6. Mechanical Properties of Solids

Can you recall?

1. Can you name a few objects which change their shape and size on application of a force and regain their original shape and size when the force is removed ?
2. Can you name objects which do not regain their original shape and size when the external force is removed?

6.1 Introduction:

Solids are made up of atoms or a group of atoms placed in a definite geometric arrangement. This arrangement is decided by nature so that the resultant force acting on each constituent due to others is zero. This is the equilibrium state of a solid at room temperature. The given equilibrium arrangement does not change with time. It can change only when an external stimulus, like compressive force from all sides, is applied to a solid. The constituents vibrate about their equilibrium positions even at very low temperatures but cannot leave their fixed positions. This fact provides the solids a definite shape and size (allows the solids to maintain a definite shape and size).

If an external force is applied to a solid the constituents are slightly displaced and restoring forces are developed in it. These restoring forces try to bring the constituents back to their equilibrium positions so that the solid can regain its shape. When the deforming forces are removed, the interatomic forces tend to restore the original positions of the molecules and thus the body regains its original shape and size. However, as we will see later, this is possible only within certain limits.

The form of a body is decided by its size and shape, e.g., a tennis ball and a football both are spherical, i.e., they have the same shape. But a tennis ball is smaller in size than a football. When a force is applied to a solid (which is not free to move), the size or shape or both change due to changes in the relative positions of molecules. Such a force is called **deforming force**.

The change in shape or size or both of a body due to an external force is called deformation.

The larger the deforming force on a body, the larger is its deformation. Deformation could be in the form of change in length of a wire, change in volume of an object or change in shape of a body.

We know that when a deforming force (e.g. stretching) is applied to a rubber band, it gets deformed (elongated) but when the force is removed, it regains its original length. When a similar force is applied to a dough, or clay it also gets deformed but it does not regain its original shape and size after removal of the deforming force. These observations indicate that rubber and clay are different in nature. The property that decides this nature is called **elasticity/plasticity**. We will learn more about these properties of solids in this Chapter .

6.2 Elastic Behavior of Solids:

If a body regains its original shape and size after removal of the deforming force, it is called an elastic body and the property is called elasticity. Here the restoring forces are strong enough to bring the displaced molecules to their original positions. Examples of elastic materials are metals, rubber, quartz, etc.

If a body regains its original shape and size completely and instantaneously upon removal of the deforming force, then it is said to be **perfectly elastic.**

If a body does not regain its original shape and size and retains its altered shape or size upon removal of the deforming force, it is called a **plastic body** and the property is called **plasticity.** Here, the restoring forces are not strong enough to bring the molecules back to their original positions. Examples of plastic materials are clay, putty, plasticine, thick mud, etc. There is no solid which is perfectly elastic or perfectly plastic. The best example of a near ideal elastic solid is quartz fibre and that of a plastic body is putty.

6.3 Stress and Strain:

The elastic properties of a body are described in terms of stress and strain. When a body gets deformed under an applied force, restoring forces are set up internally. They oppose change in shape or size of the body. When body is in equilibrium in its altered shape or size, deforming force and restoring force are equal and opposite.

The internal restoring force per unit area of a body is called stress.

$$\text{stress} = \frac{\text{deforming force}}{\text{area}} = \frac{|\vec{F}|}{A} \quad \text{--- (6.1)}$$

where $\vec{F}$ is internal restoring force (external applied deforming force). SI unit of stress is N m^{-2} or pascal (Pa). The dimensions of a stress are [$L^{-1} M^{1} T^{-2}$].

Strain is a measure of the deformation of a body. When two equal and opposite forces are applied to an elastic body, there is a change in the dimensions of the body, **Strain is defined as the ratio of change in dimensions of the body to its original dimensions.**

$$\text{Strain} = \frac{\text{change in dimensions}}{\text{original dimensions}} \quad \text{--- (6.2)}$$

It is the ratio of two similar quantities. Hence strain is a dimensionless physical quantity. It has no units. There are three types of stress and corresponding strains.

1: Stress produced by a deforming force acting along the length of a body or a rod is called **tensile stress or a longitudinal stress**. The strain produced is called tensile strain.

A) Tensile stress or compressive stress:

Suppose a force $\vec{F}$ is applied along the length of a wire, or perpendicular to its cross section A. This produces an elongation in the wire and the length of the wire increases accordingly, as shown in Fig. 6.1 (a).

$$\text{Tensile stress} = \frac{|\vec{F}|}{A} \quad \text{--- (6.3)}$$

When a rod is pushed at two ends with equal and opposite forces, its length decreases. The restoring force per unit area is called **compressive stress** as shown in Fig. 6.1 (b).

$$\text{Compressive stress } \frac{|\vec{F}|}{A} \quad \text{--- (6.4)}$$

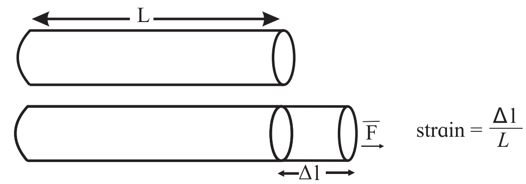

Fig. 6.1 (a): Tensile stress.

