mean proportional of $(23-x)$ and $(19-x)$ then find the value of x.

(3) Three numbers are in continued proportion, whose mean proportional is 12 and the sum of the remaining two numbers is 26, then find these numbers.

(4) If $(a + b + c)(a - b + c) = a^2 + b^2 + c^2$ show that a, b, c are in continued proportion.

(5) If $\frac{a}{b} = \frac{b}{c}$ and $a, b, c > 0$ then show that,

(i) $(a + b + c)(b - c) = ab - c^2$

(ii) $(a^2 + b^2)(b^2 + c^2) = (ab + bc)^2$

(iii) $\frac{a^2+b^2}{ab} = \frac{a+c}{b}$

(6) Find mean proportional of $\frac{x+y}{x-y}, \frac{x^2-y^2}{x^2y^2}$

Activity : Observe the political map of India from a Geography textbook. Study the scale of this map.

From the given scale find the straight line distances between various cities like

(i) New Delhi to Bengaluru (ii) Mumbai to Kolkata, (iii) Jaipur to Bhubaneshwar.

Problem set 4

(1) Select the appropriate alternative answer for the following questions.

(i) If 6 : 5 = y : 20 then what will be the value of y ?

(A) 15 (B) 24 (C) 18 (D) 22.5

(ii) What is the ratio of 1 mm to 1 cm ?

(A) 1 : 100 (B) 10 : 1 (C) 1 : 10 (D) 100 : 1

(iii*) The ages of Jatin, Nitin and Mohasin are 16, 24 and 36 years respectively. What is the ratio of Nitin's age to Mohasin's age ?

(A) 3 : 2 (B) 2 : 3 (C) 4 : 3 (D) 3 : 4

(iv) 24 Bananas were distributed between Shubham and Anil in the ratio 3 : 5, then how many bananas did Shubham get ?

(A) 8 (B) 15 (C) 12 (D) 9

(v) What is the mean proportional of 4 and 25 ?

(A) 6 (B) 8 (C) 10 (D) 12

(2) For the following numbers write the ratio of first number to second number in the reduced form.

(i) 21, 48 (ii) 36, 90 (iii) 65, 117 (iv) 138, 161 (v) 114, 133

(3) Write the following ratios in the reduced form.

(i) Radius to the diameter of a circle.

(ii) The ratio of diagonal to the length of a rectangle, having length 4 cm and breadth 3 cm.

(iii) The ratio of perimeter to area of a square, having side 4 cm.

(4) Check whether the following numbers are in continued proportion.

(i) 2, 4, 8 (ii) 1, 2, 3 (iii) 9, 12, 16 (iv) 3, 5, 8

(5) a, b, c are in continued proportion. If $a = 3$ and $c = 27$ then find b.

(6) Convert the following ratios into percentages..

(i) 37 : 500 (ii) $\frac{5}{8}$ (iii) $\frac{22}{30}$ (iv) $\frac{5}{16}$ (v) $\frac{144}{1200}$

(7) Write the ratio of first quantity to second quantity in the reduced form.

(i) 1024 MB, 1.2 GB [(1024 MB = 1 GB)]

(ii) 17 Rupees, 25 Rupees 60 paise (iii) 5 dozen, 120 units

(iv) 4 sq.m, 800 sq.cm (v) 1.5 kg, 2500 gm

(8) If $\frac{a}{b} = \frac{2}{3}$ then find the values of the following expressions.

(i) $\frac{4a+3b}{3b}$ (ii) $\frac{5a^2+2b^2}{5a^2-2b^2}$

(iii) $\frac{a^3+b^3}{b^3}$ (iv) $\frac{7b-4a}{7b+4a}$

(9) If a, b, c, d are in proportion, then prove that

(i) $\frac{11a^2+9ac}{11b^2+9bd} = \frac{a^2+3ac}{b^2+3bd}$

(ii*) $\sqrt{\frac{a^2+5c^2}{b^2+5d^2}} = \frac{a}{b}$

(iii) $\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{c^2+cd+d^2}{c^2-cd+d^2}$

(10) If a, b, c are in continued proportion, then prove that

(i) $\dfrac{a}{a+2b} = \dfrac{a-2b}{a-4c}$ (ii) $\dfrac{b}{b+c} = \dfrac{a-b}{a-c}$

(11) Solve : $\dfrac{12x^2+18x+42}{18x^2+12x+58} = \dfrac{2x+3}{3x+2}$

(12) If $\dfrac{2x-3y}{3z+y} = \dfrac{z-y}{z-x} = \dfrac{x+3z}{2y-3x}$ then prove that every ratio = $\dfrac{x}{y}$.

(13*) If $\dfrac{by+cz}{b^2+c^2} = \dfrac{cz+ax}{c^2+a^2} = \dfrac{ax+by}{a^2+b^2}$ then prove that $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$.