3 HCF and LCM

YHYJ4B

Let's recall.

- Which is the smallest prime number?
- List the prime numbers from 1 to 50. How many are they?
- Circle the prime numbers in the list below.

  17, 15, 4, 3, 1, 2, 12, 23, 27, 35, 41, 43, 58, 51, 72, 79, 91, 97

Co-prime numbers : Two numbers which have only 1 as a common factor are said to be **co-prime** or **relatively prime** or **mutually prime numbers**.

For example, 10 and 21 are co-primes, because the divisors of 10 are 1, 2, 5, 10 while the divisions of 21 are 1, 3, 7, 21 and the only factor common to both 10 and 21 is 1.

Some other co-prime numbers are (3, 8) ; (4, 9); (21, 22) ; (22, 23) ; (23, 24).

Verify that any two consecutive natural numbers are co-primes.

Let's learn. Twin Prime Numbers

If the difference between two co-prime numbers is 2, the numbers are said to be twin prime numbers.

For example : (3, 5) ; (5, 7) ; (11, 13) ; (29, 31) etc.

Practice Set 10

1. Which number is neither a prime number nor a composite number?
2. Which of the following are pairs of co-primes?

   (i) 8, 14 (ii) 4, 5 (iii) 17, 19 (iv) 27, 15
3. List the prime numbers from 25 to 100 and say how many they are.
4. Write all the twin prime numbers from 51 to 100.
5. Write 5 pairs of twin prime numbers from 1 to 50.
6. Which are the even prime numbers?

Let's learn. Factorising a Number into its Prime Factors

A simple but important rule given by Euclid is often used to find the GCD or HCF and LCM of numbers. The rule says that **any composite number can be written as the product of prime numbers**.

Let us learn how to find the prime factors of a number.

Example Write the number 24 in the form of the product of its prime factors.

Method for finding prime factors

Vertical arrangement

2	24
2	12
2	6
3	3
	1

$24 = 2 \times 2 \times 2 \times 3$

Horizontal arrangement

$24 = 2 \times 12$

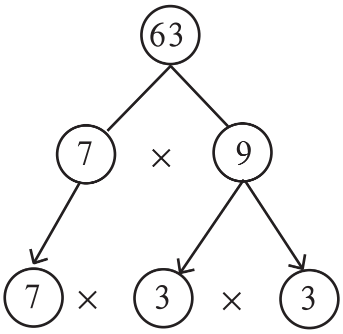
$= 2 \times 2 \times 6$ ...12 is factorised.

$= 2 \times 2 \times 2 \times 3$ ...6 is factorised.

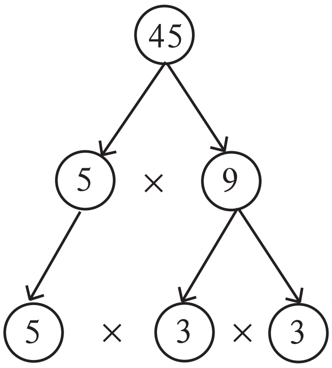
The prime factors are 2 and 3.

> **Remember :**
> **To write a given number as a product of its prime factors is to factorise it into primes.**

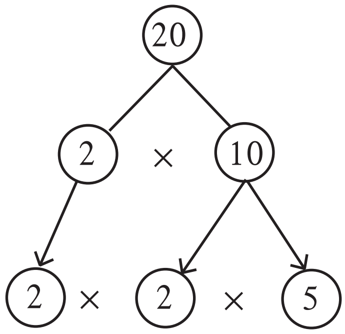
Example Write each of the given numbers as a product of its prime factors.



$63 = 7 \times 3 \times 3$



$45 = 5 \times 3 \times 3$



$20 = 2 \times 2 \times 5$

Example Factorise into primes: 117.

3	117
3	39
13	13
	1

$117 = 13 \times 9$

$= 13 \times 3 \times 3$

$117 = 3 \times 3 \times 13$

Example Factorise into primes: 250.

2	250
5	125
5	25
5	5
	1

$250 = 2 \times 125$

$= 2 \times 5 \times 25$

$= 2 \times 5 \times 5 \times 5$

$250 = 2 \times 5 \times 5 \times 5$

Example Find the prime factors of 40.

Vertical arrangement

2	40
2	20
2	10
5	5
	1

$40 = 2 \times 2 \times 2 \times 5$

Horizontal arrangement

$$\begin{aligned} 40 &= 10 \times 4 \\ &= 5 \times 2 \times 2 \times 2 \end{aligned}$$

$$\begin{aligned} 40 &= 8 \times 5 \\ &= 2 \times 2 \times 2 \times 5 \end{aligned}$$

Practice Set 11

⦿ Factorise the following numbers into primes.

