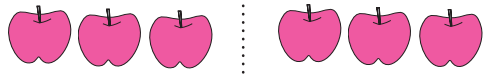
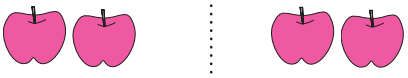
4 Operations on Fractions

Let's recall.

Let's divide the apples equally between two children.

Apples	Children		
6	2		$6 \div 2 = 3$
4	2		$4 \div 2 = 2$
1	2		$1 \div 2 = \frac{1}{2}$
7	2		$7 \div 2 = \frac{7}{2}$

Let's learn.

Conversion of an Improper Fraction into a Mixed Number

Example : If 7 apples are divided equally between 2 people, how many will each one get?

$\frac{7}{2} = 7 \div 2$

Divisor $2\overline{)7}$ Quotient 3, Dividend 7, $-\ 6$, Remainder 1

$\frac{7}{2} = 3\frac{1}{2}$

Each will get 3 full apples and $\frac{1}{2}$ apple.

Take care!

While dividing, we take care to see that the remainder is smaller than the divisor. As a result, in the mixed number, the numerator of the fractional part is smaller than its denominator.

Let's learn.

Conversion of a Mixed Number into an Improper Fraction

Example : $3\frac{2}{5}$ is a mixed number. Convert this into an improper fraction.

$$3\frac{2}{5} = 3 + \frac{2}{5} = \frac{3}{1} + \frac{2}{5} = \frac{3\times5}{1\times5} + \frac{2}{5} = \frac{3\times5+2}{5} = \frac{15+2}{5} = \frac{17}{5}$$

Practice Set 9

1. Convert into improper fractions.
   (i) $7\frac{2}{5}$ (ii) $5\frac{1}{6}$ (iii) $4\frac{3}{4}$ (iv) $2\frac{5}{9}$ (v) $1\frac{5}{7}$
2. Convert into mixed numbers.
   (i) $\frac{30}{7}$ (ii) $\frac{7}{4}$ (iii) $\frac{15}{12}$ (iv) $\frac{11}{8}$ (v) $\frac{21}{4}$ (vi) $\frac{20}{7}$
3. Write the following examples using fractions.
   (i) If 9 kg rice is shared amongst 5 people, how many kilograms of rice does each person get?
   (ii) To make 5 shirts of the same size, 11 metres of cloth is needed. How much cloth is needed for one shirt?

Let's learn.

Addition and Subtraction of Mixed Numbers

Example 1. Add. $5\frac{1}{2} + 2\frac{3}{4}$

Method I

$$5\frac{1}{2} + 2\frac{3}{4} = 5 + 2 + \frac{1}{2} + \frac{3}{4}$$

$$= 7 + \frac{1\times2}{2\times2} + \frac{3}{4}$$

$$= 7 + \frac{2}{4} + \frac{3}{4}$$

$$= 7 + \frac{2+3}{4} = 7 + \frac{5}{4}$$

$$= 7 + 1 + \frac{1}{4} = 8\frac{1}{4}$$

Method II

$$5\frac{1}{2} + 2\frac{3}{4} = \frac{5\times2+1}{2} + \frac{2\times4+3}{4}$$

$$= \frac{11}{2} + \frac{11}{4}$$

$$= \frac{11\times2}{2\times2} + \frac{11}{4}$$

$$= \frac{22}{4} + \frac{11}{4} = \frac{33}{4}$$

$$= 8\frac{1}{4}$$

Example 2. Subtract. $3\frac{2}{5} - 2\frac{1}{7}$

Method I

$$3\frac{2}{5} - 2\frac{1}{7} = (3-2) + \left(\frac{2}{5} - \frac{1}{7}\right)$$

$$= 1 + \frac{2\times7}{5\times7} - \frac{1\times5}{7\times5}$$

$$= 1 + \frac{14}{35} - \frac{5}{35}$$

$$= 1 + \frac{9}{35} = 1\frac{9}{35}$$

Method II

$$3\frac{2}{5} - 2\frac{1}{7} = \frac{17}{5} - \frac{15}{7}$$

$$= \frac{17\times7}{5\times7} - \frac{15\times5}{7\times5}$$

$$= \frac{119}{35} - \frac{75}{35} = \frac{119-75}{35}$$

$$= \frac{44}{35} = 1\frac{9}{35}$$

Think about it.

