8. Electrostatics

Can you recall?

1. What are conservative forces?
2. What is potential energy ?
3. What is Gauss' law and what is a Gaussian surface?

8.1 Introduction:

In XI[th] Std we have studied the Gauss' Law which gives the relationship between the electric charge and its electric field. It also provides equivalent methods for finding electric field intensity by relating values of the field at a closed Gaussian surface and the total charge enclosed by it. It is a powerful tool which can be applied for the calculation of the electric field when it originates from charge distribution of sufficient symmetry. The Gauss' law is written as

$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0} \qquad \text{--- (8.1)}$$

where ϕ is the total flux coming out of a closed surface and q is the total charge inside the closed surface.

Common steps involved in calculating electric field intensity by using Gauss' law:

1. Identify the charge distribution as linear/cylindrical/spherical charge density.
2. Visualize a Gaussian surface justifying its symmetry for the given charge distribution.
3. Obtain the flux by Gauss' law (Let this be Eq. (A))
4. With the electric field intensity E as unknown, obtain electric flux by calculation, using geometry of the structure and symmetry of the Gaussian surface (Let this be Eq. (B))
5. Equate RHS of Eq. (A) and Eq. (B) and calculate E.

8.2 Application of Gauss' Law:

In this section we shall see how to obtain the electric field intensity for some symmetric charge configurations with the help of some examples.

8.2.1 Electric Field Intensity due to Uniformly Charged Spherical Shell or Hollow Sphere:

Consider a sphere of radius R with its centre at O, charged to a uniform surface charge density σ (C/m^2) placed in a dielectric medium of permittivity ε $(\varepsilon = \varepsilon_0 k)$. The total charge on the sphere, $q = \sigma \times 4\pi R^2$

By Gauss' theorem, the net flux through a closed Gaussian surface

$\phi = q/\varepsilon_0$ (for air/vacuum k=1)

where q is the total charge inside the closed surface.

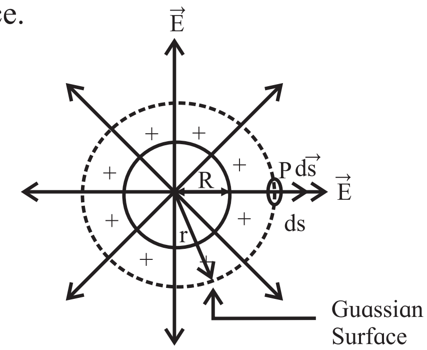


Fig. 8.1: Uniformly charged spherical shell or hollow sphere.

To find the electric field intensity at a point P, at a distance r from the centre of the charged sphere, imagine a concentric Gaussian sphere of radius r passing through P. Let ds be a small area around the point P on the Gaussian surface. Due to symmetry and spheres being concentric, the electric field at each point on the Gaussian surface has the same magnitude E and it is directed radially outward. Also, the angle between the direction of E and the normal to the surface of the sphere (ds) is zero i.e., cos θ = 1

$\therefore \ \vec{E} \cdot \vec{ds} = E \, ds \cos\theta = E \, ds$

$\therefore$ flux $d\phi$ through the area $ds = E\, ds$

Total electric flux through the Gaussian surface $\phi = \oint \vec{E} \cdot \vec{ds} = \oint E ds = E \oint ds$

$$\therefore \ \phi = E \, 4\pi r^2 \qquad \text{--- (8.2)}$$

From equations (8.1) and (8.2),

$q/\varepsilon_0 = E\, 4\pi r^2$

$\therefore E = q/4\pi\varepsilon_0 r^2$ --- (8.3)

Since $q = \sigma \times 4\pi R^2$

We have $E = \sigma \times 4\pi R^2 / 4\pi\varepsilon_0 r^2$

$\therefore E = \sigma R^2 / \varepsilon_0 r^2$ --- (8.4)

From Eqn. (8.3) it can be seen that, the electric field at a point outside the shell is the same as that due to a point charge. Thus it can be concluded that a uniformly charged sphere is equivalent to a point charge at its center.

Case (i) If point P lies on the surface of the charged sphere: $r = R$

$\therefore E = q/4\pi\varepsilon_0 R^2 = \sigma/\varepsilon_0$

Case (ii) If point P lies inside the sphere: Since there are no charges inside $\sigma = 0$,

$\therefore E = 0.$

Example : 8.1

A sphere of radius 10 cm carries a charge of 1μC. Calculate the electric field

(i) at a distance of 30 cm from the center of the sphere

(ii) at the surface of the sphere and

(iii) at a distance of 5 cm from the center of the sphere.

Solution: Given: $q = 1\mu C = 1 \times 10^{-6}$ C

(i) Electric intensity at a distance r is $E = q/4\pi\varepsilon_0 r^2$

For r = 30 cm = 0.3 m

$$E = \frac{9\times10^9 \times 1\times10^{-6}}{(0.3)^2} = 10^5 \text{ N/C}$$

(ii) E on the surface of the sphere, R =10 cm = 0.10m

$E = q/4\pi\varepsilon_0 R^2$

$$= \frac{9\times10^9 \times 1\times10^{-6}}{(0.10)^2} = 9 \times 10^5 \text{ N/C}$$

(iii) E at a point 5 cm away from the centre i.e. inside the sphere $E = 0$.

8.2.2 Electric Field Intensity due to an Infinitely Long Straight Charged Wire:

Consider a uniformly charged wire of infinite length having a constant linear charge density λ (charge per unit length), kept in a medium of permittivity ε ($\varepsilon = \varepsilon_0 k$).

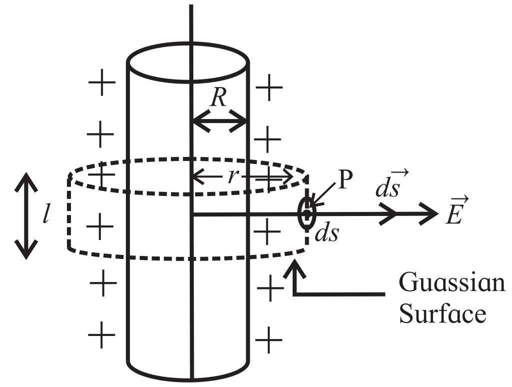


Fig. 8.2: Infinitely long straight charged wire (cylinder).

To find the electric field intensity at P ,at a distance r from the axis of the charged wire, imagine a coaxial Gaussian cylinder of length l and radius r (closed at each end by plane caps normal to the axis) passing through the point P. Consider a very small area ds at the point P on the Gaussian surface.

By symmetry, the magnitude of the electric field will be the same at all the points on the curved surface of the cylinder and will be directed radially outward. The angle between the direction of $\vec{E}$ and the normal to the curved or flat surface of the cylinder ($\vec{ds}$) is zero or ($\pi/2$) i.e., $\cos\theta = 1$ or $\cos(\pi/2) = 0$.

$\therefore \vec{E}.\vec{ds} = Eds\cos\theta = Eds$

Flux $d\phi$ through the area $ds = E\, ds$.

Total electric flux through the Gaussian cylindrical surface

$\phi = \oint \vec{E}\cdot\vec{ds} = \oint Eds = E\oint ds$

$\therefore \phi = E.\, 2\pi rl$ --- (8.5)

From equations (8.1) and (8.5)

$q/\varepsilon_0 = E\, 2\pi rl$

Since $\lambda = q/l$, $q = \lambda l$

$\therefore \lambda l/\varepsilon_0 = E\, 2\pi rl$

$E = \lambda / 2\pi\varepsilon_0 r$ --- (8.6)

The direction of the electric field E is directed outward if λ is positive and inward if is λ negative (Fig 8.3).

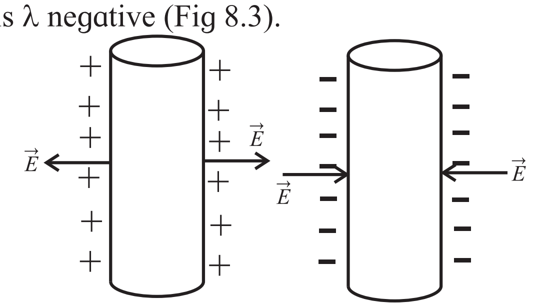


Fig. 8.3: Direction of the field for two types of charges.

Example 8.2: The length of a straight thin wire is 2 m. It is uniformly charged with a positive charge of 3μC. Calculate
(i) the charge density of the wire
(ii) the electric intensity due to the wire at a point 1.5 m away from the center of the wire

Solution: Given
charge $q = 3\ \mu C = 3 \times 10^{-6}$ C
Length $l = 2$ m, $r = 1.5$ m

(i) Charge Density λ = Charge/ length

$$= \frac{3 \times 10^{-6}}{2} = 1.5 \times 10^{-6}\ C\ m^{-1}$$

(ii) Electric Intensity $E = \lambda / 2\pi\varepsilon_0 r$

$$= \frac{1.5 \times 10^{-6}}{2 \times 3.142 \times 8.85 \times 10^{-12} \times 1.5}$$

$= 1.798 \times 10^{4}\ N\ C^{-1}$

8.2.3 Electric Field due to a Charged Infinite Plane Sheet:

Consider a uniformly charged infinite plane sheet with surface charge density σ. By symmetry electric field is perpendicular to plane sheet and directed outwards ,having same magnitude at a given distance on either sides of the sheet. Let P be a point at a distance r from the sheet and E be the electric field at P.

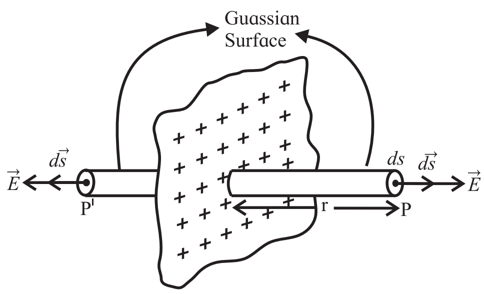


Fig. 8.4: Charged infinite plane sheet.

To find the electric field due to a charged infinite plane sheet at *P*, we consider a Gaussian surface around *P* in the form of a cylinder having cross sectional area *A* and length 2*r* with its axis perpendicular to the plane sheet. The plane sheet passes through the middle of the length of the cylinder such that the ends of the cylinder (called end caps P and P') are equidistant (at a distance *r*) from the plane sheet.

By symmetry the electric field is at right angles to the end caps and away from the plane. Its magnitude is the same at P and P'. The flux passing through the curved surface is zero as the electric field is tangential to this surface.

∴ the total flux through the closed Gaussian surface is given by

$$\phi = \left[\oint E ds\right]_{P} + \left[\oint E ds\right]_{P'} + \left[\oint E ds\right]_{\text{curved surface}}$$

(since $\theta = 0$, $\cos\theta = 1$)

$= EA + EA$

$\therefore \phi = 2EA$ --- (8.7)

If σ is the surface charge density then

$\sigma = q/A,\ q = \sigma A$

∴ Eq. (8.1) can be written as

$\phi = \sigma A/\varepsilon_0$ --- (8.8)

From Eq. (8.7) and Eq. (8.8)

$2EA = \sigma A/\varepsilon_0 \quad \therefore E = \sigma/2\varepsilon_0$

Example: 8.3 The charge per unit area of a large flat sheet of charge is 3μC/m². Calculate the electric field intensity at a point just near the surface of the sheet, measured from its midpoint.

Solution: Given
Surface Charge Density = $\sigma = 3 \times 10^{-6}\ Cm^{-2}$
Electric Intensity $E = \sigma/2\varepsilon_0$

$$= \frac{3 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} = 1.695 \times 10^{5}\ N\ C^{-1}$$

Can you recall?
What is gravitational Potential ?

8.3 Electric Potential and Potential Energy:

We have studied earlier that the potential energy of a system is the stored energy that depends upon the relative positions of its constituents. Electrostatic potential energy is the work done against the electrostatic forces to achieve a certain configuration of charges in a given system. Since every system tries to attain the lowest potential energy, work is always required to be done to change the configuration.

We know that like charges repel and unlike charges attract each other. A charge exerts a force on any other charge in its vicinity. Some work is always done to move a charge in the presence of another charge. Thus, any collection of charges possesses potential energy. Consider a positive charge Q fixed at some point in space. For bringing any other positive charge close to it, work is necessary. This work is equal to the change in the potential energy of the system of two charges.

Thus, work done against a electrostatic force = Increase in the potential energy of the system.

$$\therefore \vec{F} \cdot d\vec{r} = dU,$$

where dU is the increase in potential energy when the charge is displaced through $d\vec{r}$ and $\vec{F}$ is the force exerted on the charge.

Expression for potential energy:

Let us consider the electrostatic field due to a source charge +Q placed at the origin O. Let a small charge + q_0 be brought from point A to point B at respective distances r_1 and r_2 from O, against the repulsive forces on it.