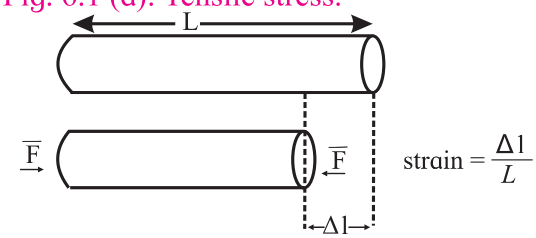

Fig. 6.1 (b): Compressive stress.

B) Tensile strain:

The strain produced by a tensile deforming force is called tensile strain or longitudinal strain or linear strain.

If L is the original length and Δl is the change in length due to the deforming force, then

$$\text{Tensile strain} = \frac{\Delta l}{L} \quad \text{--- (6.5)}$$

2 : When a deforming force acting on a body produces change in its volume, the stress is called volume stress and the strain produced is called **volume strain**.

A) Volume stress or hydraulic stress:

Let $\vec{F}$ be a force acting perpendicular to the entire surface of the body. It acts normally and uniformly all over the surface area A of the body. Such a stress which produces change in size but no change in shape is called volume stress.

$$\text{Volume stress} = \frac{|\vec{F}|}{A} \quad \text{--- (6.6)}$$

Volume stress produces change in size without change in shape of body, it is called hydraulic or hydrostatic volume stress as shown in Fig. 6.2.

B) Volume strain:

A deforming force acting perpendicular to the entire surface of a body produces a volume strain. Let V be the original volume and ΔV be the change in volume due to deforming force, then

$$\text{Volume stain} = \frac{\Delta V}{V} \quad \text{--- (6.7)}$$

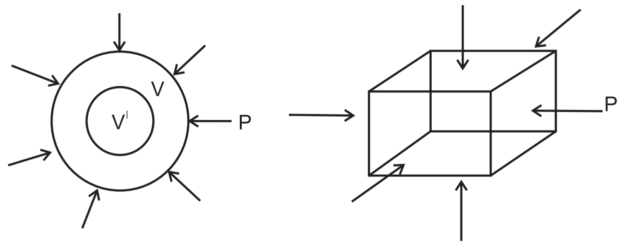

Fig. 6.2 : Volume stress.

Do you know ?

When a balloon is filled with air at high pressure, its walls experience a force from within. This is also volume stress. It tries to expand the balloon and change its size without changing shape. When the volume stress exceeds the limit of bulk elasticity, the balloon explodes. Similarly, a gas cylinder explodes when the pressure inside it exceeds the limit of bulk elasticity of its material.

A submarine when submerged under water is under volume stress.

3 : When a deforming force acting on a body produces change in the shape of a body, shearing stress and shearing strain are produced.

A) Shearing stress:

Let $\vec{F}$ be a tangential force acting on a surface area A. This force produces change in shape of the body without changing its size as shown in Fig. 6.3.

$$\text{Shearing stress} = \frac{\text{Tangential force}}{\text{Area}} \quad \text{--- (6.8)}$$

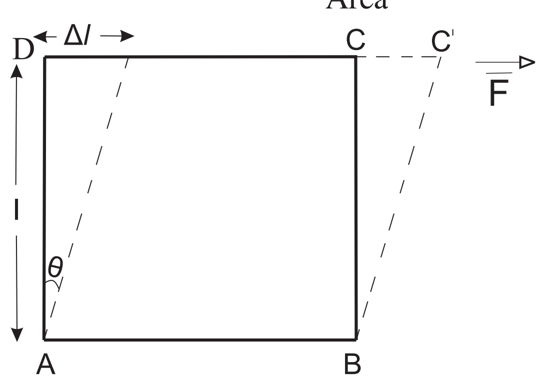

Fig. 6.3 : Tangential force produces shearing stress.

Suppose ABCD is the front face of a cube. A force $\vec{F}$ is applied to the cube so that the bottom of the cube is fixed and only the top surface is slightly displaced. Such force is called tangential force. **Tangential force is parallel to the top and the bottom surface of the block. The restoring force per unit area developed due to the applied tangential force is called shearing stress or tangential stress.**

B) Shearing strain:

There is a relative displacement, Δl, of the bottom face and the top face of the cube. Such relative displacement of two surfaces is called shear strain. It can be calculated as follows,

$$\text{Shearing strain } \frac{\Delta l}{l} = \tan\theta = \theta \quad \text{--- (6.9)}$$

when the relative displacement Δl is very small.

6.4 Hooke's Law:

Robert Hooke (1635-1703), an English physicist, studied the tension in a wire and strain produced in it. His study led to a law now known as Hooke's law.

Statement: Within elastic limit, stress is directly proportional to strain.

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

The constant is called the modulus of elasticity. **The modulus of elasticity of a material is the ratio of stress to the corresponding strain.** It is defined as the slope of the stress-strain curve in the elastic deforming region and depends on the nature of the material. The maximum value of stress up to which stress is directly proportional to strain is called the **elastic limit**. The stress-strain curve within elastic limit is shown in Fig. 6.4

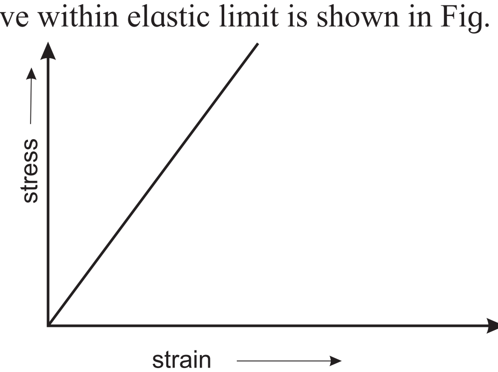

Fig 6.4: Stress versus strain graph within elastic limit for an elastic body.