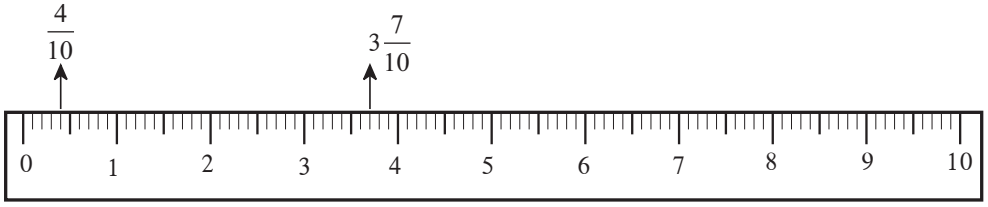
How to do this subtraction : $4\frac{1}{4} - 2\frac{1}{2}$? Is it the same as $[4 - 2 + \frac{1}{4} - \frac{1}{2}]$?

Practice Set 10

1. Add.
   (i) $6\frac{1}{3} + 2\frac{1}{3}$ (ii) $1\frac{1}{4} + 3\frac{1}{2}$ (iii) $5\frac{1}{5} + 2\frac{1}{7}$ (iv) $3\frac{1}{5} + 2\frac{1}{3}$
2. Subtract.
   (i) $3\frac{1}{3} - 1\frac{1}{4}$ (ii) $5\frac{1}{2} - 3\frac{1}{3}$ (iii) $7\frac{1}{8} - 6\frac{1}{10}$ (iv) $7\frac{1}{2} - 3\frac{1}{5}$
3. Solve.
   (1) Suyash bought $2\frac{1}{2}$ kg of sugar and Ashish bought $3\frac{1}{2}$ kg. How much sugar did they buy altogether? If sugar costs 32 rupees per kg, how much did they spend on the sugar they bought?
   (2) Aradhana grows potatoes in $\frac{2}{5}$ part of her garden, greens in $\frac{1}{3}$ part and brinjals in the remaining part. On how much of her plot did she plant brinjals?
   (3)* Sandeep filled water in $\frac{4}{7}$ of an empty tank. After that, Ramakant filled $\frac{1}{4}$ part more of the same tank. Then Umesh used $\frac{3}{14}$ part of the tank to water the garden. If the tank has a maximum capacity of 560 litres, how many litres of water will be left in the tank?

Let's learn.

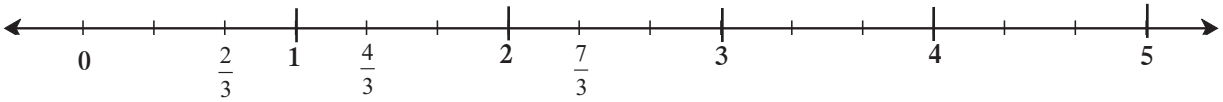
Showing Fractions on the Number Line



It is easy to mark the fractions $\frac{4}{10}$ and $3\frac{7}{10}$ on the number line because on the scale, every centimetre is divided into 10 equal parts. In the first unit, the fourth mark from zero shows the fraction $\frac{4}{10}$. The 7^{th} mark of the 10 equal parts after 3, between the numbers 3 and 4, shows the fraction $3\frac{7}{10}$.

Example : Let us show the fractions $\frac{2}{3}$, $\frac{4}{3}$, $\frac{7}{3}$ on the number line.

On the number line below, every unit is divided into 3 equal parts.



Now I know –

If a fraction has to be shown on a number line, every unit on the number line must be divided into as many equal parts as the denominator of the fraction.

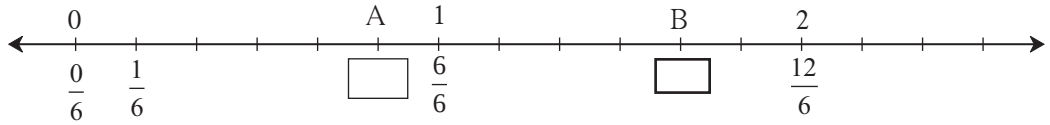
Think about it.

If we want to show the fractions $\frac{3}{10}$, $\frac{9}{20}$, $\frac{19}{40}$ on the number line, how big should the unit be?