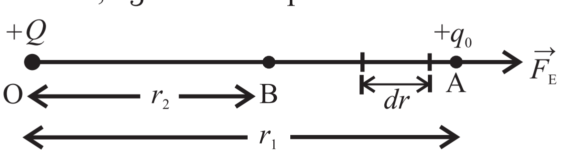


Fig. 8.5: Change $+q_0$ displaced by dr towards charge $+Q$.

Work done against the electrostatic force $\vec{F}_E$, in displacing the charge q_0 through a small displacement $d\vec{r}$ appears as an increase in the potential energy of the system.

$$dU = \vec{F}_E \cdot d\vec{r} = -F_E.dr$$

Negative sign appears because the displacement $d\vec{r}$ is against the electrostatic force $\vec{F}_E$.

For the displacement of the charge from the initial position A to the final position B, the change in potential energy ΔU, can be obtained by integrating dU

$$\therefore \Delta U = \int_A^B dU = \int_A^B -\vec{F}_E . d\vec{r}$$

The electrostatic force (Coulomb force) between the two charges separated by distance r is

$$\vec{F}_E = -\left(\frac{1}{4\pi\epsilon_0}\right)\frac{Qq_0}{r^2}\hat{r}$$

where $\hat{r}$ is the unit vector in the direction of $\vec{r}$. Negative sign shows $\vec{r}$ and $\vec{F}_E$ are oppositely directed.

$\therefore$ For a system of two point charges,

$$\Delta U = \int_A^B dU = \int_{r_1}^{r_2} -\left(\frac{1}{4\pi\epsilon_0}\right)\frac{Qq_0}{r^2}\hat{r}.d\vec{r}$$

$$\therefore \Delta U = -\left(\frac{1}{4\pi\epsilon_0}\right)Qq_0\left(\frac{-1}{r}\right)_{r_1}^{r_2}$$

$$= \left(\frac{1}{4\pi\epsilon_0}\right)Qq_0\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

The change in the potential energy depends only upon the end points and is independent of the path taken by the charge. The change in potential energy is equal to the work done W_{AB} against the electrostatic force.

$$W_{AB} = \Delta U = \left(\frac{1}{4\pi\epsilon_0}\right)Qq_0\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

So far we have defined/calculated the change in the potential energy for system of charges. It is convenient to choose infinity to be the point of zero potential energy as the electrostatic force is zero at $r = \infty$.

Thus, the potential energy U of the system of two point charges q_1 and q_2 separated by r can be obtained from the above equation by using $r_1 = \infty$ and $r_2 = r$. It is then given by

$$U(r) = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{r}\right) \quad \text{--- (8.9)}$$

Units of potential energy :

SI unit= joule (J)

"One joule is the energy stored in moving a charge of 1C through a potential difference of 1 volt. Another convenient unit of energy is electron volt (eV), which is the change in the kinetic energy of an electron while crossing two points maintained at a potential difference of 1 volt."

1 eV = 1.6×10^{-19} joule

Other related units are:

1 meV = 1.6×10^{-22} J

1 kev = 1.6×10^{-16} J

Concept of Potential:

Equation (8.9) gives the potential energy of a two particles system at a distance r from each other.

$$U(r) = \left(\frac{1}{4\pi\epsilon_0 r}\right)\left(\frac{q_1 q_2}{r}\right)$$

$$= \left(\frac{q_1}{4\pi\epsilon_0 r}\right)q_2 = \left(\frac{q_2}{4\pi\epsilon_0 r}\right)q_1$$

The quantity $V(r) \equiv \left(\frac{q}{4\pi\epsilon_0 r}\right)$ depends upon the charge q and location of a point at a distance r from it. This is defined as the electrostatic potential of the charge q at a distance r from it. In terms of potential, we can write the potential energy of the system of two charges as $U(r) = V_1(r)q_2 = V_2(r)q_1$,

where $V_1(r)$ and $V_2(r)$ are the respective potentials of charges q_1 and q_2 at distance r from either.

$\therefore$ Electrostatic potential energy (U) = electric potential $V \times$ charge q

Or, Electrostatic Potential (V) = Electrostatic Potential Energy per unit charge.

i.e., $V = U/q$

Electrostatic potential difference between any two points in an electric field can be written as

$$V_2 - V_1 = \frac{U_2 - U_1}{q} = \frac{dW}{q} = \text{work done } dW$$

(or change in PE) per unit charge to move the charge from point 2 to point 1.

Relation between electric field and electric potential:

Consider the electric field produced by a charge $+q$ kept at point O (see Fig. 8.6). Let us calculate the work done to move a unit positive charge from point M to point N which is at a small distance dx from M. The direction of the electric field at M is along $\overrightarrow{OM}$. Thus the force acting on the unit positive charge is along $\overrightarrow{OM}$. The work done = $dW = -Fdx = -Edx$. The negative sign indicates that we are moving the charge against the force acting on it. As it is the work done on a unit positive charge, dw = dV = difference in potential between M and N.

$$\therefore dV = -Edx$$

$$E = -\frac{dV}{dx}$$

Thus the electric field at a point is the negative gradient of the potential at that point.

Zero potential:

The nature of potential is such that its zero point is arbitrary. This does not mean that the choice of zero point is insignificant. Once the zero point of the potential is set, then every potential is measured with respect to that reference. The zero potential is set conveniently.

In case of a point charge or localised collection of charges, the zero point is set at infinity. For electrical circuits the earth is usually taken to be at zero potential.

Thus the potential at a point A in an electric field is the amount of work done to bring a unit positive charge from infinity to point A.

Example 8.4: Potential at a point A in space is given as 4×10^5 V.

(i) Find the work done in bringing a charge of 3 μC from infinity to the point A.

(ii) Does the answer depend on the path along which the charge is brought ?

Solution : Given

Potential (V) at the point A = 4×10^5 V

Charge $q_0 = 3$ μC $= 3 \times 10^{-6}$ C

(i) Work done in bringing the charge from infinity to the point A is

$W_\infty = q_0 V$

$= 3 \times 10^{-6} \times 4 \times 10^5$

$= 12 \times 10^{-1}$

$W_\infty = 1.2$ J

(ii) No, the work done is independent of the path.

Example 8.5 If 120 μ J of work is done in carrying a charge of 6 μ C from a place where the potential is 10 volt to another place where the potential is V, find V

Solution: Given : $W_{AB} = 120\ \mu J$,

$q_0 = 6\ \mu C$, $V_A = 10\ V$, $V_B = V$

As $V_B - V_A = \frac{W_{AB}}{q_0}$

$V - (10) = \frac{120\times10^{-6} J}{6\times10^{-6} C}$

$V - (10) = 20$

$\therefore V = 30$ volt

8.4 Electric Potential due to a Point Charge, a Dipole and a System of Charges:

a) Electric potential due to a point charge:

Here, we shall derive an expression for the electrostatic potential due to a point charge.

Figure 8.6 shows a point charge $+q$, located at point O. We need to determine its potential at a point A, at a distance r from it.

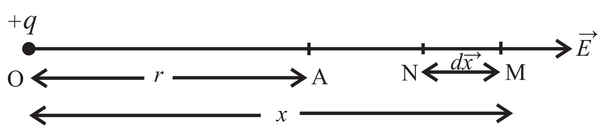


Fig. 8.6: Electric potential due to a point charge.

As seen above the electric potential at a point A is the amount of work done per unit positive charge, which is displaced from ∞ to point A. As the work done is independent of the path, we choose a convenient path along the line extending OA to ∞.

Let M be an intermediate point on this path where OM = x. The electrostatic force on a unit positive charge at M is of magnitude

$$F = \frac{1}{4\pi\epsilon_0}\times\frac{q}{x^2} \qquad \text{--- (8.10)}$$

It is directed away from O, along OM. For infinitesimal displacement dx from M to N, the amount of work done is given by

$$\therefore dW = -Fdx \qquad \text{--- (8.11)}$$

The negative sign appears as the displacement is directed opposite to that of the force.

∴ Total work done in displacing the unit positive charge from ∞ to point A is given by

$$W = \int_{\infty}^{r} -Fdx = \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\,dx$$

$$= \frac{-q}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}\,dx$$

$$= \frac{-q}{4\pi\varepsilon_0}\left[\frac{-1}{x}\right]_{\infty}^{r} \qquad \left(\because \int x^{-2}\,dx = \frac{-1}{x}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{\infty}\right] \qquad \left(\because \frac{1}{\infty}=0\right)$$

$$W = \frac{q}{4\pi\varepsilon_0 r} \qquad \text{--- (8.12)}$$

By definition this is the electrostatic potential at A due to charge q.

$$\therefore V = W = \frac{q}{4\pi\varepsilon_0 r} \qquad \text{--- (8.13)}$$

A positively charged particle produces a positive electric potential and a negatively charged particle produces a negative electric potential

At $r = \infty$, $V = \frac{q}{\infty} = 0$

This shows that the electrostatics potential is zero at infinity.

Equation (8.13) shows that for any point at a distance r from the point charge q, the value of V is the same and is independent of the direction of r. Hence electrostatic potential due to a single charge is spherically symmetric.

Figure 8.7 shows how electric potential ($V \propto \frac{1}{r}$) and electric field ($E \propto \frac{1}{r^2}$) vary with r, the distance from the charge.

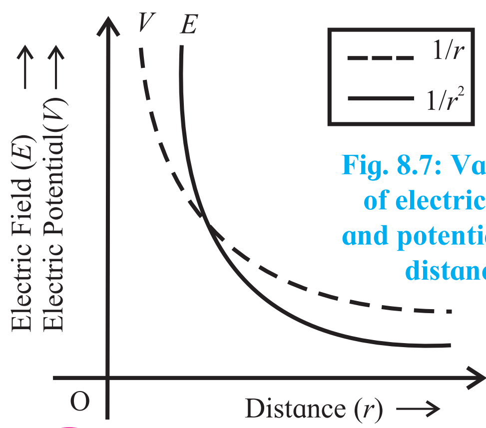


Fig. 8.7: Variation of electric field and potential with distance

Remember this

Due to a single charge at a distance r, Force (F) α $1/r^2$, Electric field (E) α $1/r^2$ but Potential (V) α $1/r$.

Example 8.6: A wire is bent in a circle of radius 10 cm. It is given a charge of 250μC which spreads on it uniformly. What is the electric potential at the centre ?

Solution : Given :

$q = 250\ \mu C = 250 \times 10^{-6}\ C$

$R = 10\ cm = 10^{-1}\ m$

$V = ?$

As $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{9\times10^9 \times 250\times10^{-6}}{10^{-1}}$

$= 2.25 \times 10^7$ volt

b) Electric potential due to an electric dipole:

We have studied electric and magnetic dipoles in XI[th] Std. Figure 8.8 shows an electric dipole AB consisting of two charges $+q$ and $-q$ separated by a finite distance $2l$. Its dipole moment is $\vec{p}$, of magnitude $p = q \times 2l$, directed from $-q$ to $+q$. The line joining the centres of the two charges is called dipole axis. A straight line drawn perpendicular to the axis and passing through centre O of the electric dipole is called equator of dipole.

In order to determine the electric potential due to a dipole, let the origin be at the centre (O) of the dipole.

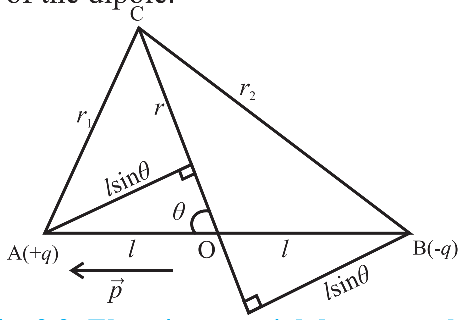


Fig. 8.8: Electric potential due to an electric dipole.

Let C be any point near the electric dipole at a distance r from the centre O inclined at an angle θ with axis of the dipole. r_1 and r_2 are the distances of point C from charges $+q$ and $-q$, respectively.

Potential at C due to charge $+q$ at A is,

$$V_1 = \frac{+q}{4\pi\varepsilon_0 r_1}$$

Potential at C due to charge $-q$ at B is,

$$V_2 = \frac{-q}{4\pi\varepsilon_0 r_2}$$

The electrostatic potential is the work done by the electric field per unit charge, $\left(V = \frac{W}{Q}\right)$.

