12 Perimeter and Area

Let's recall.

Perimeter

The sum of the lengths of all sides of a closed figure is the perimeter of that figure.
Perimeter of a polygon = sum of lengths of all sides.

$\therefore$ Perimeter of a square = 4 × side
Perimeter of square of side a = $4a$

Perimeter of a rectangle = 2 length + 2 breadth
Perimeter of a rectangle with length l and breadth b is $2l + 2b$

Example The perimeter of a rectangle is 64 cm. If its length is 17 cm, what is its breadth?

Solution : Let its breadth be x cm.

2 length + 2 breadth = perimeter

2 (length + breadth) = 64

$2\,(17 + x) = 64$

$\frac{2(17+x)}{2} = \frac{64}{2}$

$17 + x = 32$

$x = 15$

The breadth of the rectangle is 15 cm.

Example The perimeter of a rectangle of length 28 cm and breadth 20 cm is equal to the perimeter of a square. What is the length of the side of that square?

Solution : Perimeter of rectangle

$= 2$ (length + breadth)

$= 2\,(28 + 20)$

$= 96$

If the side of that square is a then $4a = 96$

Perimeter of square = 96

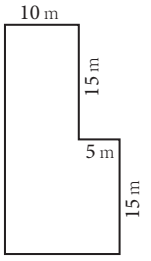
$4a = 96$

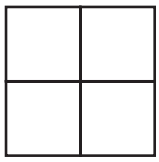
$\therefore\ a = \frac{96}{4} = 24$

Side of the square is 24 cm.

Practice Set 44

1. If the length and breadth of a rectangle are doubled, how many times the perimeter of the old rectangle will that of the new rectangle be?
2. If the side of a square is tripled, how many times the perimeter of the first square will that of the new square be?
3. Given alongside is the diagram of a playground. It shows the length of its sides. Find the perimeter of the playground.





4. As shown in the figure, four napkins all of the same size were made from a square piece of cloth of length 1 m. What length of lace will be required to trim all four sides of all the napkins ?

Let's recall.

Area

- Area of square = side × side = $(\text{side})^2$
- Area of rectangle = length × breadth = $l \times b$

Area is measured in square metres, square cm, square km, etc.

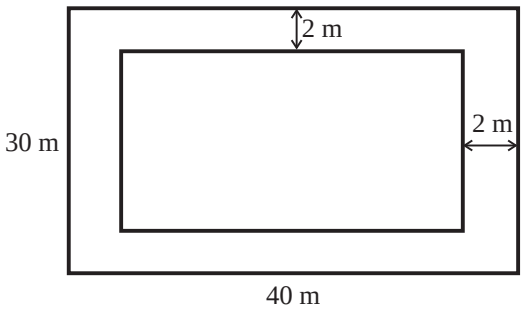
Activity I

Measure the length and breadth of the courts laid out for games such as kho-kho, kabaddi, tennis, badminton, etc. Find out their perimeters and areas.

Activity II

A wall in Aniruddha's house is to be painted. The wall is 7 m long and 5 m high. If the painter charges 120 rupees per square metre, how much will he have to be paid?

Example A rectangular garden is 40 m long and 30 m wide. A two-metre wide path is to be paved inside the garden along its boundary, using tiles 25 cm × 20 cm in size. How many such tiles will be required ?



100 cm = 1 m

25 cm = $\frac{25}{100}$ m

Let us find the area to be paved.

Area of garden = 40 × 30 = 1200 sqm

Area not to be paved = 36 × 26 = 936 sqm

∴ Area to be paved = 1200 − 936 = 264 sqm

Area of each tile = $\frac{25}{100} \times \frac{20}{100} = \frac{1}{20}$ sqm

Area of one tile is $\frac{1}{20}$ sqm.