(i) 32 (ii) 57 (iii) 23 (iv) 150 (v) 216

(vi) 208 (vii) 765 (viii) 342 (ix) 377 (x) 559

Let's recall.

Greatest Common Divisor (GCD) or Highest Common Factor (HCF)

We are familiar with the HCF and LCM of positive integers. Let us learn something more about them. The **HCF** or the **GCD** of given numbers is their greatest common divisor or factor.

In each of the following examples, write all the factors of the numbers and find the greatest common divisor.

(i) 28, 42 (ii) 51, 27 (iii) 25, 15, 35

Let's learn. Prime Factors Method

It is easy to find the HCF of numbers by first factorising all the numbers.

Example Find the HCF of 24 and 32 by the prime factors method.

2	24
2	12
2	6
3	3
	1

$$\begin{aligned} 24 &= 4 \times 6 \\ &= \underline{2} \times \underline{2} \times \underline{2} \times 3 \end{aligned}$$

2	32
2	16
2	8
2	4
2	2
	1

$$\begin{aligned} 32 &= 8 \times 4 \\ &= \underline{2} \times \underline{2} \times \underline{2} \times 2 \times 2 \end{aligned}$$

The common factor 2 occurs thrice in each number. Therefore, the HCF = $2 \times 2 \times 2 = 8$.

Example Find the HCF of 195, 312, 546.

$$195 = 5 \times 39 = 5 \times \underline{3} \times \underline{13}$$

$$312 = 4 \times 78 = 2 \times 2 \times 2 \times 39 = 2 \times 2 \times 2 \times \underline{3} \times \underline{13}$$

$$546 = 2 \times 273 = 2 \times 3 \times 91 = 2 \times \underline{3} \times 7 \times \underline{13}$$

The common factors 3 and 13 each occur once in all the numbers.

$\therefore$ HCF $= 3 \times 13 = 39$

Example Find the HCF of 10, 15, 12.

$10 = 2 \times 5$ $\quad$ $15 = 3 \times 5$ $\quad$ $12 = 2 \times 2 \times 3$

No number except 1 is a common divisor.

Hence, HCF = 1

Example Find the HCF of 60, 12, 36.

$$60 = 4 \times 15 = \underline{2} \times \underline{2} \times \underline{3} \times 5$$

$$12 = 2 \times 6 = \underline{2} \times \underline{2} \times \underline{3}$$

$$36 = 3 \times 12 = 3 \times 3 \times 4 = \underline{2} \times \underline{2} \times \underline{3} \times 3$$

$\therefore$ HCF $= 2 \times 2 \times 3 = 12$

Let us work out this example in the vertical arrangement. We write all the numbers in one line and find their factors.

2	60	12	36
2	30	6	18
3	15	3	9
	5	1	3

$\therefore$ HCF $= 2 \times 2 \times 3 = 12$

Note that 12 is a divisor of 36 and 60.

Now I know!

- If one of the given numbers is a divisor of all the others, then it is the HCF of the given numbers.
- If no prime number is a common divisor of all the given numbers, then 1 is their HCF because it is the only common divisor.

✻ **Something more**

2 is the HCF of any two consecutive even numbers and 1 is the HCF of any two consecutive odd numbers.

Verify the rule, by taking many different examples.

The Division Method for Finding the HCF

Example Find the HCF of 144 and 252.

```
144) 252( 1
    -144
    108)144( 1
       -108
         36)108(3
           -108
            000
```

1. Divide the bigger number by the smaller one.
2. Divide the previous divisor by the remainder in this division.
3. Divide the divisor of step 2 by the remainder obtained in the division in step 2.
4. Continue like this till the remainder becomes zero. The divisor in the division in which the remainder is zero is the HCF of the given numbers.

$\therefore$ The HCF of 144 and 252 = 36

Example Reduce $\frac{209}{247}$ to its simplest form.

```
209) 247 ( 1
    -209
     38)209 (5
       -190
        19)38(2
          -38
           00
```

To reduce the number to its simplest form, we will find the common factors of 209 and 247. Let us find their HCF by the division method.

Here, 19 is the HCF. That is, the numerator and denominator are both divisible by 19.

$$\therefore \quad \frac{209}{247} = \frac{209 \div 19}{247 \div 19} = \frac{11}{13}$$

Practice Set 12

1. Find the HCF.

(i) 25, 40 (ii) 56, 32 (iii) 40, 60, 75 (iv) 16, 27
(v) 18, 32, 48 (vi) 105, 154 (vii) 42, 45, 48 (viii) 57, 75, 102
(ix) 56, 57 (x) 777, 315, 588

2. Find the HCF by the division method and reduce to the simplest form.

(i) $\frac{275}{525}$ (ii) $\frac{76}{133}$ (iii) $\frac{161}{69}$

Let's recall.