The potential at C due to the dipole is,

$$V_C = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

By geometry,

$$r_1^2 = r^2 + \ell^2 - 2r\ell\cos\theta$$

$$r_2^2 = r^2 + \ell^2 + 2r\ell\cos\theta$$

$$r_1^2 = r^2\left(1 + \frac{\ell^2}{r^2} - 2\frac{\ell}{r}\cos\theta\right)$$

$$r_2^2 = r^2\left(1 + \frac{\ell^2}{r^2} + 2\frac{\ell}{r}\cos\theta\right)$$

For a short dipole, $2\ell << r$ and

If $r >> \ell$ ℓ/r is small $\therefore \frac{\ell^2}{r^2}$ can be neglected

$$\therefore r_1^2 = r^2\left(1 - 2\frac{\ell}{r}\cos\theta\right)$$

$$r_2^2 = r^2\left(1 + \frac{2\ell}{r}\cos\theta\right)$$

$$\therefore r_1 = r\left(1 - \frac{2\ell}{r}\cos\theta\right)^{1/2}$$

$$r_2 = r\left(1 + \frac{2\ell}{r}\cos\theta\right)^{1/2}$$

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2\ell}{r}\cos\theta\right)^{-1/2} \text{ and}$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2\ell}{r}\cos\theta\right)^{-1/2}$$

$$\therefore V_C = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 - \frac{2\ell\cos\theta}{r}\right)^{-1/2} - \frac{1}{r}\left(1 + \frac{2\ell\cos\theta}{r}\right)^{-1/2}\right]$$

Using binomial expansion, $(1 + x)^n = 1 + nx$, $x << l$ and retaining terms up to the first order of $\frac{\ell}{r}$ only, we get

$$V_C = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[\left(1+\frac{\ell}{r}\cos\theta\right)-\left(1-\frac{\ell}{r}\cos\theta\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_o r}\left[1+\frac{\ell}{r}\cos\theta - 1 + \frac{\ell}{r}\cos\theta\right]$$

$$= \frac{q}{4\pi\varepsilon_o r}\left[\frac{2\ell}{r}\cos\theta\right]$$

$$\therefore V_C = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad (\because p = q\times 2\ell)$$

Electric potential at C, can also be expressed as,

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\vec{r}}{r^3}$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\hat{r}}{r^2}, \quad \left(\hat{r} = \frac{\vec{r}}{r}\right)$$

where $\hat{r}$ is a unit vector along the position vector, $\overrightarrow{OC} = \hat{r}$

i) Potential at an axial point, $\theta = 0^0$ (towards $+q$) or 180^0 (towards $-q$)

$$V_{axial} = \frac{\pm 1}{4\pi\varepsilon_o}\frac{p}{r^2}$$

i.e. This is the maximum value of the potential.

ii) Potential at an equatorial point, $\theta = 90°$ and $V = 0$

Hence, the potential at any point on the equatorial line of a dipole is zero. This is the minimum value of the magnitude of the potential of a dipole.

Thus the plane perpendicular to the line between the charges at the midpoint is an equipotential plane with potential zero. The work done to move a charge anywhere in this plane (potential difference being zero) will be zero.

Example 8.7: A short electric dipole has dipole moment of 1×10^{-9} C m. Determine the electric potential due to the dipole at a point distance 0.3 m from the centre of the dipole situated

a) on the axial line b) on the equatorial line

c) on a line making an angle of 60° with the dipole axis.

Solution: Given

$p = 1\times10^{-9}$ Cm

$r = 0.3$ m

a) Potential at a point on the axial line

$$V = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2} = \frac{9\times10^9\times 1\times10^{-9}}{(0.3)^2} = 100 \text{ volt}$$

b) Potential at a point on the equatorial line $= 0$

c) Potential at a point on a line making an angle of 60° with the dipole axis is

$$V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} = \frac{9\times10^9\times 1\times10^{-9}\cos 60^\circ}{(0.3)^2}$$

$= 50$ volt

c) Electrostatics potential due to a system of charges:

We now extend the analysis to a system of charges.

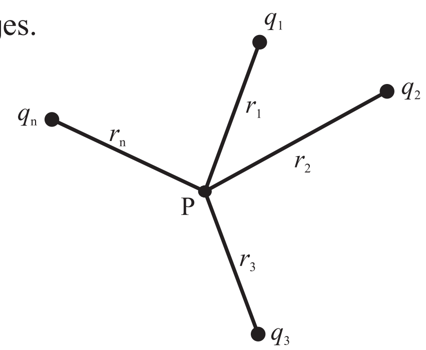


Fig. 8.9: System of charges.

Consider a system of charges q_1, q_2 .... q_n at distances r_1, r_2 ...... r_n respectively from point P. The potential V_1 at P due to the charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

Similarly the potentials V_2, V_3 ....V_n at P due to the individual charges q_2, q_3 ....q_n are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}, \; V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}, \; V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}$$

By the superposition principle, the potential V at P due to the system of charges is the algebraic sum of the potentials due to the individual charges.

$$\therefore V = V_1 + V_2 + ... + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \text{-----} + \frac{q_n}{r_n}\right)$$

Or, $V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$

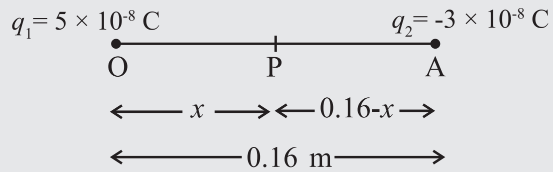
For a continuous charge distribution, summation should be replaced by integration.

Use your brain power

Is electrostatic potential necessarily zero at a point where electric field strength is zero? Justify.

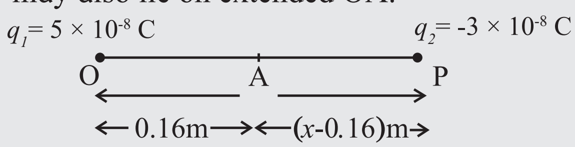
Example 8.8: Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point (s) on the line joining the two charges is the electric potential zero ? Take the potential at infinity to be zero.

Solution : As shown below, suppose the two point charges are placed on *x*- axis with the positive charge located at the origin O.



Let the potential be zero at the point P and OP = *x*. For $x < 0$ (i.e. to the left of O), the potentials of the two charges cannot add up to zero. Clearly, *x* must be positive. If *x* lies between O and A, then

$V_1 + V_2 = 0$, where V_1 and V_2 are the potentials at point P due to O and A, respectively. The other possibility is that x may also lie on extended OA.



$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{0.16-x}\right] = 0$$

$$9 \times 10^9 \left[\frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{0.16-x}\right] = 0$$

$$\Rightarrow 9 \times 10^9 \times 10^{-8}\left[\frac{5}{x} - \frac{3}{0.16-x}\right] = 0$$

$$\Rightarrow \frac{5}{x} - \frac{3}{0.16-x} = 0$$

$\therefore x = 0.10$ m

As $V_1 + V_2 = 0$

$$\frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{x-0.16}\right] = 0$$

$$9 \times 10^9 \left[\frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{x-0.16}\right] = 0$$

$\therefore x = 0.40$ m, $x = 40$ cm

8.5 Equipotential Surfaces:

An equipotential surface is that surface, at every point of which the electric potential is the same. We know that,

The potential (*V*) for a single charge *q* is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

If *r* is constant then *V* will be constant. Hence, equipotential surfaces of single point charge are concentric spherical surfaces centered at the charge. For a line charge, the shape of equipotential surface is cylindrical.

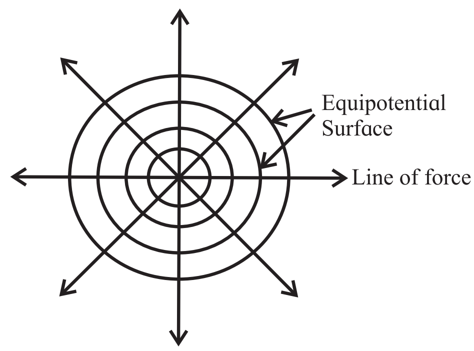


Fig. 8.10 : Equipotential surfaces.

Equipotential surfaces can be drawn through any region in which there is an electric field.

By definition the potential difference between two points P and Q is the work done per unit positive charge displaced from Q to P.

$\therefore V_P - V_Q = W_{QP}$

If points P and *Q* lie on an equipotential surface, $V_P = V_Q$.

$\therefore W_{QP} = 0$

Thus, no work is required to move a test charge along an equipotential surface.

a) If *dx* is the small distance over the equipotential surface through which unit positive charge is carried then

$$dW = \vec{E}\,.\,d\vec{x} = E\,dx \cos\theta = 0$$

$\therefore$ $\cos\theta = 0$ or $\theta = 90^0$

i.e. $\vec{E} \perp d\vec{x}$ as shown in Fig. 8.11

Hence electric field intensity $\vec{E}$ is always normal to the equipotential surface i.e., for any charge distribution, the equipotential surface through a point is normal to the electric field at that point.

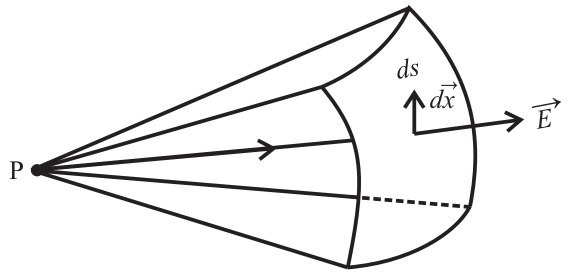


Fig. 8.11: Equipotential surface $\perp$ to $\vec{E}$

b) If the field is not normal, it would have a nonzero component along the surface. So to move a test charge against this component work would have to be done. But by the definition of equipotential surfaces, there is no potential difference between any two points on an equipotential surface and hence no work is required to displace the charge on the surface. Therefore, we can conclude that the electrostatic field must be normal to the equipotential surface at every point, and vice versa.

Do you know?

Equipotential surfaces do not intersect each other as it gives two directions of electric fields at intersecting point which is not possible.

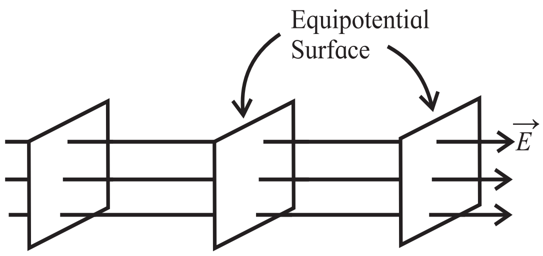


Fig. 8.12: Equipotential surfaces for a uniform electric field.

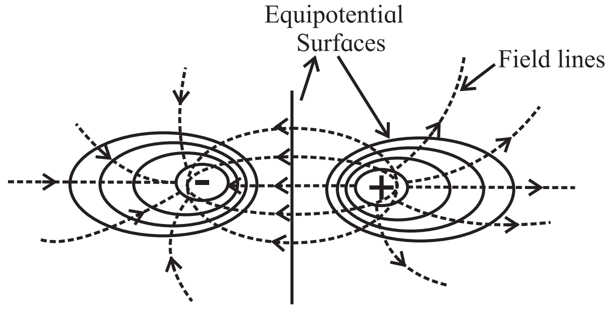


Fig. 8.13: Equipotential surfaces for a dipole.

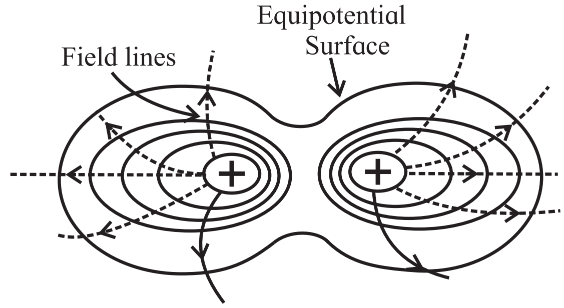


Fig. 8.14: Equipotential surfaces for two identical positive charges.

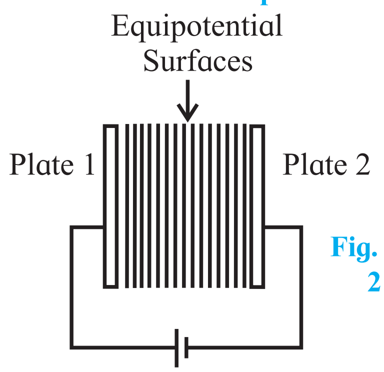
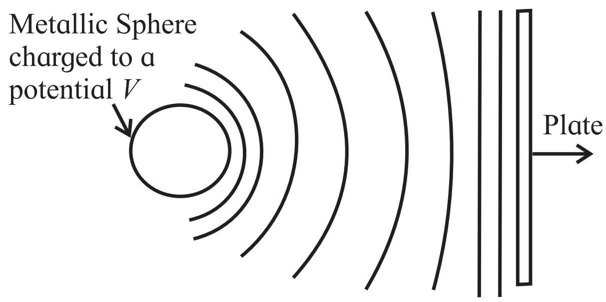


Fig. 8.15: (a) Between 2 plane metallic sheets.



(b) When one of the sheet is replaced by a charged metallic sphere.