6.5 Elastic modulus:

There are three types of stress and strain related to change in length, change in volume and change in shape. Hence, we have three moduli of elasticity corresponding to each type of stress and strain.

6.5.1 Young's modulus (Y):

It is the modulus of elasticity related to change in length of an object like a metal wire, rod, beam, etc., due to the applied deforming force. Hence it is also called as elasticity of length. It is named after the British physicist Thomas Young (1773-1829).

Consider a metal wire of length L having radius r suspended from a rigid support. A load Mg is attached to the free end of the wire. Due to this, deforming force is applied at the free end of the wire in downward direction. In its equilibrium position,

$$Longitudinal\ stress = \frac{Applied\ force}{Area}$$

$$= \frac{F}{A}$$

$$= \frac{Mg}{\pi r^2} \quad \text{--- (6.10)}$$

It produces a change in length of the wire. If (L+l) is the new length of wire, then l is the extension or elongation in wire.

$$Longitudinal\ strain = \frac{change\ in\ length}{original\ length}$$

$$= \frac{l}{L} \quad \text{--- (6.11)}$$

Young's modulus is the ratio of longitudinal stress to longitudinal strain.

$$Young's\ modulus = \frac{longitudinal\ stress}{longitudinal\ strain} \quad \text{-- (6.12)}$$

$$Y = \frac{\frac{Mg}{\pi r^2}}{\frac{l}{L}}$$

$$Y = \frac{MgL}{\pi r^2 l} \quad \text{---(6.13)}$$

SI unit of Young's modulus is N/m^2. Its dimensions are [$L^{-1} M^1 T^{-2}$].

Young's modulus indicates the resistance of an elastic solid to elongation or compression. Young's modulus of a material is useful for characterization of an object subjected to compression or tension. Young's modulus is the property of solids only.

Table 6.1: Young's modulus of some familiar materials

Material	Young's modulus Y $\times 10^{10}$ Pa (N/m^2)
Lead	1.5
Glass (crown)	6.0
Aluminium	7.0
Silver	7.6
Gold	8.1
Brass	9.0
Copper	11.0
Steel	21.0

Example 6.1: A brass wire of length 4.5m with crosssectional area of 3×10^{-5} m^2 and a copper wire of length 5.0 m with cross sectional area 4×10^{-5} m^2 are stretched by the same load. The same elongation is produced in both the wires. Find the ratio of Young's modulus of brass and copper.

Solution: For brass,

$L_B = 4.5$m, $A_B = 3\times10^{-5}$ m^2

$l_B = l$, $F_B = F$

$$Y_B = \frac{F_B L_B}{A_B l_B}$$

$$\therefore Y_B = \frac{F\times4.5}{3\times10^{-5}\times l}$$

For copper,

$L_C = 5$m, $A_C = 4\times10^{-5}$ m^2

$l_C = l$, $F_C = F$

$$Y_C = \frac{F_C L_C}{A_C l_C}$$

$$\therefore Y_c = \frac{F\times5.0}{4\times10^{-5}\times l}$$

$$\frac{Y_B}{Y_C} = \frac{F\times4.5}{3\times10^{-5}\times l}\times\frac{4\times10^{-5}\times l}{F\times5}$$

$$= \frac{18\times10^{-5}}{15\times10^{-5}} = 1.2$$

Example 6.2: A wire of length 20 m and area of cross section 1.25×10^{-4} m^2 is subjected to a load of 2.5 kg. (1 kgwt = 9.8 N). The elongation produced in wire is 1×10^{-4}m. Calculate Young's modulus of the material.

Solution: Given,

L = 20 m

A = 1.25×10^{-4} m^2

$F = mg = 2.5 \times 9.8N$
$L = 10^{-4}$ m
To find: Y

$$Y = \frac{FL}{Al} = \frac{2.5 \times 9.8 \times 20}{1.25 \times 10^{-4} \times 10^{-4}}$$

$$= 3.92 \times 10^{10}\ N\ m^{-2}$$

6.5.2 Bulk modulus (K):

It is the modulus of elasticity related to change in volume of an object due to applied deforming force. Hence it is also called as elasticity of volume. Bulk modulus of elasticity is a property of solids, liquids and gases.

If a sphere made from rubber is completely immersed in a liquid, it will be uniformly compressed from all sides. Suppose this compressive force is F. Let the change in pressure on the sphere be dP and let the change in its volume be dV. If the original volume of the sphere is V, then volume strain is defined as

$$\text{Volume strain} = \frac{\text{change in volume}}{\text{original volume}}$$

$$= -\frac{dV}{V} \qquad \text{--- (6.14)}$$

The negative sign indicates that there is a decrease in volume. The magnitude of the volume strain is $\frac{dV}{V}$

Bulk modulus is defined as the ratio of volume stress to volume strain.

$$\text{Bulk modulus} = \frac{\text{volume stress}}{\text{volume strain}} = K$$

$$K = \frac{dP}{\left(\frac{dV}{V}\right)} = V\frac{dP}{dV} \qquad \text{--- (6.17)}$$

SI unit of bulk modulus is N/m^2. Dimensions of K are [$L^{-1} M^1 T^{-2}$].

