7 Variation

Let's recall.

If the rate of notebooks is ₹ 240 per dozen, what is the cost of 3 notebooks? Also find the cost of 9 notebooks ; 24 notebooks and 50 notebooks and complete the following table.

Number of notebooks (x)	12	3	9	24	50	1
Cost (In Rupees) (y)	240					20

From the above table we see that the ratio of number of notebooks (x) and their cost (y) in each pair is $\frac{1}{20}$. It is constant. The number of notebooks and their cost are in the same proportion. In such a case, if one number increases then the other number increases in the same proportion.

Let's learn.

Direct variation

The statement 'x and y are in the same proportion' can be written as 'x and y are in direct variation' or 'there is a direct variation between x and y'. Using mathematical symbol it can be written as $x \propto y$. [α(alpha) is a greek letter, used to denote variation.]

$x \propto y$ is written in the form of equation as $x = \text{k}y$, where k is a constant.

$x = \text{k}y$ or $\frac{x}{y} = \text{k}$ is the equation form of direct variation where k is the constant of variation.

Observe how the following statements are written using the symbol of variation.

(i) Area of a circle is directly proportional to the square of its radius.

If the area of a circle = A, its radius = r, the above statement is written as $\text{A} \propto r^2$.

(ii) Pressure of a liquid (p) varies directly as the depth (d) of the liquid ; this statement is written as $p \propto d$.

To understand the method of symbolic representation of direct variation, study the following examples.

Ex. (1) x varies directly as y, when $x = 5$, $y = 30$. Find the constant of variation and equation of variation.

Solution: x varies directly as y, that is as $x \propto y$

$\therefore\ x = ky$ .... k is constant of variation.

when $x = 5$, $y = 30$, is given

$\therefore\ 5 = k \times 30\ \therefore\ k = \frac{1}{6}$ (constant of variation)

$\therefore$ equation of variation is $x = ky$, that is $x = \frac{y}{6}$ or $y = 6x$

Ex. (2) Cost of groundnuts is directly proportional to its weight. If cost of 5 kg groundnuts is ₹ 450 then find the cost of 1 quintal groundnuts.

(1 quintal = 100 kg)

Solution: Let the cost of groundnuts be x and weight of groundnuts be y.

It is given that x varies directly as y $\therefore\ x \propto y$ or $x = ky$

It is given that when $x = 450$ then $y = 5$, hence we will find k.

$x = ky$ $\quad\therefore\ 450 = 5k$ $\quad\therefore\ k = 90$ (constant of variation)

$\therefore$ equation of variation is $x = 90y$.

$\therefore$ if $y = 100$, $x = 90 \times 100 = 9000$

$\therefore$ cost of 1 quintal groundnut is ₹ 9000.

Practice Set 7.1

1. Write the following statements using the symbol of variation.

(1) Circumference (c) of a circle is directly proportional to its radius (r).

(2) Consumption of petrol (l) in a car and distance travelled by that car (d) are in direct variation.

2. Complete the following table considering that the cost of apples and their number are in direct variation.

Number of apples (x)	1	4	. . .	12	. . .
Cost of apples (y)	8	32	56	. . .	160

3. If $m \propto n$ and when $m = 154$, $n = 7$. Find the value of m, when $n = 14$

4. If n varies directly as m, complete the following table.

m	3	5	6.5	. . .	1.25
n	12	20	. . .	28	. . .

5. y varies directly as square root of x. When $x = 16$, $y = 24$. Find the constant of variation and equation of variation.

6. The total remuneration paid to labourers, employed to harvest soyabeen is in direct variation with the number of labourers. If remuneration of 4 labourers is ₹ 1000, find the remuneration of 17 labourers.

Let's recall.

The following table shows the number of rows and number of students in each row when they are made to stand for drill.

Number of students in a row	40	10	24	12	$\boxed{8}$
Number of Rows	6	24	$\boxed{10}$	$\boxed{20}$	30

From the table we observe that the product of number of students in each row and total number of rows in each pair is 240 ; which is constant. It means, number of students in a row and number of rows are in inverse proportion.

In a pair of numbers, if the increase in one number causes decrease in the other number in the same proportion, the pair is in inverse variation. In such an example, if one number of the pair is doubled, the other is halved.

Let's learn.

Inverse variation

The statement 'x is inversely proportional to y' can also be expressed as 'there is inverse variation in x and y.' If x and y are in inverse proportion, $x \times y$ is constant. Assuming the constant to be k, it is easy to solve a problem.

If x varies inversely as y then $x \times y$ is constant.

'x inversely varies as y' is written as $x \propto \frac{1}{y}$.

If $x \propto \frac{1}{y}$, $x = \frac{k}{y}$ or $x \times y = \text{k}$; this is the equation of variation. k, is the constant of variation.

Solved Examples

Ex. (1) If a varies inversely as b then complete the following table.

a	6	12	15	. . .
b	20	. . .	. . .	4
$a \times b$	120	120	. . .	. . .

Solution: (i) $a \propto \frac{1}{b}$, that is $a \times b = \text{k}$

when $a = 6$, $b = 20$ $\therefore$ $\text{k} = 6 \times 20 = 120$ (constant of variation)

(ii) If $a = 12$, $b = ?$

$a \times b = 120$

$\therefore 12 \times b = 120$

$\therefore \quad b = 10$

(iii) If $a = 15$, $b = ?$

$a \times b = 120$

$\therefore 15 \times b = 120$

$\therefore \quad b = 8$

(iv) If $b = 4$, $a = ?$

$a \times b = 120$

$\therefore a \times 4 = 120$

$\therefore \quad a = 30$

Ex. (2) $f \propto \frac{1}{d^2}$, when $d = 5$, $f = 18$

Hence, (i) if $d = 10$ find f. (ii) when $f = 50$ find d.