Hence, let us find the number of tiles required to tile 264 sqm.

$$\text{Number of tiles} = \frac{\text{Total area}}{\text{Area of one paver}}$$

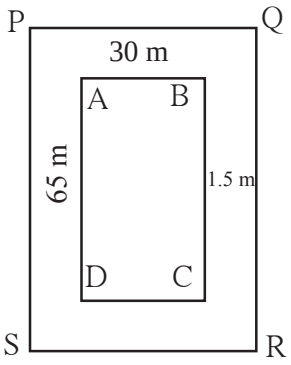
$$= 264 \div \frac{1}{20}$$

$$= 264 \times 20 = 5280$$

Therefore, 5280 tiles will be required.

Example A rectangular playground is 65 m long and 30 m wide. A pathway of 1.5 m width goes all around the ground, outside it. Find the area of the pathway.

Solution: The playground is rectangular.



□ ABCD is the playground. Around it is a pathway 1.5 m wide.

Around □ ABCD we get the rectangle □ PQRS

Length of new rectangle PQRS = 65 + 1.5 + 1.5 = 68 m

Breadth of new rectangle PQRS = 30 + 1.5 + 1.5 = 33 m

Area of path = Area of rectangle PQRS − Area of rectangle ABCD

= 68 × 33 − 65 × 30 = ☐ − ☐ = ☐ sqm.

Let's discuss.

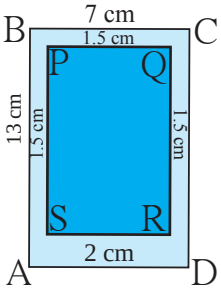
- Is there another way to find the area of the pathway in the problem above?

Example The length and the width of a mobile phone are 13 cm and 7 cm respectively. It has a screen PQRS as shown in the figure.
What is the area of the screen?

Solution: ABCD is the rectangle formed by the edges of the mobile. PQRS is the rectangle formed by leaving a 1.5 cm wide edge alongside AB, BC, and DC, and a 2 cm edge alongside DA.

Length of rectangle PQRS = ☐ cm

Breadth of rectangle PQRS = ☐ cm



Area of screen = Area of rectangle PQRS = ... × ..... = ☐ sqm

Activity

Take mobile handsets of different sizes and find the area of their screens.

Practice Set 45

1. If the side of a square is 12 cm, find its area.
2. If the length of a rectangle is 15 cm and breadth is 5 cm, find its area.
3. The area of a rectangle is 102 sqcm. If its length is 17 cm, what is its perimeter ?
4*. If the side of a square is tripled, how many times will its area be as compared to the area of the original square ?

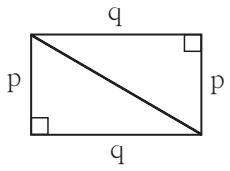
Let's learn.

Area of a Right-angled Triangle

Activity Cut out two right-angled triangles having the same measures. Join them as shown in the figure. See how they form a rectangle. The sides of length p and q that form the right angles of the triangles are also the ones that form the sides of the rectangle. From the figure we see that

Area of a rectangle = 2 × area of right-angled triangle

∴ 2 × area of right-angled triangle = p × q



$$\text{Area of right-angled triangle} = \frac{p \times q}{2}$$

Now I know!

Area of a right-angled triangle = $\frac{1}{2}$ × product of sides forming the right angle

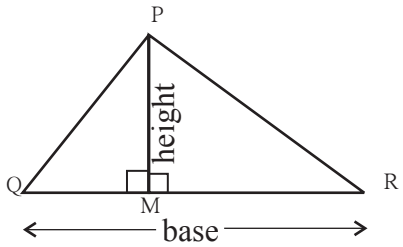
If one of the sides of a right-angled triangle forming the right angle is taken to be the base, the other becomes the height of the triangle.

Thus, the area of a right-angled triangle = $\frac{1}{2}$ base × height

If ΔABC is any triangle, then any side can be taken as the base. Then the measure of the perpendicular on the base from the apex opposite to it, is the height of the triangle.

Take any ΔPQR and take QR as the base. PM is the perpendicular from P on QR.

Figure 1 : Point M is in seg QR.



ΔPMR and ΔPMQ are right-angled triangles.