Table 6.2 gives bulk moduli of some familiar materials

Bulk modulus measures the resistance offered by gases, liquids or solids while an attempt is made to change their volume.

The reciprocal of bulk modulus of elasticity is called compressibility of the material.

$$\text{Compressibility} = \frac{1}{\text{Bulk modulus}} \qquad \text{--- (6.18)}$$

Compressibility is the fractional decrease in volume, $-\Delta V/V$ per unit increase in pressure. SI unit of compressibility is m^2/N or Pa^{-1} and its dimensions are [$L^1 M^{-1} T^2$].

Do you know ?

The bulk modules of water is 2.18×10^8 Pa and its compressibility is 45.8×10^{-10} Pa^{-1}. Materials with small bulk modulus and large compressibility are easier to compress.

Example 6.3: A metal cube of side 1m is subjected to a force. The force acts normally on the whole surface of cube and its volume changes by 1.5×10^{-5} m^3. The bulk modulus of metal is 6.6×10^{10} N/m^2. Calculate the change in pressure.

Solution: Given,
volume of cube $= V = l^3 = (1)^3 = 1m^3$
Change in volume $= dV = 1.5 \times 10^{-5}$ m^3
Bulk modulus $= K = 6.6 \times 10^{10}$ N/m^2.
To find: Change in pressure dP

$$K = V\frac{dP}{dV}$$

$$dP = K\frac{dV}{V}$$

$$dP = \frac{6.6 \times 10^{10} \times 1.5 \times 10^{-5}}{1}$$

$dP = 9.9 \times 10^5$ N/m^2.

Table 6.2: Bulk modulus of some familiar materials

Material	Bulk modulus K $\times 10^{10}$ Pa (N/m^2)
Lead	4.1
Brass	6.0
Glass (crown)	6.0
Aluminium	7.5
Silver	10.0
Copper	14.0
Steel	16.0
Gold	18.0

6.5.3 Modulus of rigidity (η):

The modulus of elasticity related to change in shape of an object is called rigidity modulus. It is the property of solids only as they alone possess a definite shape.

The block shown in Fig. 6.5 is made of a uniform isotropic material. It has a uniform crosssection area A and height l. A cross section of the block is defined as any plane parallel to the top and the bottom surface and cuts the block. Two forces of magnitude 'F' are applied along top and bottom surface as shown in Fig. (6.5). They constitute a couple. The upper surface is displaced relative to the lower surface by a small distance Δl and corresponding angles change by a small amount $\theta = \Delta l/l$.

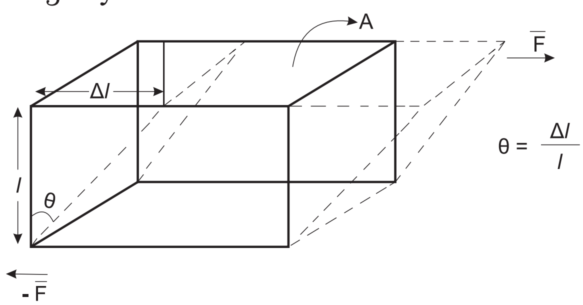

Fig. 6.5: Modulus of rigidity, tangential force F and shear strin θ.

A couple is applied by pushing the top and the bottom surfaces as shown in Fig. 6.5. Similar couple would be applied if the bottom of the block is fixed and only the top is pushed.

The forces $\vec{F}$ and - $\vec{F}$ are parallel to the cross section. This is different than the tensile stress where the force is normal to the cross section.

As a result of the way in which the forces are applied the block is subjected to a shear stress defined by **shear stress = *F/A***.

The SI unit of shear stress is N/m^2 or Pa. The block is distorted as a result of the shear stress. The top and bottom surface are relatively displaced by a small distance Δl. The corner angle changes by a small amount θ which is called shear strain and is expressed in radian. Shear strain 'θ' is given by $\theta = \Delta l/l$, (for small Δl).

Shear modulus or modulus of rigidity: It is defined as the ratio of shear stress to shear strain within elastic limits.

$$\eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} \quad \text{--- (6.17)}$$

Table 6.3 gives values of rigidity modulus η of some familiar materials.

Table 6.3: Rigidity modulus η of some familiar materials

Material	Rigidity modulus η $\times 10^{10}$ Pa (N/m^2)
Lead	0.6
Aluminium	2.5
Glass (crown)	2.5
Silver	2.7
Gold	2.9
Brass	3.5
Copper	4.4
Steel	8.3

Rigidity modulus indicates the resistance offered by a solid to change in its shape.

Example 6.4: Calculate the modulus of rigidity of a metal, if a metal cube of side 40 cm is subjected to a shearing force of 2000 N. The upper surface is displaced through 0.5cm with respect to the bottom. Calculate the modulus of rigidity of the metal.

