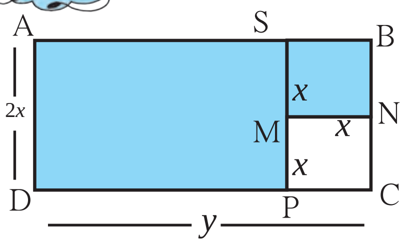
14 Algebraic Formulae - Expansion of Squares

Let's recall.



A rectangle ABCD is shown in the figure alongside. Its length is y units and its breadth, $2x$ units. A square of side x units is cut out from this rectangle. We can use operations on algebraic expressions to find the area of the shaded part. Let us write the area of rectangle ABCD as A(□ ABCD)

Area of the shaded part $=$ A(□ ABCD) $-$ A(□ MNCP)

$= 2xy - x^2$

Area of the shaded part $=$ A(□ ASPD) $+$ A(□ SBNM)

$= (y - x) \times 2x + x^2$

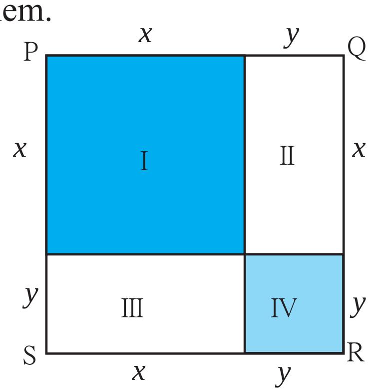
$= 2xy - 2x^2 + x^2$

$= 2xy - x^2$

Let's learn. **The Expanded Form of the Square of a Binomial**

The product of algebraic expressions is called their 'expansion' or their 'expanded form'. There are some formulae which help in writing certain expansions. Let's consider some of them.

Activity I



- In the figure alongside, the side of the square PQRS is $(x + y)$.

  $\therefore$ A (□PQRS) $= (x + y)^2$

  The square PQRS is divided into 4 rectangles : I, II, III, IV

  A(□ PQRS) = Sum of areas of rectangles I, II, III, IV.

$\therefore$ A (□PQRS)= A (Rectangle I) + A (Rectangle II) + A (Rectangle III) + A (Rectangle IV)

$(x + y)^2 = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$

$\therefore \quad (x + y)^2 = x^2 + 2xy + y^2$

Now, let us multiply $(x + y)^2$ as algebraic expressions.

$(x + y)\ (x + y) = x\ (x + y) + y\ (x + y)$

$= x^2 + xy + yx + y^2 \qquad \therefore \qquad (x + y)^2 = x^2 + 2xy + y^2$

The expression obtained by squaring the binomial $(x + y)$ is equal to the expression obtained by finding the area of the square. Therefore, $(x + y)^2 = x^2 + 2xy + y^2$ is the formula for the expansion of the square of a binomial.

Activity II In the figure alongside, the square with side a is divided into 4 rectangles, namely, square with side $(a-b)$, square with side b and two rectangles of sides $(a-b)$ and b.

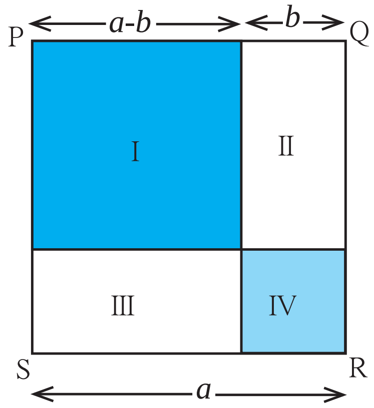
A (square I) + A (rectangle II) + A (rectangle III) + A (square IV) = A (□PQRS)

$$(a-b)^2 + (a-b)\,b + (a-b)\,b + b^2 = a^2$$

$$(a-b)^2 + 2ab - 2b^2 + b^2 = a^2$$

$$(a-b)^2 + 2ab - b^2 = a^2$$

$$\therefore\ (a-b)^2 = a^2 - 2ab + b^2$$



Let us multiply the algebraic expressions and obtain the formula.

$$\begin{aligned}(a - b)^2 &= (a - b) \times (a - b)\\ &= a\,(a - b) - b\,(a - b)\\ &= a^2 - ab - ab + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Now I know !

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$

We can verify the formulae by substituting a and b with any numbers.

Thus, if $a = 5$, $b = 3$, then

$(a + b)^2 = (5 + 3)^2 = 8^2 = 64$

$a^2 + 2ab + b^2 = 5^2 + 2 \times 5 \times 3 + 3^2$

$= 25 + 30 + 9 = 64$

$(a - b)^2 = (5 - 3)^2 = 2^2 = 4$

$a^2 - 2ab + b^2 = 5^2 - 2 \times 5 \times 3 + 3^2$

$= 25 - 30 + 9 = 4$

Use the given values to verify the formulae for squares of binomials.

(i) $a = -7$, $b = 8$ (ii) $a = 11$, $b = 3$ (iii) $a = 2.5$, $b = 1.2$

Expand.