Like the lines of force, the equipotential surface give a visual picture of both the direction and the magnitude of electric field in a region of space.

Example 8.9: A small particle carrying a negative charge of 1.6×10^{-19} C is suspended in equilibrium between two horizontal metal plates 10 cm apart having a potential difference of 4000 V across them. Find the mass of the charged particle.

Solution: Given :

$q = 1.6 \times 10^{-19}$ C

$dx = 10$ cm $= 10 \times 10^{-2}$ m $= 10^{-1}$ m

$dV = 4000\ V$

$$E = \frac{-dV}{dx} = \frac{-4000}{10^{-1}}$$

$= -4 \times 10^{4}$ Vm^{-1}

As the charged particle remains suspended in equilibrium,

$F = mg = qE$

$$\therefore m = \frac{qE}{g} = \frac{\left(-1.6\times10^{-19}\right)\left(-4\times10^{4}\right)}{9.8}$$

$= 0.653 \times 10^{-15}$ kg

$m = 6.53 \times 10^{-16}$ kg

8.6 Electrostatic Potential Energy of Two Point Charges and of a Dipole in an Electrostatic Field:

When two like charges lie infinite distance apart, their potential energy is zero because no work has to done in moving one charge at infinite distance from the other. But when they are brought closer to one another, work has to be done against the force of repulsion. As electrostatic force is conservative, this work gets stored as the potential energy of the two charges. **Electrostatic potential energy of a system of point charges is defined as the total amount of work done to assemble the system of charges by bringing them from infinity to their present locations**.

a) Potential energy of a system of two point charges:

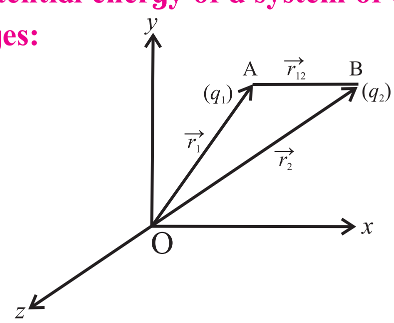


Fig. 8.16: System of two point charges.

Let us consider 2 charges q_1 and q_2 with position vectors r_1 and r_2 relative to the origin (O).

To calculate the electric potential energy of the two charge system, we assume that the two charges q_1 and q_2 are initially at infinity. We then determine the work done in bringing the charges to the given location by an external agency.

In bringing the first charge q_1 to position A $\left(\vec{r}_1\right)$, no work is done because there is no external field against which work is required to be done as charge q_2 is still at infinity i.e., $W_1 = 0$. This charge produces a potential in space given by

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} \qquad \text{--- (8.14)}$$

where r_1 is the distance of point A from the origin.

When we bring charge q_2 from infinity to B $\left(\vec{r}_2\right)$ at a distance r_{12}, from q_1, work done is

W_2 = (potential at B due to charge q_1) $\times q_2$

$$= \frac{q_1}{4\pi\varepsilon_0 r_{12}} \times q_2, \text{ (where AB} = r_{12}) \qquad \text{--- (8.15)}$$

This work done in bringing the two charges to their respective locations is stored as the potential energy of the configuration of two charges.

$$\therefore U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} \qquad \text{--- (8.16)}$$

Equation (8.16) can be generalised for a system of any number of point charges.

Example 8.10: Two charges of magnitude 5 nC and −2 nC are placed at points (2 cm, 0, 0) and (20 cm, 0, 0) in a region of space, where there is no other external field. Find the electrostatic potential energy of the system.

Solution : Given

$q_1 = 5$ nC $= 5 \times 10^{-9}$ C

$q_2 = -2$ nC $= -2 \times 10^{-9}$ C

$r = (20 - 2)$ cm $= 18$ cm $= 18 \times 10^{-2}$ m

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$= \frac{9\times10^{9}\times5\times10^{-9}\times-2\times10^{-9}}{18\times10^{-2}}$$

$= -5 \times 10^{-7}$ J $= -0.5 \times 10^{-6}$ J $= -0.5$ μJ

b) Potential energy for a system of N point charges:

Equation (8.16) gives an expression for potential energy for a system of two charges. We now analyse the situation for a system of N point charges.

In bringing a charge q_3 from ∞ to C $(\vec{r_3})$ work has to be done against electrostatic forces of both q_1 and q_2

$\therefore W_3 =$ (potential at C due to q_1 and q_2)$\times q_3$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right] \times q_3$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

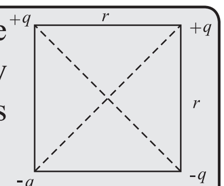
Similarly in bringing a charge q_4 from ∞ to D $\left(\vec{r_4}\right)$ work has to be done against electrostatic forces of q_1, q_2, and q_3

$$W_4 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1\, q_4}{r_{14}} + \frac{q_2\, q_4}{r_{24}} + \frac{q_3\, q_4}{r_{34}}\right]$$

Proceeding in the same way, we can write the electrostatic potential energy of a system of N point charges at $\vec{r_1}, \vec{r_2} \ldots.\vec{r_N}$ as

$$U = \frac{1}{4\pi\varepsilon_0} \sum_{\text{all pairs}} \frac{q_j\, q_k}{r_{jk}}$$

Example 8.11: Calculate the electrostatic potential energy of the system of charges shown in the figure.



Solution : Taking zero of potential energy at ∞, we get potential energy (PE) of the system of charges

$$\text{PE} = \frac{1}{4\pi\epsilon_0} \sum \frac{q_j q_k}{r_{jk}}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q\,q}{r} + \frac{q(-q)}{r} + \frac{(-q)(-q)}{r} + \frac{(-q)(+q)}{r} + \frac{q(-q)}{r\sqrt{2}} + \frac{q(-q)}{r\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{r} - \frac{q^2}{r} + \frac{q^2}{r} - \frac{q^2}{r} - \frac{q^2}{r\sqrt{2}} - \frac{q^2}{r\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-2q^2}{r\sqrt{2}}\right] = \left[\frac{-\sqrt{2}q^2}{4\pi\varepsilon_0 r}\right]$$

(c) Potential energy of a single charge in an external electric field:

Above, we have obtained an expression for potential energy of a system of charges when the source of the electric field, i.e., charges and their locations, were specified.

In this section, we determine the potential energy of a charge (or charges) in an external field $\vec{E}$ which is not produced by the given charge (or charges) whose potential energy we wish to calculate. The external sources could be known, unknown or unspecified, but what is known is the electric field E or the `electrostatic potential V due to the external sources.

Here we assume that the external field is not affected by the charge q, if q is very small. The external electric field E and the corresponding external potential V may vary from point to point.

If $V(\vec{r})$ is the external potential at any point P having position vector $\vec{r}$, then by definition, work done in bringing a unit positive charge from ∞ to the point P is equal to V.

$\therefore$ Work done in bringing a charge q, from ∞ to the given point in the external field is $qV(\vec{r})$.

This work is stored in the form of potential energy of a charge q at a given point in the external electric field.

$\therefore$ PE of a system of a single charge q at $\vec{r}$ in an external field is given by

$$\text{PE} = qV\left(\vec{r}\right) \qquad \text{--- (8.17)}$$

(d) Potential energy of a system of two charges in an external electric field:

In order to find the potential energy of a system of two charges q_1 and q_2 located at r_1 and r_2 respectively in an external electric field, we calculate the work done in bringing the charge q_1 from ∞ to r_1.

From Eq. (8.17), in the said process the work done

$$W = q_1 V(\vec{r_1}) \qquad \text{--- (8.18)}$$

To bring the charge q_2 to r_2, the work is done not only against the external field E but also against the field due to q_1.

$\therefore$ Work done on q_2 against the external field $= q_2\, V(\vec{r_2})$ and Work done on q_2 against the field due to $q_1 = \dfrac{q_1\, q_2}{4\pi\varepsilon_0\, r_{12}}$,
where r_{12} = distance between q_1 and q_2.

By the Principle of superposition for fields, we add up the work done on q_2 against the two fields.

$\therefore$ Work done in bringing q_2 to r_2

$$= q_2\, V\left(\vec{r_2}\right) + \frac{q_1\, q_2}{4\pi\varepsilon_0\, r_{12}} \quad \text{--- (8.19)}$$

Thus from (8.18) and (8.19) potential energy of the system

= Total work done in assembling the configuration

$$= q_1\, V\left(\vec{r_1}\right) + q_2\, V\left(\vec{r_2}\right) + \frac{q_1\, q_2}{4\pi\varepsilon_0 r_{12}}$$

Example 8.12: Two charged particles having equal charge of 3×10^{-5} C each are brought from infinity to a separation of 30 cm. Find the increase in electrostatic potential energy during the process.

Solution : Taking the potential energy (PE) at ∞ to be zero,

Increase in PE = present PE

$$V = \frac{q_1 q_2}{4\pi\epsilon_0\, r} = \frac{9\times10^9 \times (3\times10^{-5})^2}{0.3}$$

$$= \frac{9\times9\times10^9\times10^{-10}}{3\times10^{-1}} = \frac{81}{3} = 27 \text{ J}$$

Example 8.13:

a) Determine the electrostatic potential energy of a system consisting of two charges -2 μC and +4 μC (with no external field) placed at (-8 cm, 0, 0) and (+8 cm, 0, 0) respectively.

b) Suppose the same system of charges is now placed in an external electric field $E = A\,(1/r^2)$, where $A = 8 \times 10^5$ cm^{-2}, what would be the electrostatic potential energy of the configuration

Solution: Given :

q_1 = -2 μC = -2×10^{-6} C, r_1= 0.08 cm

q_2= +4 μC = $+4 \times 10^{-6}$ C, r_2 = 0.08 cm

r = 16 cm = 0.16 m

a) Electrostatic potential energy of the system of two charges is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$= \frac{9\times10^9\times(-2)\times10^{-6}\times4\times10^{-6}}{0.16}$$

$= 0.45$ J

b) In the electric field, total potential energy

$$(\text{PE}) = \frac{q_1 q_2}{4\pi\epsilon_0\, r} + q_1\, V(\vec{r_1}) + q_2\, V(\vec{r_2})$$

$$E = \frac{-dV}{dr} \quad \therefore\ V = \int -E dr = \int \frac{-A}{r^2}\, dr, \; V = \frac{A}{r}$$

$$\therefore \text{ Total PE } = \frac{q_1 q_2}{4\pi\epsilon_0\, r} + \frac{Aq_1}{r_1} + \frac{Aq_2}{r_2}$$

$$= -0.45 + \frac{8\times10^5\times\left(-2\times10^{-6}\right)}{0.08} + \frac{8\times10^5\times\left(4\times10^{-6}\right)}{0.08}$$

= -0.45 -20 + 40

= 19.55 J

(e) Potential energy of a dipole in an external field:

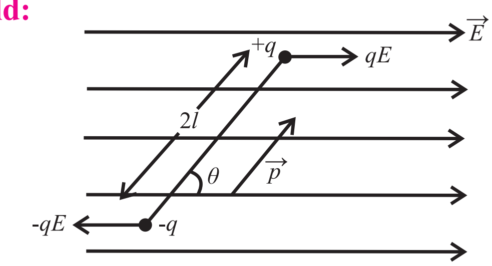


Fig. 8.17 : Couple acting on a dipole.

Consider a dipole with charges $-q$ and $+q$ separated by a finite distance 2ℓ, placed in a uniform electric field $\vec{E}$. It experiences a torque $\vec{\tau}$ which tends to rotate it.

$$\vec{\tau} = \vec{p} \times \vec{E} \text{ or } \tau = pE\sin\theta$$

In order to neutralize this torque, let us assume an external torque $\vec{\tau}_{ext}$ is applied, which rotates it in the plane of the paper from angle θ_0 to angle θ, without angular acceleration and at an infinitesimal angular speed. Work done by the external torque

$$W = \int_{\theta_0}^{\theta} \tau_{ext}(\theta)\, d\theta = \int_{\theta_0}^{\theta} pE\sin\theta\, d\theta$$

$$= pE\left[-\cos\theta\right]_{\theta_0}^{\theta}$$

$$= pE\left[-\cos\theta - (-\cos\theta_0)\right]$$

$$= pE\left[-\cos\theta + \cos\theta_0\right]$$

$$= pE\left[\cos\theta_0 - \cos\theta\right]$$

This work done is stored as the potential energy of the system in the position when the dipole makes an angle θ with the electric field. The zero potential energy can be chosen as per convenience. We can choose U (θ_0) = 0, giving

$$\therefore U(\theta) - U(\theta_0) = pE(\cos\theta_0 - \cos\theta)$$

a) If initially the dipole is perpendicular to the field $\overline{E}$ i.e., $\theta_0 = \frac{\pi}{2}$ then

$$U(\theta) = pE\left(\cos \pi/_2 - \cos\theta\right)$$

$$= -pE\cos\theta$$

$$U(\theta) = -\overline{p}.\overline{E}$$

b) If initially the dipole is parallel to the field $\overline{E}$ then $\theta_0 = 0$

$$U(\theta) = pE(\cos 0 - \cos\theta)$$

$$U(\theta) = pE(1 - \cos\theta)$$

Example 8.14: An electric dipole consists of two opposite charges each of magnitude 1 μC separated by 2 cm. The dipole is placed in an external electric field of 10^5 N C^{-1}.