Solution: Given,

Length of side of cube = l= 40 cm = 0.40 m

Shearing force = F= 2000 N = 2×10^3 N

Displacement of top face = Δl = 0.5cm = 0.005m

Area = A = l^2 = $0{\cdot}16\text{m}^2$

To find: modulus of rigidity, η

$$\eta = \frac{F}{A\theta}$$

$$\theta = \frac{\Delta l}{l} = \frac{0.005}{0.40} = 0.0125$$

$$\eta = \frac{2.0\times10^3\,\text{N}}{(0.16\text{m}^2)\cdot(0.0125)}$$

$$= 1.0\times10^6\,\text{N}/\text{m}^2$$

6.5.4 Poisson's ratio:

Suppose a wire is fixed at one end and a force is applied at its free end so that the wire gets stretched. Length of the wire increases and at the same time, its diameter decreases, i.e., the wire becomes longer and thinner as shown in Fig. 6.6 (a).

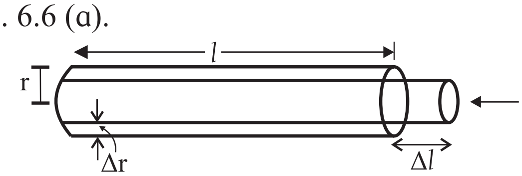

Fig. 6.6 (a): When a wire is stretched its length increases and its diameter decreases.

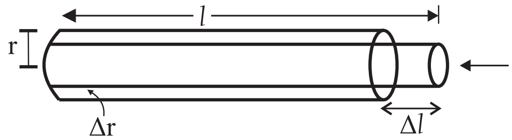

Fig. 6.6 (b): When a wire is compressed its length increases and its diameter increases.

If equal and opposite forces are applied to an object along its length inwards, the object gets compressed (Fig. 6.6 (b)). There is a decrease in dimensions along its length and at the same time there is an increase in its dimensions perpendicular to its length. When length of the wire decreases, its diameter increases.

The ratio of change in dimensions to original dimensions in the direction of the applied force is called **linear strain** while the ratio of change in dimensions to original dimensions in a direction perpendicular to the applied force is called **lateral strain.** Within elastic limit, the ratio of lateral strain to the linear strain is called the **Poisson's ratio**.

If l is the original length of wire, Δl is increase/decrease in length of wire, D is the original diameter and d is corresponding change in diameter of wire then, Poisson's ratio is given by

$$\sigma = \frac{Lateral\ strain}{Linear\ strain} = \frac{d/D}{l/L} = \frac{d.L}{D.l} \quad \text{--- (6.18)}$$

Poisson's ratio has no unit. It is dimensionless. Table 6.4 gives values of Poisson ratio, σ, of some familiar materials.

Table 6.4: Poisson ratio, σ, of some familiar materials

Material	Poisson ratio σ
Glass (crown)	0.2
Steel	0.28
Aluminium	0.36
Brass	0.37
Copper	0.37
Silver	0.38
Gold	0.42

Do you know ?

For most of the commonly used metals, the value of σ is between 0.25 and 0.35. Many times we assume that volume is constant while stretching a wire. However, in reality, its volume also increases. Using approximations it can be shown that $\sigma_{max} \approx 0.5$ if volume is unchanged. In practice, it is much less. This shows that volume also increases while stretching.

6.6 Stress-Strain Curve:

Suppose a metal wire is suspended vertically from a rigid support and stretched by applying load to its lower end. The load is gradually increased in small steps until the wire breaks. The elongation produced in the wire is measured during each step. Stress and strain is noted for each load and a graph is drawn by taking tensile strain along x-axis and tensile stress along y-axis. It is a stress-strain curve as shown in Fig. 6.7.

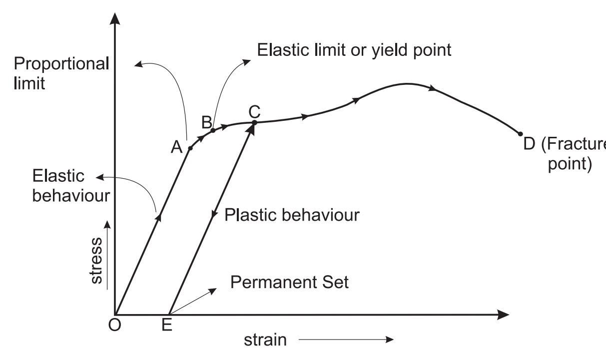

Fig. 6.7 : stress-strain curve.

The initial part of the graph is a straight line OA. This is the region in which Hooke's law is obeyed and stress is directly proportional to strain. The straight line portion ends at A. The stress at this point is called **proportional limit**. If the load is further increased till point B is reached, stress and strain are no longer proportional and Hooke's law is not valid. If the load is gradually removed starting at any point between O and B. The curve is retraced until the wire regains its original length. The change is reversible. The material of the wire shows elastic behaviour in the region OB. Point B is called the **yield point**. The corresponding point is called the **elastic limit**.

When the stress is increased beyond point B, the strain continues to increase. If the load is removed at any point beyond B, C for example, the material does not regain its original length. It follows the line CE. Length of the wire when there is no stress is greater than the original length. The deformation is irreversible and the material has acquired a **permanent set**.

Further increase in load causes a large increase in strain for relatively small increase in stress, until a point D is reached at which **fracture** takes place.