Example $(2x + 3y)^2$

$= (2x)^2 + 2(2x) \times (3y) + (3y)^2$

$= 4x^2 + 12xy + 9y^2$

Example $(5x - 4)^2$

$= (5x)^2 - 2(5x) \times (4) + 4^2$

$= 25x^2 - 40x + 16$

Example $(51)^2$

$= (50 + 1)^2$

$= 50^2 + 2 \times 50 \times 1 + 1 \times 1$

$= 2500 + 100 + 1$

$= 2601$

Example $(98)^2$

$= (100 - 2)^2$

$= 100^2 - 2 \times 100 \times 2 + 2^2$

$= 10000 - 400 + 4$

$= 9604$

Practice Set 50

1. Expand.

   (i) $(5a + 6b)^2$ (ii) $\left(\frac{a}{2}+\frac{b}{3}\right)^2$ (iii) $(2p - 3q)^2$ (iv) $\left(x-\frac{2}{x}\right)^2$

   (v) $(ax + by)^2$ (vi) $(7m - 4)^2$ (vii) $\left(x+\frac{1}{2}\right)^2$ (viii) $\left(a-\frac{1}{a}\right)^2$

2. Which of the options given below is the square of the binomial $(8 - \frac{1}{x})$?

   (i) $64 - \frac{1}{x^2}$ (ii) $64 + \frac{1}{x^2}$ (iii) $64 - \frac{16}{x} + \frac{1}{x^2}$ (iv) $64 + \frac{16}{x} + \frac{1}{x^2}$

3. Of which of the binomials given below is $m^2n^2 + 14mnpq + 49p^2q^2$ the expansion?

   (i) $(m + n)(p + q)$ (ii) $(mn - pq)$ (iii) $(7mn + pq)$ (iv) $(mn + 7pq)$

4. Use an expansion formula to find the values.

   (i) $(997)^2$ (ii) $(102)^2$ (iii) $(97)^2$ (iv) $(1005)^2$

Let's learn.

* Expansion of $(a + b)\ (a - b)$

$$
\begin{aligned}
(a + b)(a - b) &= (a + b) \times (a - b) \\
&= a(a - b) + b(a - b) \\
&= a^2 - ab + ba - b^2 \\
&= a^2 - b^2 \\
(a + b)(a - b) &= a^2 - b^2
\end{aligned}
$$

Now I know !

$$(a + b)(a - b) = a^2 - b^2$$

Example $(3x + 4y)(3x - 4y) = (3x)^2 - (4y)^2 = 9x^2 - 16y^2$

Example $102 \times 98 = (100 + 2)(100 - 2) = (100)^2 - (2)^2 = 10000 - 4 = 9996$

Practice Set 51

1. Use the formula to multiply the following.

   (i) $(x + y)(x - y)$ (ii) $(3x - 5)(3x + 5)$

   (iii) $(a + 6)(a - 6)$ (iv) $\left(\frac{x}{5}+6\right)\left(\frac{x}{5}-6\right)$

2. Use the formula to find the values.

   (i) 502×498 (ii) 97×103 (iii) 54×46 (iv) 98×102

Let's learn. Factorising Algebraic Expressions

We have learnt to factorise whole numbers. Now let us learn to factorise algebraic expressions. First, let us factorise a monomial.

$15 = 3 \times 5$, that is, 3 and 5 are factors of 15.

Similarly, $3x = 3 \times x$, Hence, 3 and x are factors of $3x$

Consider $5t^2$. $5t^2 = 5 \times t^2 = 5 \times t \times t$

$1, 5, t, t^2, 5t, 5t^2$ are all factors of $5t^2$.

$6\,ab^2 = 2 \times 3 \times a \times b \times b$

When factorising a monomial, first factorise the coefficient if possible and then factorise the part with variables.

Practice Set 52

- Factorise the following expressions and write them in the product form.

  (i) $201\,a^3 b^2$, (ii) $91\,xyt^2$, (iii) $24\,a^2 b^2$, (iv) tr^2s^3

Let's learn. Factorising a Binomial

4, x and y are factors of every term in the binomial $4xy + 8xy^2$

$\therefore\ 4xy + 8xy^2 = 4(xy + 2xy^2) = 4x\,(y + 2xy) = 4xy\,(1 + 2y)$

We can factorise a binomial by identifying the factors common to both terms and writing them outside the brackets in product form.

This is how we factorise $9a^2bc + 12abc^2 = 3(3a^2bc + 4abc^2) = 3abc\,(3a + 4c)$

$(a + b)(a - b) = a^2 - b^2$ is a formula we have already learnt.

Hence, we also get the factors $a^2 - b^2 = (a + b)(a - b)$

Factorise:

Example $a^2 - 4b^2 = a^2 - (2b)^2$
$= (a + 2b)\,(a - 2b)$

Example $3a^2 - 27b^2 = 3(a^2 - 9b^2)$
$= 3(a + 3b)\,(a - 3b)$

Practice Set 53

- Factorise the following expressions.

  (i) $p^2 - q^2$ (ii) $4x^2 - 25y^2$ (iii) $y^2 - 4$ (iv) $p^2 - \frac{1}{25}$

  (v) $9x^2 - \frac{1}{16}y^2$ (vi) $x^2 - \frac{1}{x^2}$ (vii) $a^2b - ab$ (viii) $4x^2y - 6x^2$

  (ix) $\frac{1}{2}y^2 - 8z^2$ (x) $2x^2 - 8y^2$

❑❑❑