Find:

(i) The maximum torque exerted by the field on the dipole

(ii) The work the external agent will have to do in turning the dipole through 180° starting from the position θ = 0°

Solution: Given :

$p = q \times 2l = 10^{-6} \times 2 \times 10^{-2} = 2 \times 10^{-8}$ cm

E = 10^5 NC^{-1}

(i) $\tau_{max} = p\,E \sin 90° = 2 \times 10^{-8} \times 10^5 \times 1$

$= 2 \times 10^{-3}$ Nm

(ii) $W = pE(\cos\theta_1 - \cos\theta_2)$

$= 2 \times 10^{-8} \times 10^5 \times (\cos 0 - \cos 180°)$

$= 2 \times 10^{-3}(1 + 1) = 4 \times 10^{-3}$ J

8.7 Conductors and Insulators, Free Charges and Bound Charges Inside a Conductor:

a) Conductors and Insulators:

When you come in contact with wires in wet condition or while opening the window of your car, you might have experienced a feeling of electric shock. Why don't you get similar experiences with wooden materials?

The reason you get a shock is that there occurs a flow of electrons from one body to another when they come in contact via rubbing or moving against each other. Shock is basically a wild feeling of current passing through your body.

Conductors are materials or substances which allow electricity to flow through them. This is because they contain a large number of free charge carriers (free electrons). In a metal, the electrons in the outermost orbit (valence electrons) are loosely bound to the nucleus and are thus free for conduction, when an external electric field is applied.

Metals, humans, earth and animal bodies are all conductors. The main reason we get electric shocks is that being a good conductor our human body allows a resistance-free path for the current to flow from the wire to our body.

Under electrostatic conditions the conductors have following properties.

1. In the interior of a conductor, net electrostatic field is zero.
2. Potential is constant within and on the surface of a conductor.
3. The interior of a conductor does not have any charge.
4. Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.
5. Surface charge density of a conductor could be different at different points.

Electrostatic shielding :

- To protect a delicate instrument from the disturbing effects of other charged bodies near it, place the instrument inside a hollow conductor where $E = 0$. This is called electrostatic shielding.
- Thin metal foils are used in making the shields.
- During lightning and thunder storm it is always advisable to stay inside the car than near a tree in open ground, since the car acts as a shield.

Faraday Cages:

- It is an enclosure which is used to block the external electric fields in conductive materials.
- Electro-magnetic shielding: MRI scanning rooms are built in such a manner that they prevent the mixing of the external radio frequency signals with the MRI machine.

b) Free charges and Bound charges inside materials:

The electrical behaviour of conductors and insulators can be understood on the basis of free and bound charges.

In metallic conductors, the electrons in the outermost shells of the atoms are loosely bound to the nucleus and hence can easily get detached and move freely inside the metal. When an external electric field is applied, they drift in a direction opposite to the direction of the applied electric field. These charges are called free charges.

The nucleus, which consist of the positively charged protons, and the inner shell electrons keeps the charges fixed in their positions. These immobile charges are called bound charges.

In electrolytic conductors, positive and negative ions act as charge carriers but their movements are restricted by the electrostatic force between them and the external electric field.

In insulators, the electrons are tightly bound to the nucleus and are thus not available for conductions and hence are poor conductors of electricity. There are no free charges since all the charges are bound to the nucleus. An insulating material can be considered as a collection of molecules that are not easily ionized. An insulator can carry any distribution of external electric charges on its surface or in its interior and the electric field in the interior can have non-zero values unlike conductors.

8.8 Dielectrics and Electric Polarisation:

Dielectrics are insulates which can be used to store electrical energy. This is because when such substances are placed in an external field, their positive and negative charges get displaced in opposite directions and the molecules develop a net dipole moment. This is called polarization of the material and such materials are called dielectrics.

In every atom there is a positively charged nucleus and there are negatively charged electrons surrounding it. The negative charges form an electron cloud around the positive charge. These two oppositely charged regions have their own centres of charge (where the effective charge is located). The centre of negative charge is the centre of mass of negatively charged electrons and that of positive charge is the centre of mass of positively charged protons in the nucleus.

Thus, dielectrics are insulating materials or non-conducting substances which can be polarized through small localized displacement of charges. e.g. glass, wax, water, wood , mica, rubber, stone, plastic etc. Dielectric constants of various materials are given in Table 8.1(pp203).

Dielectrics can be classified as polar dielectrics and non-polar dielectrics as described below.

Polar dielectrics:

A molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons), because of the asymmetric shape of the molecules, is called a polar molecule as shown in Fig. 8.18 (a). They have permanent dipole moments of the order of 10^{-30} Cm. They act as tiny electric dipoles, as the charges are separated by a small distance. The dielectrics like HCl, water, alcohol, NH_3 etc are made of polar molecules and are called polar dielectrics. Water molecule has a bent shape with its two O - H bonds which are inclined at an angle of about 105°. It has a very high dipole moment of 6.1×10^{-30} Cm. Fig. 8.18 (b) and (c) show the structure of HCl and H_2O, respectively.

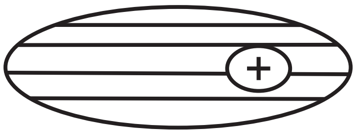


Fig. 8.18. (a) A polar molecule.

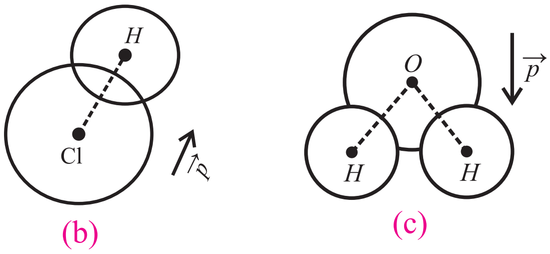


Fig. 8.18. Examples of Polar molecules (b) HCI (c) H_2O.

Non Polar dielectrics:

A molecule in which the centre of mass of the positive charges coincides with the centre of mass of the negative charges is called a non polar molecule as shown in Fig. 8.19 (a). These have symmetrical shapes and have zero dipole moment in the normal state. The dielectrics like hydrogen, nitrogen, oxygen, CO_2, benzene, methane are made up of nonpolar molecules and are called non polar dielectrics. Structures of H_2 and CO_2 are shown in Fig. 8.19 (b) and (c), respectively.

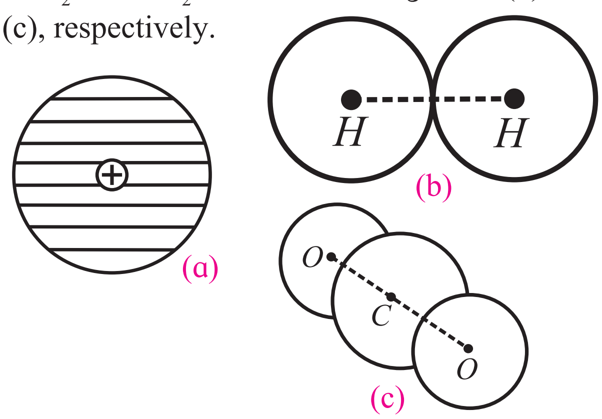


Fig. 8.19. (a) Nonpolar molecule. Examples of Nonpolar molecules (b) H_2 (c) CO_2.

Polarization of a non-polar dielectric in an external electric field:

In the presence of an external electric field E_o, the centres of the positive charge in each molecule of a non-polar dielectric is pulled in the direction of E_o, while the centres of the negative charges are displaced in the opposite direction. Therefore, the two centres are separated and the molecule gets distorted. The displacement of the charges stops when the force exerted on them by the external field is balanced by the electric field of induced dipole of the molecule.

Each molecule becomes a tiny dipole having a dipole moment. The induced dipole moments of different molecules add up giving a net dipole moment to the dielectric in the presence of the external field.

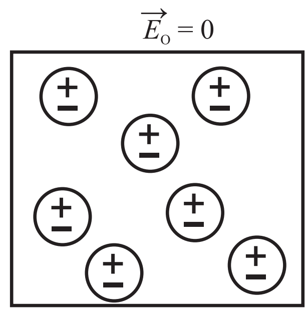


Fig. 8.20 (a) A non-polar dielectric material in absence of electric field.

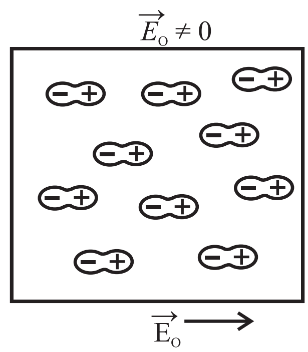


Fig. 8.20 (b) A non-polar dielectric material in the presence of an external field.

Polarization of a polar dielectric in an external electric field:

The molecules of a polar dielectric have tiny permanent dipole moments. Due to thermal agitation in the material in the absence of any external electric field, these dipole moments are randomly oriented as shown in Fig. 8.21

(a). Hence the total dipole moment is zero. When an external electric field is applied the dipole moments of different molecules tend to align with the field. As a result the dielectric develops a net dipole moment in the direction of the external field. Hence the dielectric is polarized. The extent of polarization depends on the relative values of the following two opposing fields :

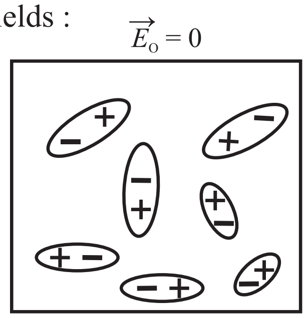


Fig. 8.21 (a) A polar dielectric in absence of electric field.

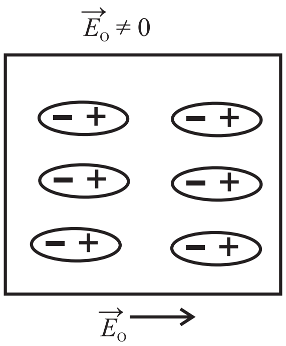


Fig. 8.21 (b) A polar dielectric in presence of an external field.

1. The applied external electric field which tends to align the dipole with the field.
2. The electric field due to induced dipole.

The polarization in presence of a strong external electric field is shown in Fig. 8.21 (b)

Thus, both polar and non-polar dielectric materials develop net dipole moment in the presence of an electric field.

The dipole moment per unit volume is called polarization and is denoted by $\vec{P}$. For linear isotropic dielectrics $\vec{P} = \chi_e \vec{E}$.

χ_e is a constant called electric susceptibility of the dielectric medium. It describes the electrical behaviour of a dielectric. It has different values for different dielectric materials.

For vacuum $\chi_e = 0$.

Reduction of electric field due to polarization of a dielectric:

When a dielectric is placed in an external electric field, the value of the field inside the dielectric is less than the external field as a result of polarization. Consider a rectangular dielectric slab placed in a uniform electric field $\vec{E}$ acting parallel to two of its faces. Since the electric charges are not free to move about in a dielectric, no current results when it is placed in an electric field. Instead of moving the charges, the electric field produces a slight rearrangement of charges within the atoms, resulting in aligning them with the field. This is shown in Fig. 8.20 and Fig. 8.21. During the process of alignment charges move only over distances that are less than an atomic diameter.

As a result of the alignment of the dipole moments there is an apparent sheet of positive charges on the right side and negative charges on the left side of the dielectric. These two sheets of induced surface charges produce an electric field $\vec{E}_0$ called the polarization field in the insulator which opposes the applied electric field $\vec{E}$. The net field $\vec{E}'$, inside the dielectric is the vector sum of the applied field $\vec{E}$ and the polarization field $\vec{E}_0$

$\therefore E' = E - E_0$ (in magnitude)

This is shown in Fig. 8.22 (a), (b) and (c).

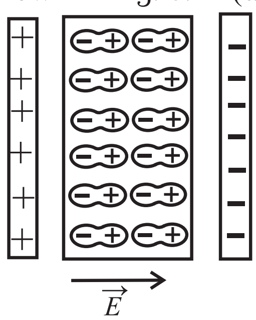


Fig. 8.22 (a) A dielectric slab placed between the plates of a capacitor.

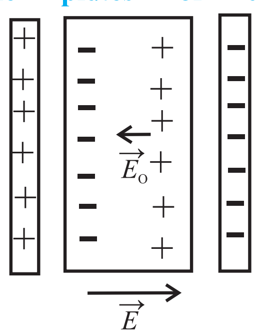


Fig. 8.22 (b) Induced surface charges and the polarization field in dielectric material placed between the plates of a capacitor.