The material shows **plastic flow or plastic deformation** from point B to point D. The material does not regain its original state when the stress is removed. **The deformation is called plastic deformation.**

The curve described above shows all the possibilities for an elastic substance. In particular, many metallic wires (copper, aluminum, silver, etc) exhibit this type of behavior. However, majority of materials in every day life exhibit only some part of it.

Materials such as glass, ceramics, etc., break within the elastic limit. They are called **brittle**.

Metals such as copper, aluminum, wrought iron, etc. have large plastic range of extension. They lengthen considerably and undergo plastic deformation till they break. They are called **ductile**.

Metals such as gold, silver which can be hammered into thin sheets are called **malleable**.

Rubber has large elastic region. It can be stretched so that its length becomes many times its original length, after removal of the stress it returns to its original state but the stress strain curve is not a straight line. A material that can be elastically stretched to a larger value of strain is called an **elastomer**.

In case of some materials like vulcanized rubber, when the stress applied on a body decreases to zero, the strain does not return to zero immediately. The strain lags behind the stress. This lagging of strain behind the stress is called **elastic hysteresis**. Figure 6.8 shows the stress-strain curve for increasing and decreasing load. It encloses a loop. Area of loop gives the energy dissipated during deformation of a material.

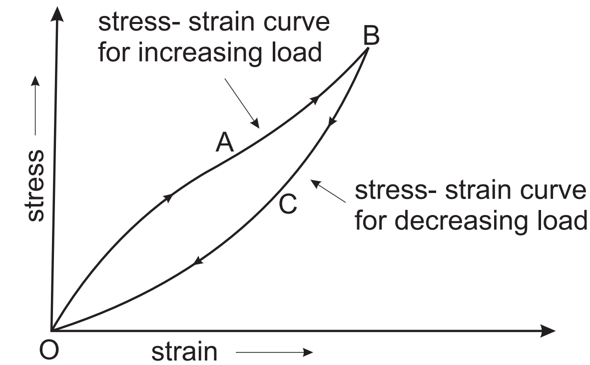

Fig. 6.8: Stress-stain curve for increasing and decreasing load.

Can you tell?

Why does a rubber band become loose after repeated use?

6.7 Strain Energy:

The elastic potential energy gained by a wire during elongation by a stretching force is called as strain energy.

Consider a wire of original length L and cross sectional area A stretched by a force F acting along its length. The wire gets stretched and elongation l is produced in it. The stress and the strain increase proportionately.

Longitudinal stress = $\frac{F}{A}$

Longitudinal strain = $\frac{l}{L}$

Young's modulus = $\frac{longitudinal\ stress}{longitudinal\ strain}$

$$Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{l}{L}\right)} = \frac{FL}{Al}$$

$$\therefore F = \frac{YAl}{L} \qquad \text{--- (6.19)}$$

The magnitude of stretching force increases from zero to F during elongation of wire. At a certain stage, let 'f' be the force applied and 'x' be the corresponding extension. The force at this stage is given by Eq. (6.19) as

$$f = \frac{YAx}{L}$$

For further extension dx in the wire, the work done is given by

Work = (force)·(displacement).

dW = f dx

$$\therefore dW = \frac{YAx}{L}dx$$

When the wire gets stretched from x = 0 to x = l, the total work done is given as

$$W = \int_0^l dW$$

$$\therefore W = \int_0^l \frac{YAx}{L}dx$$

$$\therefore W = \frac{YA}{L}\int_0^l x dx$$

$$\therefore W = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^l$$

$$\therefore W = \frac{YA}{L}\left[\frac{l^2}{2} - \frac{0^2}{2}\right]$$

$$W = \frac{YAl^2}{2L}$$

$$W = \frac{1}{2}\frac{YAl}{L}l$$

$$W = \frac{1}{2}Fl$$

Work done = $\frac{1}{2}$ (load)·(extension) --- (6.20)

This work done by stretching force is equal to energy gained by the wire. This energy is strain energy.

Strain energy = $\frac{1}{2}$ (load)·(extension) --- (6.21)

Strain energy per unit volume can be obtained by using Eq. (6.20) and various formula of stress, strain and young's modulus.

Work done per unit volume

$$= \frac{work\ done\ in\ streching\ wire}{volume\ of\ wire.}$$

$$= \frac{1}{2}\frac{F.l}{A.L}$$

$$= \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{l}{L}\right)$$

Work done per unit volume

$$= \frac{1}{2}\ (\text{stress})\cdot(\text{strain})$$

Strain energy per unit volume

$$= \frac{1}{2}\ (\text{stress})\cdot(\text{strain}) \qquad \text{--- (6.22)}$$

As $Y = \frac{\text{stress}}{\text{strain}}$,

Stress = Y· (strain) and

$$\text{strain} = \frac{\text{stress}}{Y}$$

∴ Strain energy per unit volume

$$= \frac{1}{2} Y\cdot(\text{strain})^2 \qquad \text{--- (6.23)}$$

Also, strain energy per unit volume

$$= \frac{1}{2}\frac{(\text{stress})^2}{Y} \qquad \text{--- (6.24)}$$

Thus Eq. (6.22), (6.23) and (6.24) give strain energy per unit volume in various forms.

