14) Compound interest

A person takes a loan from institutes like bank or *patapedhi* with fixed rate of interest. After stipulated time he repays the loan with some more money. The additional money paid is called interest.

We have learnt the formula for finding the interest, $I = \dfrac{PNR}{100}$.

In this formula I = Interest, P = Principal, N = Number of years and R = Rate of interest at p.c.p.a. This is how the simple interest is calculated.

Let's learn.

Compound Interest

We will learn how bank charges compound interest for a fixed deposit or a loan.

Teacher : Sajjanrao takes a loan of ₹10,000 from a bank at the rate of 10 p.c.p.a. for 1year. How much money including the interest he will have to pay after one year?

Student : Here P = ₹10,000 ; R = 10; N = 1 year

∴ $I = \dfrac{PNR}{100} = \dfrac{10000 \times 10 \times 1}{100} = ₹\ 1000$

∴ After one year Sajjanrao will have to pay 10,000 + 1000 = ₹ 11,000 with interest.

Student : If a borrower fails to pay the amount and interest after one year?

Teacher : Bank calculates the interest after each year and it is expected that the interest should be paid by the borrower every year. If the borrower fails to pay the interest for one year then the bank considers the principal and the interest of first year together as the loan for second year. Thus for second year, the interest is calculated on the amount formed by principal of first year together with its interest. That is for second year the principal for charging interest is the amount of the first year. The interest charged by this method is called as compound interest.

Student : If Sajjanarao increases the duration of loan repayment by one more year?

Teacher : Then for second year considering ₹ 11,000 as principal, interest and amount is to be found out.

Student : For this can we use the ratio $\dfrac{\text{Amount}}{\text{Principal}} = \dfrac{110}{100}$ which is learnt in the previous standard?

Teacher : Surely ! For every year the ratio $\frac{\text{Amount}}{\text{Principal}}$ is constant. While finding compound interest every year, the amount (A) of previous year is the principal for next year. Hence it is convenient to find amount rather than the interest. Let us write the amount after first year as A_1, after second year A_2 and after third year as A_3.

Originally the principal is P.

$$\therefore \frac{A_1}{P} = \frac{110}{100} \quad \therefore A_1 = P \times \frac{110}{100}$$

For finding the amount A_2 of second year,

$$\frac{A_2}{A_1} = \frac{110}{100} \quad \therefore A_2 = A_1 \times \frac{110}{100} = P \times \frac{110}{100} \times \frac{110}{100}$$

Student: Then for finding the amount A_3 of third year,

$$\frac{A_3}{A_2} = \frac{110}{100} \quad \therefore A_3 = A_2 \times \frac{110}{100} = P \times \frac{110}{100} \times \frac{110}{100} \times \frac{110}{100}$$

Teacher : Very Good ! This is the formula for finding the amount by compound interest. Here $\frac{110}{100}$ is the amount of ₹1 after one year. To find amount after the given number of years multiply the principal by those many times by this ratio.

Student : If the ratio $\frac{\text{Amount}}{\text{Principal}}$ is assumed to be M then the amount after one year is $P \times M$, after second year is $P \times M^2$, after third year PM^3. In this way we can find the amount after any number of years.

Teacher : Correct ! If the interest rate is R p.c.p.a. then,

the amount of ₹1 after 1 year $= 1 \times M = 1 \times \frac{100+R}{100} = 1 \times \left(1+\frac{R}{100}\right)$

the amount of ₹ P after 1 year $= P \times \frac{100+R}{100} = P \times \left(1+\frac{R}{100}\right)$

$\therefore$ if the principal P, interest rate R p.c.p.a. and duration is N years then

the amount after N years, $A = P \times \left(\frac{100+R}{100}\right)^N = P \times \left(1+\frac{R}{100}\right)^N$

Solved Example

Ex. **(1)** Find the compound interest if ₹4000 are invested for 3 years at the rate of $12\frac{1}{2}$ p.c.p.a.

Solution: Here, P = ₹ 4000; R = $12\frac{1}{2}$%; N = 3 years.

$$A = P\left(1+\frac{R}{100}\right)^N = P\left(1+\frac{12.5}{100}\right)^3 \qquad\qquad A = 4000\left(\frac{1125}{1000}\right)^3 = 4000\left(\frac{9}{8}\right)^3$$

$$= 4000\left(1+\frac{125}{1000}\right)^3 \qquad\qquad\qquad\qquad = 5695.31 \text{ Rupees}$$

$\therefore$ Compound Interest after three years, I $\quad$ = Amount – Principal

$$= 5695.31 - 4000 = 1695.31 \text{ rupees}$$

Practice Set 14.1

1. Find the amount and the compound interest.

No.	Principal (₹)	Rate (p.c.p.a.)	Duration (years)
1	2000	5	2
2	5000	8	3
3	4000	7.5	2

2. Sameerrao has taken a loan of ₹12500 at a rate of 12 p.c.p.a. for 3 years. If the interest is compounded annually then how many rupees should he pay to clear his loan?

3. To start a business Shalaka has taken a loan of ₹8000 at a rate of $10\frac{1}{2}$ p.c.p.a. After two years how much compound interest will she have to pay?

For more information

1. Some times the interest is calculated at an interval of six months. For the duration of N years, if rate is R and if the interest to be calculated six monthly then the rate is to be taken as $\frac{R}{2}$ and the duration is considered as 2N stages of six months.

2. Many banks charge the compound interest monthly. At that time they take the interest rate as $\frac{R}{12}$ monthly and the duration is taken 12 × N stages of months and interest is calculated.