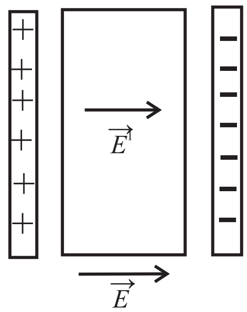


Fig. 8.22 (c) The net field $\vec{E'}$ is a vector sum of $\vec{E}$ and $\vec{E_0}$.

Do you know?

If we apply a large enough electric field, we can ionize the atoms and create a condition for electric charge to flow like a conductor. The fields required for the breakdown of dielectric is called dielectric strength.

The greater the applied field, greater is the degree of alignment of the dipoles and hence greater is the polarization field.

The induced dipole moment disappears when the field is removed. The induced dipole moment is often responsible for the attraction of a charged object towards an uncharged insulator such as charged comb and uncharged bits of paper.

Table 1:Dielectric constants of various materials:

Material	Min	Max
Air	1	1
Ebonite	2.7	2.7
Glass	3.8	14.5
Mica	4	9
Paper	1.5	3
Paraffin	2	3
Porcelain	5	6.5
Quartz	5	5
Rubber	2	4
Wood dry	1.4	2.9
Metals	∞	∞

8.9 Capacitors and Capacitance, Combination of Capacitors in Series and Parallel:

In XIth Std. you have studied about resistors, resistance and conductance. A resistor is an electrical component which allows current to pass through it and dissipates heat but can't store electrical energy. So there was a need to develop a device that can store electrical energy. The most common arrangement for this consists of a set of conductors (conducting plates) having opposite charges on them and separated by a dielectric material.

The conductors 1 and 2 shown in the Fig. 8.23 have charges $+Q$ and $-Q$ with potential difference, $V = V_1 - V_2$ between them. The electric field in the region between them is proportional to the magnitude of charge Q.

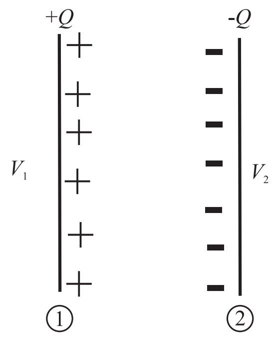


Fig. 8.23: A capacitor formed by two conductors.

The potential difference V is the work done to carry a unit positive test charge from the conductor 2 to conductor 1 against the field. As this work done will be proportional to Q, then $V \propto Q$ and the ratio $\frac{Q}{V}$ is a constant.

$$\therefore C = \frac{Q}{V}$$

The constant C is called the capacitance of the capacitor, which depends on the size, shape and separation of the system of two conductors.

The SI unit of capacitance is farad (F). Dimensional formula is $[M^{-1} L^{-2} T^4 A^2]$.

1 farad = 1 coulomb/1volt

A capacitor has a capacitance of one farad, if the potential difference across it rises by 1volt when 1 coulomb of charge is given to it. In practice farad is a big unit, the most commonly used units are its submultiples.

$1\mu F = 10^{-6} F$

$1 nF = 10^{-9} F$

$1 pF = 10^{-12} F$

Uses of Capacitors

Principle of a capacitor:

To understand the principle of a capacitor let us consider a metal plate P_1 having area A. Let some positive charge $+Q$ be given to this plate. Let its potential be V. Its capacitance is given by $C_1 = \frac{Q}{V}$

Now consider another insulated metal plate P_2 held near the plate P_1. By induction a negative charge is produced on the nearer face and an equal positive charge develops on the farther face of P_2 (Fig. 8.24 (a)). The induced negative charge lowers the potential of plate P_1, while the induced positive charge raises its potential.

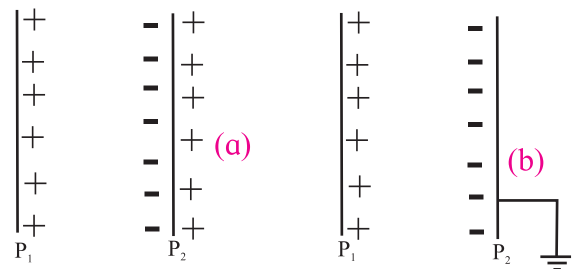


Fig. 8.24: (a) and (b) Parallel plate capacitor.

As the induced negative charge is closer to P_1 it is more effective, and thus there is a net reduction in potential of plate P_1. If the outer surface of P_2 is connected to earth, the induced positive charges on P_2 being free, flows to earth. The induced negative charge on P_2 stays on it, as it is bound to positive charge of P_1. This greatly reduces the potential of P_2, (Fig 8.24 (b)). If V_1 is the potential on plate P_2 due to charge (- Q) then the net potential difference between P_1 and P_2 will now be $+V-V_1$.

Hence the capacitance $C_2 = \frac{Q}{V - V_1}$ $\therefore C_2 > C_1$

Thus capacitance of metal plate P_1, is increased by placing an identical earth connected metal plate P_2 near it.

Such an arrangement is called capacitor. It is symbolically shown as ⊣⊢.

If the conductors are plane sheets then it is called parallel plate capacitor. We also have spherical capacitor, cylindrical capacitor etc. based on the shape of the conductors.

Combination of Capacitors:

When there is a combination of capacitors to be used in a circuit we can sometimes replace it with an equivalent capacitor or a single capacitor that has the same capacitance as the actual combination of capacitors. The effective capacitance depends on the way the individual capacitors are combined. Here we discuss two basic combinations of capacitors which can be replaced by a single equivalent capacitor.

(a) Capacitors in series:

When a potential difference (V) is applied across several capacitors connected end to end in such a way that sum of the potential difference across all the capacitors is equal to the applied potential difference V, then the capacitors are said to be connected in series.

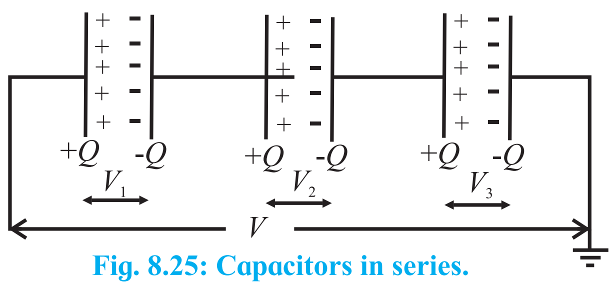


Fig. 8.25: Capacitors in series.

In series arrangement as shown in Fig. 8.25, the second plate of first conductor is connected to the first plate of the second conductor and so on. The last plate is connected to earth. In a series combination, charges on the plates $(\pm Q)$ are the same on each capacitor.

Potential difference across the series combination of capacitor is V volt,

where $V = V_1 + V_2 + V_3$

$$\therefore V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

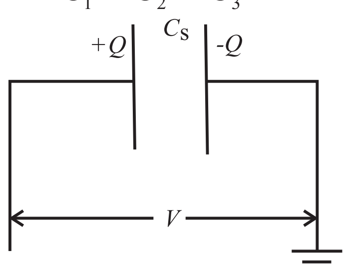


Fig. 8.26: Effective capacitance of three capacitors in series.

Let C_s represent the equivalent capacitance shown in Fig. 8.26, then $V = \frac{Q}{C_S}$

$$\therefore \frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

(for 3 capacitors in series)

This argument can be extended to yield an equivalent capacitance for n capacitors connected in series. The reciprocal of equivalent capacitance is equal to the sum of the reciprocals of individual capacitances of the capacitors.

$$\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

If all capacitors are equal then

$$\frac{1}{C_{eq}} = \frac{n}{C} \text{ or } C_{eq} = \frac{C}{n}$$

Remember this

Series combination is used when a high voltage is to be divided on several capacitors. Capacitor with minimum capacitance has the maximum potential difference between the plates.

b) Capacitors in Parallel:

The parallel arrangement of capacitors is as shown in Fig. 8.27 below, where the insulated plates are connected to a common terminal A which is joined to the source of potential, while the other plates are connected to another common terminal B which is earthed.

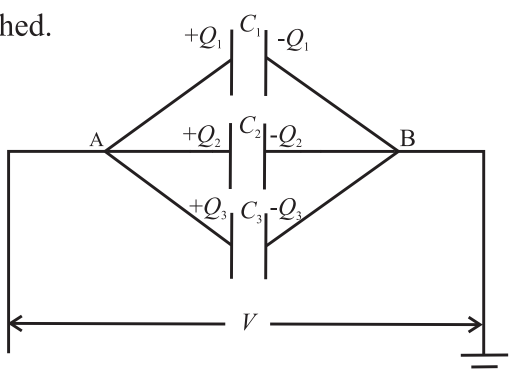


Fig. 8.27: Parallel combination of capacitors.

In this combination all the capacitors have the same potential difference but the plate charges ($\pm Q_1$) on capacitor1, ($\pm Q_2$) on the capacitor 2 and ($\pm Q_3$) on capacitor 3 are not necessarily the same. If charge Q is applied at point A then it will be distributed to the capacitors depending on the capacitances.

$\therefore$ Total charge Q can be written as $Q = Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V$

Let C_p be the equivalent capacitance of the combination then $Q = C_p V$

$$\therefore C_p V = C_1 V + C_2 V + C_3 V$$

$$\therefore C_p = C_1 + C_2 + C_3$$

The general formula for effective capacitance C_p for parallel combination of n capacitors follows similarly

$$C_p = C_1 + C_2 + ... + C_n$$

If all capacitors are equal then $C_{eq} = nC$

Remember this

Capacitors are combined in parallel when we require a large capacitance at small potentials.

Example 8.15 When 10^8 electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Find the capacitance of the two conductors.

Solution : Given :

Number of electrons $n = 10^8$

$V = 10$ volt

$\therefore$ charge transferred

$Q = ne = 10^8 \times 1.6 \times 10^{-19}$

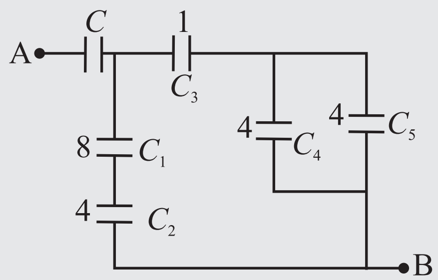
$(\because e = 1.6 \times 10^{-19}$ C)

$= 1.6 \times 10^{-11}$ C

$\therefore$ Capacitance between two conductors

$$C = \frac{Q}{V} = \frac{1.6 \times 10^{-11}}{10} = 1.6 \times 10^{-12} \text{ F}$$

Example 8.16: From the figure given below find the value of the capacitance C if the equivalent capacitance between A and B is to be 1 µF. All other capacitors are in micro farad.



Solution : Given :

$C_1 = 8\ \mu F$, $C_2 = 4\ \mu F$, $C_3 = 1\mu F$,
$C_4 = 4\ \mu F$, $C_5 = 4\ \mu F$

The effective capacitance of C_4 and C_5 in parallel

$= C_4 + C_5 = 4 + 4 = 8\ \mu F$

The effective capacitance of C_3 and 8 μF in series

$$= \frac{1 \times 8}{1+8} = \frac{8}{9}\ \mu F$$

The capacitance 8 μF is in parallel with the series combination of C_1 and C_2. Their effective combination is

$$\frac{C_1 C_2}{C_1 + C_2} + \frac{8}{9} \Rightarrow \frac{8 \times 4}{12} + \frac{8}{9} \Rightarrow \frac{32}{9}\ \mu F$$

This capacitance of $\frac{32}{9}$ μF is in series with C and their effective capacitance is given to be 1μF

$$\frac{\frac{32}{9} \times C}{\frac{32}{9} + C} = 1$$

$$\therefore \frac{32}{9} \times C = \frac{32}{9} + C$$

$= 1.392\ \mu F$

8.10 Capacitance of a Parallel Plate Capacitor Without and With Dielectric Medium Between the Plates:

In section 8.8 we have studied the behaviour of dielectrics in an external field. Let us now see how the capacitance of a parallel plate capacitor is modified when a dielectric is introduced between its plates.

a) Capacitance of a parallel plate capacitor without a dielectric:

A parallel plate capacitor consists of two thin conducting plates each of area *A*, held parallel to each other, at a suitable distance *d* apart. One of the charged plates is isolated and charged and the other is earthed as shown in Fig. 8.28.

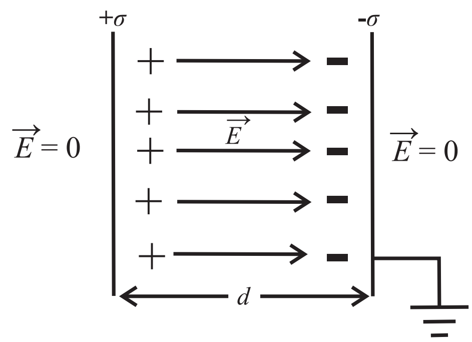


Fig. 8.28: Capacitor with dielectric.