$A(\Delta PQR) = A(\Delta PMQ) + A(\Delta PMR)$

$= \frac{1}{2} \times l(QM) \times l(PM) + \frac{1}{2} \times l(MR) \times l(PM)$

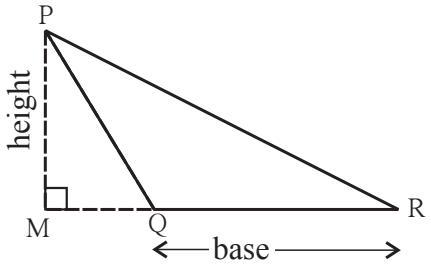
$= \frac{1}{2} [l(QM) + l(MR)] \times l(PM)$

$= \frac{1}{2} l(QR) \times l(PM)$

$= \frac{1}{2} \times \text{base} \times \text{height}$

$A(\Delta PQR) = \frac{1}{2} \times \text{base} \times \text{height}$

Figure 2 : Point M is outside seg QR.



ΔPMR and ΔPMQ are right-angled triangles.

$A(\Delta PQR) = A(\Delta PMR) - A(\Delta PMQ)$

$= \frac{1}{2} \times l(MR) \times l(PM) - \frac{1}{2} \times l(MQ) \times l(PM)$

$= \frac{1}{2} [l(MR) - l(MQ)] \times l(PM)$

$= \frac{1}{2} \times l(QR) \times l(PM)$

$= \frac{1}{2} \times \text{base} \times \text{height}$

$A(\Delta PQR) = \frac{1}{2} \times \text{base} \times \text{height}$

Now I know!

Area of triangle $= \frac{1}{2} \times$ base $\times$ height

Example If the sides that form the right angle of a triangle are 3.5 cm and 4.2 cm long, find the area of the triangle.

Solution : Area of a right-angled triangle

$= \frac{1}{2} \times$ product of sides forming right angle

$= \frac{1}{2} \times 3.5 \times 4.2$

$= 7.35$ sqcm

Example If the base of a triangle is 5.6 cm and height is 4.5 cm, what is its area?

Solution : Area of a triangle

$= \frac{1}{2} \times$ base $\times$ height

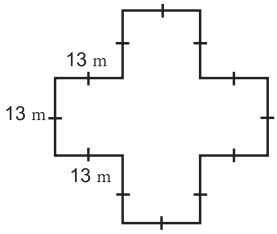
$= \frac{1}{2} \times 5.6 \times 4.5$

$= 12.6$ sqcm

(Note that sqcm is also written as cm^2).

Pratice set 46

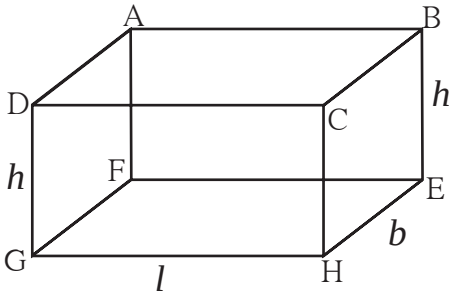
1. A page of a calendar is 45 cm long and 26 cm wide. What is its area ?
2. What is the area of a triangle with base 4.8 cm and height 3.6 cm ?
3. What is the value of a rectangular plot of land 75.5 m long and 30.5 m broad at the rate of 1000 rupees per square metre?
4. A rectangular hall is 12 m long and 6 m broad. Its flooring is to be made of square tiles of side 30 cm. How many tiles will fit in the entire hall? How many would be required if tiles of side 15 cm were used?
5. Find the perimeter and area of a garden with measures as shown in the figure alongside.



Let's learn. Surface Area

The surface area of any three-dimensional object is the sum of the areas of all its faces.

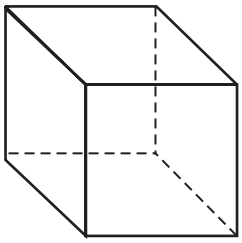
✸ Surface Area of a Cuboid



- A cuboid has six faces.
- Each face is a rectangle.
- Opposite faces have the same area.
- Each edge is perpendicular to the two other edges it meets.
- Let l be the length of the horizontal face of the cuboid and b be the breadth. Let h be the height of its vertical sides.