Least Common Multiple (LCM)

The Least Common Multiple of the given numbers is the smallest number that is divisible by each of the given numbers.

- Write the tables of the given numbers and find their LCM.

(i) 6, 7 (ii) 8, 12 (iii) 5, 6, 15

Let's learn.

Example Find the LCM of 60 and 48.

Let us find the prime factors of each number.

$60 = 2 \times 2 \times 3 \times 5$ $\qquad$ $48 = 2 \times 2 \times 2 \times 2 \times 3$

Let us consider each prime number in these multiplications.

2 occurs a maximum of 4 times. (in the factors of 48)

3 occurs only once (in the factors of 60)

5 occurs only once (in the factors of 60)

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 10 \times 24 = 240$

Example Find the LCM of 18, 30, 50.

18 = 2 × 9	30 = 2 × 15	50 = 2 × 25
= 2 × 3 × 3	= 2 × 3 × 5	= 2 × 5 × 5

2, 3, 5 are the prime numbers that occur in the multiplications above.

In the products above, the number 2 occurs a maximum of $\boxed{}$ times, 3 occurs a maximum of $\boxed{}$ times and 5 a maximum of $\boxed{}$ times.

$\therefore$ LCM $= 2 \times 3 \times 3 \times 5 \times 5 = 450$ $\qquad$ $\therefore$ The LCM of 18, 30, 50 is 450.

Example Find the LCM of 16, 28, 40.

Vertical arrangement

2	16	28	40
2	8	14	20
2	4	7	10
	2	7	5

- Use the tests of divisibility to find the prime number that divides all the numbers and then divide the given numbers. Repeat this process for the quotients as many times as possible.
- Now find the number that divides at least two of the numbers obtained and divide those numbers by the number you find. Do this as many times as possible. If a number cannot be divided, leave it as it is.
- Stop when the only common divisor you get is 1.
- Find the product of the numbers in the column on the left. Multiply this product by the numbers in the last row.
  LCM $= 2 \times 2 \times 2 \times 2 \times 5 \times 7 = 560$

Example Find the LCM and HCF of 18 and 30. Compare the product of the LCM and HCF with the product of the given numbers.

2	18	30
3	9	15
	3	5

HCF $= 2 \times 3 = 6$

LCM $= 2 \times 3 \times 3 \times 5 = 90$

HCF $\times$ LCM $= 6 \times 90 = 540$

Product of the two given numbers $= 18 \times 30 = 540$

Product of the two given numbers $=$ HCF $\times$ LCM

We see that the product of two numbers is equal to the product of their GCD and LCM. Verify this statement for the following pairs of numbers : (15, 48), (14, 63), (75, 120)

Example Find the LCM and HCF of 15, 45 and 105.

3	15	45	105
5	5	15	35
	1	3	7

$15 = \underline{3} \times \underline{5}$

$45 = \underline{3} \times 3 \times \underline{5}$

$105 = \underline{3} \times \underline{5} \times 7$

$\text{GCD} = 3 \times 5 = 15$

$\text{LCM} = 3 \times 3 \times 5 \times 7 = 315$

Example The product of two 2-digit numbers is 1280 and the GCD = 4. What is their LCM?

$$\text{GCD} \times \text{LCM} = \text{Product of given numbers}$$

$$4 \times \text{LCM} = 1280$$

$$\therefore \text{LCM} = \frac{1280}{4} = 320$$

Practice Set 13

1. Find the LCM.

(i) 12, 15 (ii) 6, 8, 10 (iii) 18, 32 (iv) 10, 15, 20 (v) 45, 86
(vi) 15, 30, 90 (vii) 105, 195 (viii) 12, 15, 45 (ix) 63, 81
(x) 18, 36, 27

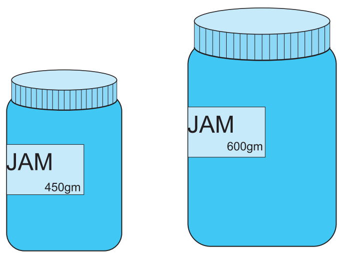
2. Find the HCF and LCM of the numbers given below. Verify that their product is equal to the product of the given numbers.