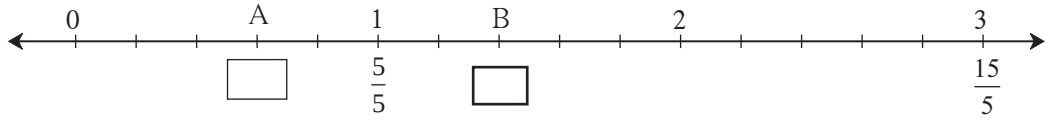
Practice Set 11

1. What fractions do the points A and B show on the number lines below?

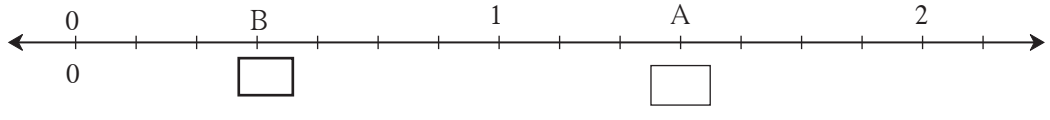
(1)



(2)



(3)



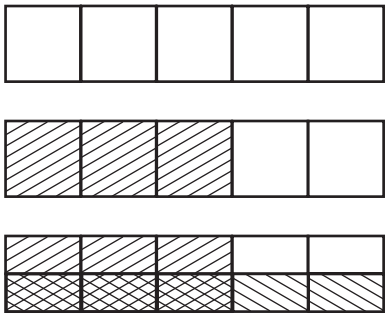
2. Show the following fractions on the number line.

(1) $\frac{3}{5}$, $\frac{6}{5}$, $2\frac{3}{5}$ (2) $\frac{3}{4}$, $\frac{5}{4}$, $2\frac{1}{4}$

Let's learn.

Multiplication of Fractions

See how the multiplication $\frac{3}{5} \times \frac{1}{2}$ is done with the help of the rectangular strip.



- Draw vertical lines to divide a rectangular strip into 5 equal parts.
- Shade the part that shows the fraction $\frac{3}{5}$.
- We have to show $\frac{1}{2}$ of $\frac{3}{5}$. So, draw a horizontal line to divide the strip into two equal parts.
- Shade one of the two horizontal parts in a different way.

When we divided the strip into 2 equal parts, we also divided the $\frac{3}{5}$ part into 2 equal parts. To take one of those parts, consider the parts shaded twice.

We have 10 equal boxes. Of these, 3 boxes have been shaded twice.

These boxes, i.e., the part shaded twice can be written as the fraction $\frac{3}{10}$.

$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$.

We can carry out the above multiplication like this : $\frac{3}{5} \times \frac{1}{2} = \frac{3 \times 1}{5 \times 2} = \frac{3}{10}$

Now I know –

When multiplying two fractions, the product of the numerators is written in the numerator and that of the denominators, in the denominator.

Example : Sulochanabai owns 42 acres of farm land. If she planted wheat on $\frac{2}{7}$ of the land, on how many acres has she planted wheat?

We must find out $\frac{2}{7}$ of 42 acres $\therefore \frac{42}{1} \times \frac{2}{7} = \frac{42 \times 2}{1 \times 7} = \frac{6 \times 7 \times 2}{7} = 12$

Sulochanabai has planted wheat on 12 acres of land.

Practice Set 12

1. Multiply.

(i) $\frac{7}{5} \times \frac{1}{4}$ (ii) $\frac{6}{7} \times \frac{2}{5}$ (iii) $\frac{5}{9} \times \frac{4}{9}$ (iv) $\frac{4}{11} \times \frac{2}{7}$

(v) $\frac{1}{5} \times \frac{7}{2}$ (vi) $\frac{9}{7} \times \frac{7}{8}$ (vii) $\frac{5}{6} \times \frac{6}{5}$ (viii) $\frac{6}{17} \times \frac{3}{2}$

2. Ashokrao planted bananas on $\frac{2}{7}$ of his field of 21 acres. What is the area of the banana plantation?

3*. Of the total number of soldiers in our army, $\frac{4}{9}$ are posted on the northern border and one-third of them on the north-eastern border. If the number of soldiers in the north is 540000, how many are posted in the north-east?

Let's learn.

Reciprocals or Multiplicative Inverses

Look at these multiplications.

(1) $\frac{5}{6} \times \frac{6}{5} = \frac{30}{30} = 1$ (2) $4 \times \frac{1}{4} = \frac{4}{1} \times \frac{1}{4} = \frac{4}{4} = 1$

(3) $\frac{3}{2} \times \frac{2}{3} = \frac{6}{6} = 1$ (4) $\frac{71}{3} \times \frac{3}{71} = 1$

What is the peculiarity you see in all of them?