6.8 Hardness:

Hardness is the property of a material which enables it to resist plastic deformation. Hard materials have little ductility and they are brittle to some extent. **The term hardness also refers to stiffness or resistance to bending, scratching abrasion or cutting**. It is the property of a material which gives it the ability to resist permanent deformation when a load is applied to it. **The greater the hardness, greater is the resistance to deformation**.

The most well-known example of the hard materials is diamond. It is incredibly difficult to scratch a diamond. Metal with very low hardness is aluminium.

Hardness of material is different from its strength and toughness.

If a force is applied to a body it produces deformation in it. Higher is the force required for deformation, the stronger is the material, i.e., the material has more **strength**.

Steel has high strength whereas plasticine clay is not strong because it gets easily deformed even by a small force.

Toughness is the ability of a material to resist fracturing when a force is applied to it. Plasticine clay is relatively tough as it can be stretched and deformed due to applied force without breaking.

A single material may be hard, strong and tough, e.g.,

1) Bulletproof glass is hard and tough but not strong.
2) Drill bits must be hard, strong and tough for their work.
3) Anvils are very tough and strong but they are not hard.

6.9 Friction in Solids:

Whenever the surface of one body slides over another, each body exerts a certain amount of force on the other body. These forces are tangential to the surfaces. The force on each body is opposite to the direction of motion between the two bodies. It prevents or opposes the relative motion between the two bodies. It is a common experience that an object placed on any surface does not move easily when a small force is applied to it. This is because of certain force of opposition acting between the surface of the object and the surface on which it is placed. Even a rolling ball comes to rest after covering a finite distance on playground because of such opposing force. Our foot ware is provided with designs at the bottom of its sole so as to produce force of opposition to avoid slipping. It is difficult to walk without such opposing force. You know what happens when you try to walk fast on polished flooring at home with soap water spread on it. There is a possibility of slipping due to lack of force of opposition. To initiate any motion between a pair of surfaces, we need a certain minimum force. Also after the motion begins, it is constantly opposed by some natural force. This mechanical force between two solid surfaces in contact with each other is called as frictional force. **The property which resists the relative motion between two surfaces in contact is called friction.**

In some cases it is necessary to avoid friction, because friction causes dissipation of energy in machines due to which efficiency of machines decreases. In such cases friction should be reduced by using polished surfaces, lubricants, etc. Relative motion between solids and fluids (i.e. liquids and gases) is also naturally opposed by friction, e.g., a boat on the surface of water experiences opposition to its motion. In this section we are going to study friction in solids only.

6.9.1 Origin of friction:

If smooth surfaces are observed under powerful microscope, many irregularities and projections are observed. Friction arises due to interlocking of these irregularities between two surfaces in contact. The surfaces can be made extremely smooth by polishing to avoid irregularities but it is noticed that in this case also, friction does not decrease but may increase. Hence the interlocking of irregularities is not the real cause of friction.

According to modern theory, cause of friction is the force of attraction between molecules of two surfaces in actual contact in addition to the force due to the interlocking between the two surfaces. When one body is in contact with another body, the real microscopic area in contact is very small due to irregularities in contact. Figure 6.9 shows the microscopic view of two polished surfaces in contact.

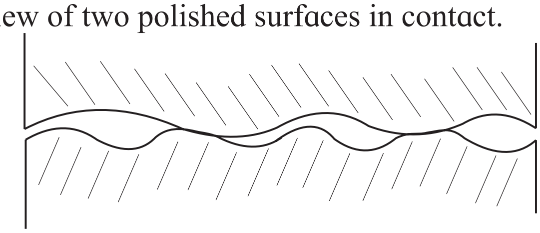

Fig. 6.9: Microscopic view of polished surfaces in contact.

Due to small area, pressure at points of contact is very high. Hence there is a strong force of attraction between the surfaces in contact. If both the surfaces are of the same material the force of attraction is called **cohesive force** while if the surfaces are of different materials the force of attraction is called **adhesive force**. When the surfaces in contact become more and more smooth, the actual area of contact goes on increasing. Due to this, the force of attraction between the molecules increases and hence the friction also increases. Putting some grease or other lubricant (a different material) between the two surfaces reduces the friction.

6.9.2 Types of friction:

1. Static friction:

Suppose a wooden block is placed on a horizontal surface as shown in Fig 6.10. A small horizontal force F is applied to it. The block does not move with this force as it cannot overcome the frictional force between the block and horizontal surface. In this case, the force of static friction is equal to F and balances it. **The frictional force which balances applied force when the body is static is called force of static friction, F_s. In other words, static friction prevents sliding motion.**

If we keep increasing $\underline{F}$, a stage will come when, for $\underline{F} = \underline{F}_{max}$, the object will start moving. For $\underline{F} < \underline{F}_{max}$, the force of static friction is equal to $\underline{F}$. For $\underline{F} \geq \underline{F}_{max}$, the kinetic friction comes into play. Static friction opposes impending motion i.e. the motion that would take place in absence of frictional force under the applied force.

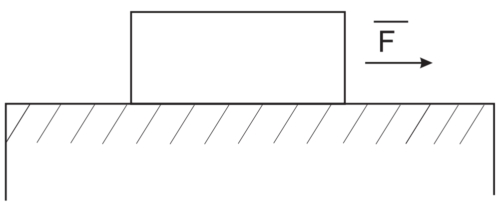

Fig. 6.10: Static friction.