(i) 32, 37 (ii) 46, 51 (iii) 15, 60 (iv) 18, 63 (v) 78, 104

The Use of LCM and HCF

Example A shop sells a 450 g bottle of jam for 96 rupees and a bigger bottle of 600 g for 124 rupees. Which bottle is it more profitable to buy?

Solution: We have learnt the unitary method. Using that we can find the cost of 1 gm jam in each bottle and compare. However, the calculation is easier if we use a bigger common factor rather than a smaller one.



Let us use 150, the HCF of 450 and 600 to compare.

$450 = 150 \times 3,$ $600 = 150 \times 4$

$\therefore$ In the small bottle, 150 g of jam costs $\frac{96}{3}$ = 32 rupees.

In the large bottle, 150 g of jam costs $\frac{124}{4}$ = 31 rupees.

$\therefore$ Thus, it is more profitable to buy the 600 g bottle of jam.

Example Add $\frac{17}{28} + \frac{11}{35}$

Method 1 In order to add, let us make the denominators of the fractions equal.

Solution: $\frac{17}{28}+\frac{11}{35} = \frac{17\times35+11\times28}{28\times35} = \frac{595+308}{28\times35} = \frac{903}{28\times35} = \frac{903}{980} = \frac{129}{140}$

Method 2 Let us find the LCM of 28 and 35 in order to add the fractions.

LCM = 7 × 4 × 5 = 140

Solution: $\frac{17}{28} + \frac{11}{35} = \frac{17\times5}{28\times5} + \frac{11\times4}{35\times4} = \frac{85+44}{140} = \frac{129}{140}$

Taking the LCM rather than multiplying the denominators made our calculations so much easier!

Example On dividing a certain number by 8, 10, 12, 14 the remainder is always 3. Which is the smallest such number?

Solution: To find this multiple, let us find the LCM of the given divisors.

2	8	10	12	14
2	4	5	6	7
	2	5	3	7

LCM = 2 × 2 × 2 × 5 × 3 × 7 = 840

To the LCM we add the remainder obtained every time.

Hence, that number = LCM + remainder

= 840 + 3 = 843

Example Find the LCM of the numbers 16, 20, 80.

Solution: 16 = 2 × 2 × 2 × 2

20 = 2 × 2 × 5

80 = 2 × 2 × 2 × 2 × 5

LCM = 4 × 4 × 5 = 80

4	16	20	80
4	4	5	20
5	1	5	5
	1	1	1

Did you notice that here 80 is one of the given numbers and that the other numbers 16 and 20, are its divisors.

Remember:

If the greatest of the given numbers is divisible by the other numbers, then that greatest number is the LCM of the given numbers.

In order to verify the above rule, examine these groups of numbers (18,90) (35,140,70).

Example Shreyas, Shalaka and Snehal start running from the same point on a circular track at the same time and complete one lap of the track in 16 minutes, 24 minutes and 18 minutes respectively. What is shortest period of time in which they will all reach the starting point together ?

Solution: The number of minutes they will take to reach together will be a multiple of 16, 24 and 18. To find out the smallest such number, we will find the LCM.

$16 = 2 \times 2 \times 2 \times 2$ $\quad$ $24 = 2 \times 2 \times 2 \times 3$ $\quad$ $18 = 2 \times 3 \times 3$

$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

They will come together in 144 minutes or 2 hours 24 minutes.

Practice Set 14

1. Choose the right option.
   (i) The HCF of 120 and 150 is ....
   (1) 30 (2) 45 (3) 20 (4) 120
   (ii) The HCF of this pair of numbers is not 1.
   (1) 13, 17 (2) 29, 20 (3) 40, 20 (4) 14, 15
2. Find the HCF and LCM.
   (i) 14, 28 (ii) 32, 16 (iii) 17, 102, 170 (iv) 23, 69 (v) 21, 49, 84
3. Find the LCM.
   (i) 36, 42 (ii) 15, 25, 30 (iii) 18, 42, 48 (iv) 4, 12, 20 (v) 24, 40, 80, 120
4. Find the smallest number which when divided by 8, 9, 10, 15, 20 gives a remainder of 5 every time.
5. Reduce the fractions $\frac{348}{319}, \frac{221}{247}, \frac{437}{551}$ to the lowest terms.
6. The LCM and HCF of two numbers are 432 and 72 respectively. If one of the numbers is 216, what is the other?
7. The product of two two-digit numbers is 765 and their HCF is 3. What is their LCM?
8. A trader has three bundles of string 392 m, 308 m and 490 m long. What is the greatest length of string that the bundles can be cut up into without any left over string?

9*. Which two consecutive even numbers have an LCM of 180?