Area of rectangle ABCD = Area of rectangle GHEF = $l \times b$
Area of rectangle ADGF = Area of rectangle BCHE = $b \times h$
Area of rectangle CHGD = Area of rectangle ABEF = $l \times h$
Total surface area of cuboid = Sum of area of all rectangles
Total surface area of cuboid = 2 (length × breadth + breadth × height + length × height)
$= 2 (l \times b + b \times h + l \times h) = 2 (lb + bh + lh)$

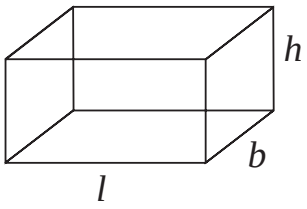
✸ Surface Area of a Cube



- A cube has 6 faces.
- Each face is a square.
- Area of all faces is equal.
- Let the side of each square be l.
- Area of one face of the cube = Area of square
- Total surface area of the cube = Sum of areas of 6 squares.
  $= 6 \times \text{side}^2$
  $= 6 \times l^2$

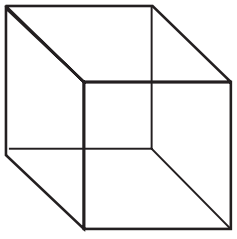
Example How much sheet metal is required to make a closed rectangular box of length 1.5 m, breadth 1.2 m and height 1.3 m ?

Solution : length of box = l = 1.5 m, breadth = b = 1.2 m, height = h = 1.3 m.
Surface area of box = $2 (l \times b + b \times h + l \times h)$
$= 2 (1.5 \times 1.2 + 1.2 \times 1.3 + 1.5 \times 1.3)$
$= 2 (1.80 + 1.56 + 1.95)$
$= 2 (5.31)$
$= 10.62$ sqm



10.62 sqm of sheet metal will be needed to make the box.

Example One side of a cubic box is 0.4 m. How much will it cost to paint the outer surface of the box at the rate of 50 rupees per sqm?



Solution : side = l = 0.4 m.
Total surface area of cube = $6 \times (l)^2$
$= 6 \times (0.4)^2$
$= 6 \times 0.16 = 0.96$ sqm
Cost of painting 1 sqm is 50 rupees.
∴ Cost of painting 0.96 sqm will be = 0.96×50
= 48 rupees
It will cost 48 rupees to paint the outer surface of the box.

Practice Set 47

1. Find the total surface area of cubes having the following sides.
   (i) 3 cm (ii) 5 cm (iii) 7.2 m (iv) 6.8 m (v) 5.5 m

2. Find the total surface area of the cuboids of length, breadth and height as given below:
   (i) 12 cm, 10 cm, 5 cm (ii) 5 cm, 3.5 cm, 1.4 cm
   (iii) 2.5 cm, 2 m, 2.4 m (iv) 8 m, 5 m, 3.5 m

3. A matchbox is 4 cm long, 2.5 cm broad and 1.5 cm in height. Its outer sides are to be covered exactly with craft paper. How much paper will be required to do so ?

4. An open box of length 1.5 m, breadth 1 m, and height 1 m is to be made for use on a trolley for carrying garden waste. How much sheet metal will be required to make this box? The inside and outside surface of the box is to be painted with rust proof paint. At a rate of 150 rupees per sqm, how much will it cost to paint the box?

Maths is fun!

There are some three-digit numbers which can be divided by the product of their digits without leaving a remainder.

Example (i) Take the number 175, $1 \times 7 \times 5 = 35$, $\frac{175}{35} = 5$

(ii) Take the number 816, $8 \times 1 \times 6 = 48$, $\frac{816}{48} = 17$

(iii) Take the number 612, $6 \times 1 \times 2 = 12$, $\frac{612}{12} = 51$

The numbers 135, 312, 672 are some more numbers like these.

Can you find other such numbers?