3. Now a days banks calculate compound interest daily.

Activity: Visit the bank nearer to your house and get the information regarding the different schemes and rate of interests. Make a chart and display in your class.

Application of formula for compound interest

The formula for amount by compound interest is very useful to solve the problems in other fields related to our practical life; for example increase in the population, the reduction in the value of vehicle or machine etc. If any article is sold after using it for some time its value reduces as compared to its cost price. This reduction in price is called 'depreciation'.

The depreciation takes place by a specific rate for a specific time. For example every year price of a machine reduces by a definite percentage. For finding the reduced price over a period of time the formula for compound interest is used. To find the reduced price, rate of depreciation should be known. In such a case the value of the article decreases, so the rate of depreciation R is taken as negetive.

Ex. (1) The population of a city increases at compounding rate of 8% per year. Find the population in the year 2012 if populatin in the year 2010 was 2,50,000.

Solution: P = Population in the year 2010 = 2,50,000

A = Population in the year 2012;

R = Rate of increase of population per year = 8%

N = 2 years

A = Population in the year 2012, that is population after 2 years

$$A = P \times \left(1 + \frac{R}{100}\right)^N = 250000 \times \left(1 + \frac{8}{100}\right)^2$$

$$= 250000 \times \left(\frac{108}{100}\right)^2$$

$$= 250000 \times \left(\frac{108}{100}\right) \times \left(\frac{108}{100}\right)$$

$$= 2,91,600.$$

∴ In the year 2012, population of the city was 2,91,600.

Ex. (2) Rehana purchased a scooter in the year 2015 for ₹60000. If its value falls by 20% every year what will be the price of scooter after 2 years ?

Solution: P = ₹60000   A = Amount obtained after two years

R = Rate of depreciation = 20 %   N = 2 years

A = Amount obtained after 2 years

$$A = P \times \left(1 + \frac{R}{100}\right)^N$$
$$= 60000 \times \left(1 - \frac{1}{5}\right)^2$$
$$= 60000 \times \frac{4}{5} \times \frac{4}{5}$$

$$A = 60000 \times \left(1 + \frac{-20}{100}\right)^2$$
$$= 60000 \times \left(\frac{4}{5}\right)^2$$
$$A = 38400 \text{ rupees.}$$

∴ After two years the price will be Rs. 38400.

In the formula for compound interest four quantities A, P, N and R incur. If three of them are given, study how the fourth can be found out from the following examples.

Ex. (3) The amount of a certain principal is ₹6655 in 3 years, compounded annually at the rate of 10 p.c.p.a. Find the principal.

Solution: A = ₹6655; R = 10 p.c.p.a ; N = 3 years

$$A = P \times \left(1 + \frac{R}{100}\right)^N$$

$$\therefore 6655 = P \times \left(1 + \frac{10}{100}\right)^3 = P \times \left(\frac{110}{100}\right)^3$$

$$\therefore P = \frac{6655 \times 10^3}{11 \times 11 \times 11} \qquad P = 5 \times 10^3 \qquad = 5000$$

∴ the principal was ₹ 5000.

Ex. (4) Find the number of years for which the compound interest of ₹9000 is ₹1890, at the rate of 10 p.c.p.a. .

Solution: R = 10%; P = ₹ 9000; compound interest = ₹ 1890

We will find the amount first.

A = P + I   = 9000 + 1890   = ₹ 10890

Write the formula for compound interest and substitute the values.

$$A = 10890 = P \times \left(1 + \frac{R}{100}\right)^N = 9000 \times \left(1 + \frac{10}{100}\right)^N = 9000 \times \left(\frac{11}{10}\right)^N$$

$$\therefore \left(\frac{11}{10}\right)^N = \frac{10890}{9000} = \frac{121}{100} \quad \therefore \left(\frac{11}{10}\right)^N = \frac{121}{100} \quad \therefore \quad N = 2$$

$\therefore$ compound interest is for 2 years.

Practice Set 14.2

1. On the construction work of a flyover bridge there were 320 workers initially. The number of workers were increased by 25% every year. Find the number of workers after 2 years.

2. A shepherd has 200 sheep with him. Find the number of sheeps with him after 3 years if the increase in number of sheeps is 8% every year.

3. In a forest there are 40,000 trees. Find the expected number of trees after 3 years if the objective is to increase the number at the rate 5% per year.

4. The cost price of a machine is 2,50,000. If the rate of depreciation is 10% per year find the depreciation in price of the machine after two years.

5. Find the compound interest if the amount of a certain principal after two years is ₹4036.80 at the rate of 16 p.c.p.a.

6. A loan of ₹15000 was taken on compound interest. If the rate of compound interest is 12 p.c.p.a. find the amount to settle the loan after 3 years.

7. A principal amounts to ₹13924 in 2 years by compound interest at 18 p.c.p.a. Find the principal.

8. The population of a suburb is 16000. Find the rate of increase in the population if the population after two years is 17640.

9. In how many years ₹ 700 will amount to ₹ 847 at a compound interest rate of 10 p.c.p.a.

10. Find the difference between simple interest and compound interest on ₹ 20000 at 8 p.c.p.a.

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Answers

Practice Set 14.1 1. (1) ₹ 2205, ₹ 205   (2) ₹ 6298.56, ₹ 1298.56

(3) ₹ 4622.5, ₹ 622.5   **2.** ₹ 17561.60   **3.** ₹ 1768.2

Practice Set 14.2 1. 500   **2.** 252 sheeps   **3.** 46,305 trees

4. ₹ 47500   **5.** ₹ 1036.80   **6.** ₹ 21073.92   **7.** ₹ 10,000

8. 5 p.c.p.a   **9.** In 2 years   **10.** ₹ 128