A fraction is multiplied by another fraction obtained by exchanging the numerator and denominator of the first fraction. Their product is 1. Each fraction of such a pair is called the reciprocal or multiplicative inverse of the other.

Example : The multiplicative inverse or reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

The multiplicative inverse of 4, that is, of $\frac{4}{1}$ is $\frac{1}{4}$.

Now I know –

When the product of two numbers is 1, each of the numbers is the multiplicative inverse or reciprocal of the other.

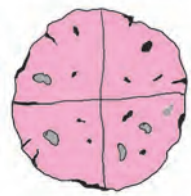
Think about it.

(1) What is the reciprocal of 1? (2) Would 0 have a reciprocal?

Let's learn.

Division of Fractions

Example : Here is one bhakari. If each one is to be given a quarter of it, how many will get a share?



A quarter means $\frac{1}{4}$.

As we can see in the picture, we can get 4 quarters from one bhakari, so it will be enough for four people.

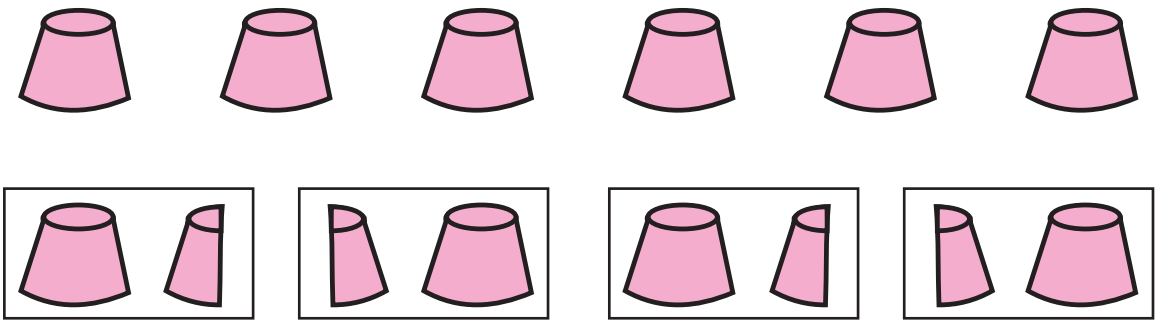
We can write this as $4 \times \frac{1}{4} = 1$.

Now, we shall convert the division of a fraction into a multiplication.

$$1 \div \frac{1}{4} = 4 = 1 \times \frac{4}{1}$$

Example : There are 6 blocks of jaggery, each of one kilogram. If one family requires one and a half kg jaggery every month, for how many families will these blocks suffice?

One and a half is $1 + \frac{1}{2} = \frac{3}{2}$



Let us divide to see how many families can share the jaggery.

$6 \div \frac{3}{2} = \frac{6}{1} \div \frac{3}{2} = \frac{6}{1} \times \frac{2}{3} = 4$ Therefore, 6 blocks will suffice for 4 families.

Example : $12 \div 4 = \frac{12}{1} \times \frac{1}{4} = \frac{12}{4} = 3$

Example : $\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2} = \frac{5 \times 3}{7 \times 2} = \frac{15}{14} = 1\frac{1}{14}$

Now I know –

To divide a number by a fraction is to multiply it by the reciprocal of the fraction.

Practice Set 13

1. Write the reciprocals of the following numbers.

   (i) 7 (ii) $\frac{11}{3}$ (iii) $\frac{5}{13}$ (iv) 2 (v) $\frac{6}{7}$

2. Carry out the following divisions.

   (i) $\frac{2}{3} \div \frac{1}{4}$ (ii) $\frac{5}{9} \div \frac{3}{2}$ (iii) $\frac{3}{7} \div \frac{5}{11}$ (iv) $\frac{11}{12} \div \frac{4}{7}$

3*. There were 420 students participating in the Swachh Bharat campaign. They cleaned $\frac{42}{75}$ part of the town, Sevagram. What part of Sevagram did each student clean if the work was equally shared by all?

Ramanujan's Magic Square

22	12	18	87
88	17	9	25
10	24	89	16
19	86	23	11

- Add the four numbers in the rows, the columns and along the diagonals of this square.
- What is the sum?
- Is it the same every time?
- What is the peculiarity?
- Look at the numbers in the first row.
  22 - 12 - 1887
  Find out why this date is special.

Obtain and read a biography of the great Indian mathematician Srinivasa Ramanujan.