When a charge +*Q* is given to the isolated plate, then a charge -*Q* is induced on the inner face of earthed plate and +*Q* is induced on its outer face. But as this face is earthed the charge +*Q* being free, flows to earth.

In the outer regions the electric fields due to the two charged plates cancel out. The net field is zero.

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the inner regions between the two capacitor plates the electric fields due to the two charged plates add up. The net field is thus

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \quad \text{--- (8.20)}$$

The direction of *E* is from positive to negative plate.

Let *V* be the potential difference between the 2 plates. Then electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed \quad \text{--- (8.21)}$$

Substituting Eq. (8.20) in Eq. (8.21) we get $V = \frac{Q}{A\varepsilon_0} d$

Capacitance of the parallel plate capacitor is given by

Remember this

(1) If there are n parallel plates then there will be (n-1) capacitors, hence

$$C = (n-1)\frac{A\varepsilon_0}{d}$$

(2) For a spherical capacitor, consisting of two concentric spherical conducting shells with inner and outer radii as *a* and *b* respectively, the capacitance *C* is given by

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

(3) For a cylindrical capacitor, consisting of two coaxial cylindrical shells with radii of the inner and outer cylinders as *a* and *b*, and length *l*, the capacitance *C* is given by

$$C = \frac{2\pi\varepsilon_0\,\ell}{\log_e \frac{b}{a}}$$

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{A\varepsilon_0}{d} \quad \text{--- (8.22)}$$

b) Capacitance of a parallel plate capacitor with a dielectric slab between the plates:

Let us now see how Eq. (8.22) gets modified with a dielectric slab in between the plates of the capacitor. Consider a parallel plate capacitor with the two plates each of area A separated by a distance *d*. The capacitance of the capacitor is given by

$$C_0 = \frac{A\varepsilon_0}{d}$$

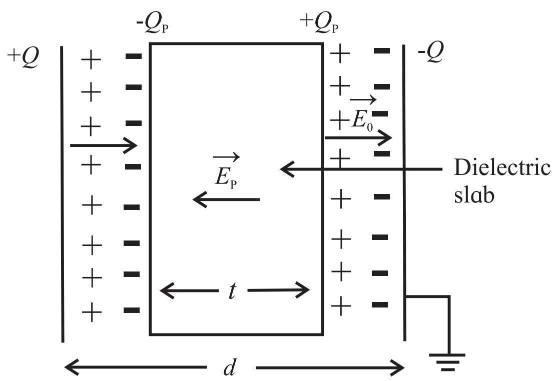


Fig. 8.29: Dielectric slab in the capacitor.

Let E_0 be the electric field intensity between the plates before the introduction of the dielectric slab. Then the potential difference between the plates is given by $V_0 = E_0 d$,

$$\text{where } E_o = \frac{\sigma}{\varepsilon_o} = \frac{Q}{A\varepsilon_o}, \text{ and}$$

σ is the surface charge density on the plates.

Let a dielectric slab of thickness *t* (*t* < *d*) be introduced between the plates of the capacitor. The field E_0 polarizes the dielectric, inducing charge - Q_p on the left side and +Q_p on the right side of the dielectric as shown in Fig. 8.29.

These induced charges set up a field E_p inside the dielectric in the opposite direction of E_0. The induced field is given by

$$E_p = \frac{\sigma_p}{\varepsilon_o} = \frac{Q_p}{A\varepsilon_o} \quad \left[\sigma_p = \frac{Q_p}{A}\right]$$

The net field (*E*) inside the dielectric reduces to E_0- E_p.

Hence,

$$E = E_o - E_p = \frac{E_o}{k} \quad \left[\because \frac{E_o}{E_o - E_p} = k\right],$$

where *k* is a constant called the dielectric constant.

$$\therefore E = \frac{Q}{A\varepsilon_0 k} \text{ or } Q = Ak\varepsilon_0 E \quad \text{--- (8.23)}$$

Remember this

The dielectric constant of a conductor is infinite.

The field E_p exists over a distance t and E_0 over the remaining distance (*d* - *t*) between the capacitor plates. Hence the potential difference between the capacitor plates is

$$V = E_o(d-t) + E(t)$$

$$= E_o(d-t) + \frac{E_o}{k}(t) \qquad \left(\because E = \frac{E_0}{k}\right)$$

$$= E_o\left[(d-t) + \frac{t}{k}\right]$$

$$= \frac{Q}{A\varepsilon_o}\left[d - t + \frac{t}{k}\right]$$

The capacitance of the capacitor on the introduction of dielectric slab becomes

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\varepsilon_0}\left(d - t + \frac{d}{k}\right)} = \frac{A\varepsilon_0}{\left(d - t + \frac{t}{k}\right)}$$

Special cases:

1. If the dielectric fills up the entire space then $t = d \therefore C = \frac{A\varepsilon_0 k}{d} = k\,C_0$

$\therefore$ capacitance of a parallel plate capacitor increases k times i.e. $k = \frac{C}{C_0}$

2. If the capacitor is filled with n dielectric slabs of thickness $t_1, t_2 .... t_n$ then this arrangement is equivalent to n capacitors connected in series as shown in Fig. 8.30.

$$\therefore C = \frac{A\varepsilon_0}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + ... + \frac{t_n}{k_n}\right)}$$

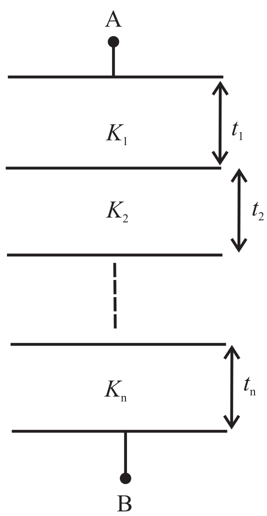


Fig. 8.30 : Capacitor filled with n dielectric slabs.

3. If the arrangement consists of n capacitors in parallel with plate areas $A_1, A_2, ... A_n$ and plate separation d

$$C = \frac{\varepsilon_0}{d}\left(A_1 k_1 + A_2 k_2 + ...+ A_n k_n\right)$$

$$\text{if } A_1 = A_2 .... A_n = \frac{A}{n} \text{ then}$$

$$C = \frac{A\varepsilon_0}{dn}\left(k_1 + k_2 + ...+ k_n\right)$$

4. If the capacitor is filled with a conducting slab ($k = \infty$) then

$$C = \left(\frac{d}{d - t}\right) C_o \qquad \therefore C > C_o$$

The capacitance thus increases by a factor $\left(\frac{d}{d - t}\right)$

8.11 Displacement Current:

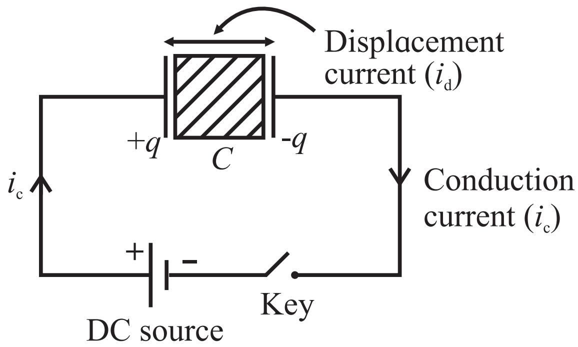


Fig. 8.31: Displacement current in the space between the plates of the capacitor.

We know that electric current in a DC circuit constitutes a flow of free electrons. In a circuit as shown in Fig 8.31, a parallel plate capacitor with a dielectric is connected across a DC source. In the conducting part of the circuit free electrons are responsible for the flow of current. But in the region between the plates of the capacitor, there are no free electrons available for conduction in the dielectric.

As the circuit is closed, the current flows through the circuit and grows to its maximum value (i_c) in a finite time (time constant of the circuit). The conduction current, i_c is found to be same everywhere in the circuit except inside the capacitor. As the current passes through the leads of the capacitor, the electric field between the plates increases and this in turn causes polarisation of the dielectric. Thus, there is a current in the dielectric due to the movement of the bound charges. The current due to bound charges is called displacement current (i_d) or charge- separation current.

We can now derive an expression between i_c and i_d.

From Eq (8.23) we can infer that the charge produced on the plates of a capacitor is due to the electric field E.

$q = Ak\varepsilon_0 E$

Differentiating the above equation, we get

$$\frac{dq}{dt} = Ak\varepsilon_0 \frac{dE}{dt} \qquad \text{--- (8.24)}$$

dq/dt is the conduction current (i_c)in the conducting part of the circuit.

$$i_c = \frac{dq}{dt} = Ak\varepsilon_0 \frac{dE}{dt}$$

$$\frac{dE}{dt} = \frac{i_c}{Ak\varepsilon_0} \therefore \frac{dE}{dt} \propto i_c \text{ (for fixed value of A)}$$

The rate of change of electric field (dE/dt) across the capacitor is directly proportional to the current (i_c) flowing in the conducting part of the circuit.

The quantity on the RHS of Eq (8.24) is having the dimension of electric current and is caused by the displacement of bound charges in the dielectric of the capacitor under the influence of the electric field. This current, called displacement current (i_d), is equivalent to the rate of flow of charge ($dq/dt=i_c$) in the conducting part of the circuit. In the absence of any dielectric between the plates of the capacitor, k=1 (for air or vacuum), the displacement current $i_d = A\varepsilon_0 (dE/dt)$.

As a broad generalization of displacement current in a circuit containing a capacitor, it can be stated that the displacement currents do not remain confined to the space between the plates of a capacitor. A displacement current (i_d) exists at any point in space where, time-varying electric field (E) exists (i.e. $dE/dt \neq 0$).

Example 8.17 A parallel plate capacitor has an area of 4 cm² and a plate separation of 2 mm

(i) Calculate its capacitance

(ii) What is its capacitance if the space between the plates is filled completely with a dielectric having dielectric constant of constant 6.7.

Solution : Given

$A = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$

$d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

(i) Capacitance $C = \frac{A\varepsilon_0}{d}$

$$= \frac{8.85\times10^{-12}\times4\times10^{-4}}{2\times10^{-3}} = 1.77\times10^{-12}\text{ F}$$

(ii) Capacitance $C' = \frac{A\varepsilon_0 k}{d}$

$$= \frac{8.85\times10^{-12}\times4\times10^{-4}\times6.7}{2\times10^{-3}}$$

$= 11.86 \times 10^{-12}$ F

Example 8.18: In a capacitor of capacitance 20 μF, the distance between the plates is 2 mm. If a dielectric slab of width 1 mm and dielectric constant 2 is inserted between the plates, what is the new capacitance ?

Solution: Given

$C = 20\ \mu\text{F} = 20 \times 10^{-6}$ F

$d = 2 \text{ mm} = 2 \times 10^{-3}$ m

$t = 1 \times 10^{-3}$ m

$k = 2$

$$C = \frac{A\varepsilon_0}{d} \text{ and } C' = \frac{A\varepsilon_0}{d - t + \frac{t}{k}}$$

$$\Rightarrow \frac{C}{C'} = \frac{d - t + \frac{t}{k}}{d}$$

$$\Rightarrow \frac{20}{C'} = \frac{\left(2\times10^{-3} - 1\times10^{-3} + \frac{1\times10^{-3}}{2}\right)}{2\times10^{-3}}$$

$\Rightarrow C' = 26.67\ \mu$F

8.12 Energy Stored in a Capacitor:

A capacitor is a device used to store energy. Charging a capacitor means transferring electron from one plate of the capacitor to the other. Hence work will have to be done by the battery in order to remove the electrons against the opposing forces. These opposing forces arise since the electrons are being pushed to the negative plate which repels them and electrons are removed from the positive plate which tends to attract them. In both the cases, the coulombian forces oppose the transfer of charges from one plate to another. As the charge on the plate increases, its opposing force also increases.

This work done is stored in the form of electrostatic energy in the electric field between the plates, which can later be recovered by discharging the capacitor.

Consider a capacitor of capacitance C being charged by a DC source of V volts as shown in Fig. 8.32.

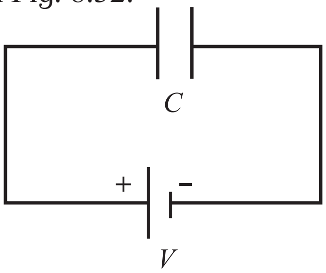


Fig. 8.32: Capacitor charged by a DC source.