Solution: $f \propto \frac{1}{d^2}$ $\quad \therefore f \times d^2 = k$, when $d = 5$ and $f = 18$.

$\therefore 18 \times 5^2 = k \quad \therefore k = 18 \times 25 = 450$ (constant of variation)

(i) if $d = 10$ then $f = ?$

$f \times d^2 = 450$

$\therefore f \times 10^2 = 450$

$\therefore f \times 100 = 450$

$\therefore \quad f = 4.5$

(ii) if $f = 50$, then $d = ?$

$f \times d^2 = 450$

$\therefore 50 \times d^2 = 450$

$\therefore \quad d^2 = 9$

$\therefore \quad d = 3$ or $d = -3$

Practice Set 7.2

1 . The information about numbers of workers and number of days to complete a work is given in the following table. Complete the table.

Number of workers	30	20		10	
Days	6	9	12		36

2. Find constant of variation and write equation of variation for every example given below.

(1) $p \propto \frac{1}{q}$; if $p = 15$ then $q = 4$ (2) $z \propto \frac{1}{w}$; when $z = 2.5$ then $w = 24$

(3) $s \propto \frac{1}{t^2}$; if $s = 4$ then $t = 5$ (4) $x \propto \frac{1}{\sqrt{y}}$; if $x = 15$ then $y = 9$

3. The boxes are to be filled with apples in a heap. If 24 apples are put in a box then 27 boxes are needed. If 36 apples are filled in a box how many boxes will be needed ?

4. Write the following statements using symbol of variation .

(1) The wavelength of sound (l) and its frequency (f) are in inverse variation.

(2) The intensity (I) of light varies inversely with the square of the distance (d) of a screen from the lamp.

5. $x \propto \frac{1}{\sqrt{y}}$ and when $x = 40$ then $y = 16$. If $x = 10$, find y.

6. x varies inversely as y, when $x = 15$ then $y = 10$, if $x = 20$ then $y = ?$

Let's learn.

Time, Work, Speed

Examples related to the number of workers and time taken to finish the work are of inverse variation. Similarly, there are some examples related to the time taken to cover a distance by a vehicle and its uniform speed. Such examples of variation are the examples related to time, work and speed.

Now we will see how to solve such examples of variation.

Ex. (1) 15 women finish the work of harvesting a groundnut crop in 8 days. Find the number of women if the same job is to be completed in 6 days.

Solution: The number of days required to finish a job is inversely proportional to the number of women employed. Let the number of days be d and number of women be n.

$d \propto \frac{1}{n}$ $\quad\quad$ $\therefore\ d \times n = \text{k}$ $\quad\quad$ (k is constant)

If $n = 15$, then $d = 8$ $\quad$ $\therefore\ \text{k} = d \times n = 15 \times 8 = 120$

Now let us find n when $d = 8$.

$d \times n = \text{k}$

$\therefore\ d \times n = 120$ $\quad\quad$ $\therefore\ 6 \times n = 120,$ $\quad$ $n = 20$

$\therefore$ 20 women should be employed to finish the work in 6 days

Ex. (2) A vehical running at a speed of 48 km/hr takes 6 hours to complete the journey. How much time will be taken to complete the journey if its speed is 72 km/hr ?

Solution : Let us assume the speed of vehicle to be s and time taken to travel be t.

There is inverse variation in speed and time.

$s \propto \frac{1}{t}$ $\therefore$ $s \times t = k$ (k is constant)

$k = s \times t = 48 \times 6 = 288$ Now, let us find t when $s = 72$.

$s \times t = 288$ $\therefore$ $72 \times t = 288$ $\therefore$ $t = \frac{288}{72} = 4$

$\therefore$ time taken to travel the same distance at the speed 72 km/hr is 4 hours.

Practice Set 7.3

1. Which of the following statements are of inverse variation ?

(1) Number of workers on a job and time taken by them to complete the job.

(2) Number of pipes of same size to fill a tank and the time taken by them to fill the tank.

(3) Petrol filled in the tank of a vehical and its cost

(4) Area of circle and its radius.

2. If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours ?

3. 120 bags of half litre milk can be filled by a machine within 3 minutes find the time to fill such 1800 bags ?

4. A car with speed 60 km/hr takes 8 hours to travel some distance. What should be the increase in the speed if the same distance is to be covered in $7\frac{1}{2}$ hours?

Answers

Practice Set 7.1 **1.** (1) $c \propto r$ (2) $l \propto d$ **2.** $x = 7$, $x = 20$, $y = 96$ **3.** 308
4. $m = 7$, $n = 26$ and 5 **5.** $k = 6$, $y = 6\sqrt{x}$ **6.** ₹ 4250

Practice Set 7.2 **1.** Number of workers 15 and 5 respectively, days = 18
2. (1) $k = 60$, $pq = 60$ (2) $k = 60$, $zw = 60$
(3) $k = 100$, $st^2 = 100$ (4) $k = 45$, $x\sqrt{y} = 45$
3. 18 boxes **4.** (1) $l \propto \frac{1}{f}$ (2) $I \propto \frac{1}{d^2}$ **5.** $y = 256$ **6.** $y = 7.5$

Practice Set 7.3 **1.** inverse variation (1), (2) **2.** 24 worker
3. 45 minutes **4.** 4 km/hr

D2PPKD