During the process of charging, let q' be the charge on the capacitor and V be the potential difference between the plates. Hence

$$C = \frac{q'}{V}$$

A small amount of work is done if a small charge dq is further transferred between the plates.

$$\therefore dW = V\, dq = \frac{q'}{C}\, dq$$

Total work done in transferring the charge

$$W = \int dw = \int_O^Q \frac{q'}{C}\, dq = \frac{1}{C}\int_O^Q q'\; dq$$

$$= \frac{1}{C}\left[\frac{(q')^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

This work done is stored as electrical potential energy U of the capacitor. This work done can be expressed in different forms as follows.

$$\therefore U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \quad (\because Q = CV)$$

Observe and discuss

The energy supplied by the battery is QV but energy stored in the electric field is $\frac{1}{2}QV$. The rest half $\frac{1}{2}QV$ of energy is wasted as heat in the connecting wires and battery itself.

Example 8.19: A parallel plate air capacitor has a capacitance of 3×10^{-9} Farad. A slab of dielectric constant 3 and thickness 3 cm completely fills the space between the plates. The potential difference between the plates is maintained constant at 400 volt. What is the change in the energy of capacitor if the slab is removed ?

Solution : Energy stored in the capacitor with air

$$E_a = \frac{1}{2}CV^2 = \frac{1}{2}\times 3\times 10^{-9}\times(400)^2$$
$$= 24 \times 10^{-5}\,\text{J}$$

when the slab of dielectric constant 3 is introduced between the plates of the capacitor, the capacitance of the capacitor increases to

$$C' = kC$$
$$C' = 3 \times 3 \times 10^{-9} = 9 \times 10^{-9}\,\text{F}$$

Energy stored in the capacitor with the dielectric (E_d)

$$E_d = \frac{1}{2}C'V^2$$

$$E_d = \frac{1}{2} \times 9 \times 10^{-9} \times (400)^2$$
$$= 72 \times 10^{-5}\,\text{J}$$

Change in energy $= E_d - E_a = (72 - 24) \times 10^{-5}$
$= 48 \times 10^{-5}$ J

There is, therefore, an increase in the energy on introducing the slab of dielectric material.

8.13 Van de Graaff Generator:

Van de Graaff generator is a device used to develop very high potentials of the order of 10^7 volts. The resulting large electric fields are used to accelerate charged particles (electrons, protons, ions) to high energies needed for experiments to probe the small scale structure of matter and for various experiments in Nuclear Physics.

It was designed by Van de Graaff (1901-1967) in the year 1931.

Principle: This generator is based on

(i) the phenomenon of Corona Discharge (action of sharp points),

(ii) the property that charge given to a hollow conductor is transferred to its outer surface and is distributed uniformly over it,

(iii) if a charge is continuously supplied to an insulated metallic conductor, the potential of the conductor goes on increasing.

Construction:

Fig. 8.33 shows the schematic diagram of Van de Graaff generator.

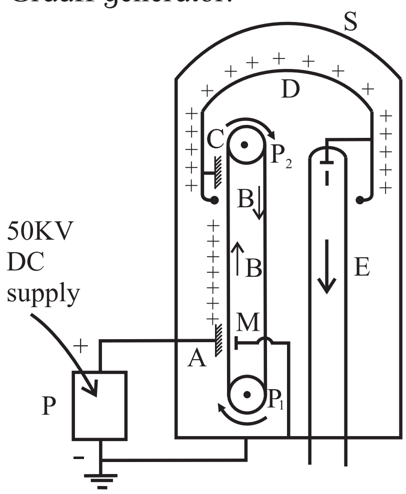


Fig. 8.33: Schematic diagram of van de Graff generator.

P_1 P_2 = Pulleys
BB = Conveyer belt
A = Spray brush
C = Collector brush
D = Dome shaped hollow conductor
E = Evacuated accelerating tube
I = Ion source
P = DC power supply
S = Steel vessel filled with nitrogen
M = Earthed metal plate

An endless conveyor belt BB made of an insulating material such as reinforced rubber or silk, can move over two pulleys P_1 and P_2. The belt is kept continuously moving by a motor (not shown in the figure) driving the lower pulley (P_1).

The spray brush A, consisting of a large number of pointed wires, is connected to the positive terminal of a high voltage DC power supply. From this brush positive charge can be sprayed on the belt which can be collected by another similar brush C. This brush is connected to a large, dome-shaped, hollow metallic conductor D, which is mounted on insulating pillars (not shown in the figure). E is an evacuated accelerating tube having an electrode I at its upper end, connected to the dome-shaped conductor.

To prevent the leakage of charge from the dome, the pulley and belt arrangement, the dome and a part of the evacuated tube are enclosed inside a large steel vessel S, filled with nitrogen at high pressure. A small quantity of freon gas is mixed with nitrogen to ensure better insulation between the vessel S and its contents. A metal plate M held opposite to the brush A on the other side of the belt is connected to the vessel S, which is earthed.

Working: The electric motor connected to the pulley P_1 is switched on, which begins to rotate setting the conveyor belt into motion. The DC supply is then switched on. From the pointed ends of the spray brush A, positive charge is continuously sprayed on the belt B. The belt carries this charge in the upward direction, which is collected by the collector brush C and sent to the dome shaped conductor.

As the dome is hollow, the charge is distributed over the outer surface of the dome. Its potential rises to a very high value due to the continuous accumulation of charges on it. The potential of the electrode I also rises to this high value.

The positive ions such as protons or deuterons from a small vessel (not shown in the figure) containing ionised hydrogen or deuterium are then introduced in the upper part of the evacuated accelerator tube. These ions, repelled by the electrode I, are accelerated in the downward direction due to the very high fall of potential along the tube, these ions acquire very high energy. These high energy charged particles are then directed so as to strike a desired target.

Uses: The main use of Van de Graff generator is to produce very high energy charged particles having energies of the order of 10 MeV. Such high energy particles are used

1. to carry out the disintegration of nuclei of different elements,
2. to produce radioactive isotopes,
3. to study the nuclear structure,
4. to study different types of nuclear reactions,
5. accelerating electrons to sterilize food and to process materials.

Internet my friend

1. https://en.m.wikipedia.org
2. hyperphyrics.phy-astr.gsu.edu
3. https://www.britannica.com/science
4. https://www.khanacademy.org>in-i

Exercises

Q1. Choose the correct option

i) A parallel plate capacitor is charged and then isolated. The effect of increasing the plate separation on charge, potential, capacitance respectively are

(A) Constant, decreases, decreases
(B) Increases, decreases, decreases
(C) Constant, decreases, increases
(D) Constant, increases, decreases

ii) A slab of material of dielectric constant k has the same area A as the plates of a parallel plate capacitor and has thickness (3/4d), where d is the separation of the plates. The change in capacitance when the slab is inserted between the plates is

(A) $C = \frac{A\varepsilon_0}{d}\left(\frac{k+3}{4k}\right)$

(B) $C = \frac{A\varepsilon_0}{d}\left(\frac{2k}{k+3}\right)$

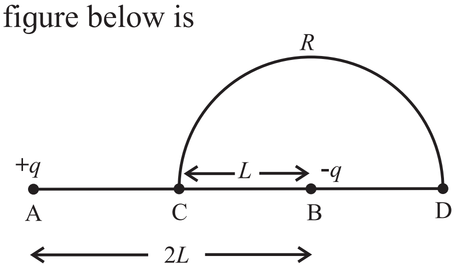
(C) $C = \frac{A\varepsilon_0}{d}\left(\frac{k+3}{2k}\right)$

(D) $C = \frac{A\varepsilon_0}{d}\left(\frac{4k}{k+3}\right)$

iii) Energy stored in a capacitor and dissipated during charging a capacitor bear a ratio.

(A) 1:1 (B) 1:2
(C) 2:1 (D) 1:3

iv) Charge $+q$ and $-q$ are placed at points A and B respectively which are distance 2L apart. C is the mid point of A and B. The work done in moving a charge +Q along the semicircle CRD as shown in the figure below is



(A) $\frac{-qQ}{6\pi\varepsilon_0 L}$ (B) $\frac{qQ}{2\pi\varepsilon_0 L}$

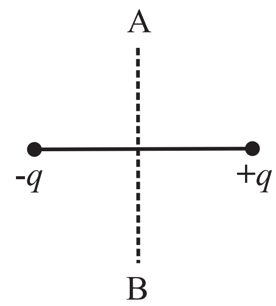
(C) $\frac{qQ}{6\pi\varepsilon_0 L}$ (D) $\frac{-qQ}{4\pi\varepsilon_0 L}$

v) A parallel plate capacitor has circular plates of radius 8 cm and plate separation 1mm. What will be the charge on the plates if a potential difference of 100 V is applied?

(A) 1.78×10^{-8} C (B) 1.78×10^{-5} C
(C) 4.3×10^{4} C (D) 2×10^{-9} C

Q2. Answer in brief.

i) A charge q is moved from a point A above a dipole of dipole moment p to a point B below the dipole in equitorial plane without acceleration. Find the work done in this process.



ii) If the difference between the radii of the two spheres of a spherical capacitor is increased, state whether the capacitance will increase or decrease.

iii) A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor?

iv) The safest way to protect yourself from lightening is to be inside a car. Justify.

v) A spherical shell of radius b with charge Q is expanded to a radius a. Find the work done by the electrical forces in the process.

3. A dipole with its charges, $-q$ and $+q$ located at the points (0, -b, 0) and (0 +b, 0) is present in a uniform electric field E whose equipotential surfaces are planes parallel to the YZ planes.

(a) What is the direction of the electric field E? (b) How much torque would the dipole experience in this field? [(a) along/parallel to X axis, (b) $\tau = 2bqE$]

4. Three charges $-q$, $+Q$ and $-q$ are placed at equal distance on straight line. If the potential energy of the system of the three charges is zero, then what is the ratio of Q:q? [(Q : q = 1 : 4)]

5. A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, the electric field, charge stored and voltage will increase, decrease or remain constant.

6. Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1:2, so that the energy stored in these two cases becomes the same. [Vp : Vs = √ 2:3]

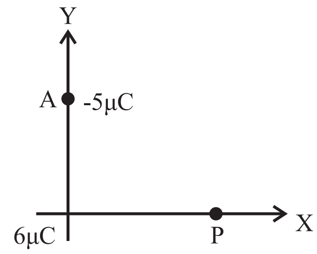
7. Two charges of magnitudes $-4Q$ and $+2Q$ are located at points (2a, 0) and (5a, 0) respectively. What is the electric flux due to these charges through a sphere of radius 4a with its centre at the origin?

8. A 6 µF capacitor is charged by a 300 V supply. It is then disconnected from the supply and is connected to another uncharged 3µF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation ? [Ans: 9×10^{-2} J]

9. One hundred twenty five small liquid drops, each carrying a charge of 0.5 µC and each of diameter 0.1 m form a bigger drop. Calculate the potential at the surface of the bigger drop. [Ans: 2.25×10^{6} V]

10. The dipole moment of a water molecule is 6.3×10^{-30} Cm. A sample of water contains 10^{21} molecules, whose dipole moments are all oriented in an electric field of strength 2.5×10^{5} N /C. Calculate the work to be done to rotate the dipoles from their initial orientation $\theta_1 = 0$ to one in which all the dipoles are perpendicular to the field, $\theta_2 = 90°$.[Ans: 1.575×10^{-3} J]

11. A charge 6 µC is placed at the origin and another charge –5 µC is placed on the y axis at a position A (0, 6.0) m.



a) Calculate the total electric potential at the point P whose coordinates are (8.0, 0) m

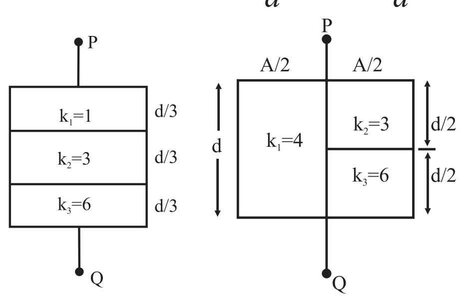
b) Calculate the work done to bring a proton from infinity to the point P. What is the significance of the sign of the work done ?

[Ans: (a) $V_p = 2.25 \times 10^{3}$ V
(b) $W = 3.6 \times 10^{-16}$ J]

12. In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the separation between the plates is 2 mm. a) Calculate the capacitance of the capacitor, b) If this capacitor is connected to 100 V supply, what would be the charge on each plate? c) How would charge on the plates be affected if a 2 mm thick mica sheet of k = 6 is inserted between the plates while the voltage supply remains connected ?
[Ans: (a) 2.655×10^{-11} F,
(b) 2.655×10^{-9} C, (c) 15.93×10^{-9} C]

13. Find the equivalent capacitance between P and Q. Given, area of each plate = A and separation between plates = d.

[Ans: (a) $\dfrac{2A\varepsilon_0}{d}$ (b) $\dfrac{4A\varepsilon_0}{d}$]



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