The force of static friction is self adjusting force. When the applied force $\underline{F}$ is very small, the block remains at rest. Here the force of friction is also small. When $\underline{F}$ is increased by a small value, the block remains still at rest as the force of friction is increased to balance the applied force. If applied force is increased, the friction also increases and reaches the maximum value. Just before the body starts sliding over another body, the value of frictional force is maximum, it is called **limiting force of friction,** F_L. If the direction of applied force is reversed, the direction of static friction is also reversed, i.e., it adjusts its direction also.

Laws of static friction:

1] The limiting force of static friction is directly proportional to the normal reaction (N) between the two surfaces in contact.

$$F_L \propto N$$

$$\therefore F_L = \mu_s N \qquad \text{--- (6.25)}$$

Where μ_s is constant of proportionality. It is called as **coefficient of static friction**.

$$\therefore \mu_s = \frac{F_L}{N} \qquad \text{--- (6.26)}$$

The coefficient of static friction is defined as the ratio of limiting force of friction to the normal reaction. Table 6.4 gives the coefficient of static fiction for some materials.

2] The limiting force of friction is independent of the apparent area between the surfaces in contact, so long as the normal reaction remains the same.

3] The limiting force of friction depends upon materials in contact and the nature of their surfaces.

Table 6.4: Coefficient of static friction

Material	Coefficient of static friction μ_s
Teflon on Teflon	0.4
Brass on steel	0.51
Copper on steel	0.53
Aluminium on steel	0.61
Steel on steel	0.74
Glass on glass	0.94
Rubber on concrete (dry)	1.0

Example 6.5: The coefficient of static friction between a block of mass 0.25 kg and a horizontal surface is 0.4. Find the horizontal force applied to it.

Solution: Given,

$\mu_s = 0.4$

$m = 0.25$ kg

To find: Force

$F = \mu_s \cdot N = \mu_s \cdot (mg)$

$F = 0.4 \times 0.25 \times 9.8$

$F = 0.98$ N

2. Kinetic friction :

Once the sliding of block on the surface starts, the force of friction decreases. The force required to keep the body sliding steadily is thus less than the force required to just start its sliding. The force of friction that comes into play when a body is in steady state of motion over another surface is called force of kinetic friction.

Friction between two surfaces in contact when one body is actually sliding over the other body, is called kinetic friction or dynamic friction.

Laws of kinetic friction :

1. The force of kinetic friction (F_k) is directly proportional to the normal reaction between two surfaces in contact.

$$\therefore F_k \propto N$$

$$\therefore F_k = \mu_k N \qquad \text{--- (6.27)}$$

Where μ_k is constant of proportionality. It is called as coefficient of kinetic friction.

$$\therefore \mu_k = \frac{F_k}{N} \qquad \text{--- (6.28)}$$

The coefficient of kinetic friction is defined as the ratio of force of kinetic friction to the normal reaction between the two surfaces in contact. Table 6.5 gives the co-efficient of kinetic friction for some materials.

2. Force of kinetic friction is independent of shape and apparent area of the surfaces in contact.
3. Force of kinetic friction depends upon the nature and material of the surfaces in contact.
4. The magnitude of the force of kinetic friction is independent of the relative velocity between the object and the surface provided that the relative velocity is neither too large nor too small.

Table 6.5: Coefficient of kinetic friction

Material	Coefficient of kinetic friction μ_k
Rubber on concrete (dry)	0.25
Glass on glass	0.40
Brass on steel	0.40
Copper on steel	0.44
Aluminium on steel	0.47
Steel on steel	0.57
Teflon on Teflon	0.80

3 Rolling friction :

Motion of a body over a surface is said to be rolling motion if the point of contact of the body with the surface keeps changing continuously.

Friction between two bodies in contact when one body is rolling over the other, is called rolling friction.

For same pair of surfaces, the force of static friction is greater than the force of kinetic friction while the force of kinetic friction is greater than force of rolling friction. As rolling friction is the minimum, ball bearings are used to reduce friction in parts of machines to increase its efficiency.

Advantages of friction:

Friction is necessary in our daily life.

- We can walk due to friction between ground and feet.
- We can hold object in hand due to static friction.
- Brakes of vehicles work due to friction; hence we can reduce speed or stop vehicles.
- Climbing on a tree is possible due to friction.

Disadvantages of friction

- Friction opposes motion.
- Friction produces heat in different parts of machines. It also produces noise.
- Automobile engines consume more fuel due to friction.

Methods of reducing friction

- Use of lubricants, oil and grease in different parts of a machine.
- Use of ball bearings converts kinetic friction into rolling friction.

Can you tell?

1) It is difficult to run fast on sand.
2) It is easy to roll than pull a barrel along a road.
3) An inflated tyre rolls easily than a flat tyre.
4) Friction is a necessary evil.

Internet my friend

1. https://opentextbc.ca>chapter>friction.
2. https://www.livescience.com
3. https://www.khanacdemy.org.physics
4. https://courses.lumenlearning.com> elastiscitychapter>elasticity
5. https://www.toper.com>guides>physics

Exercises

1. Choose the correct answer:

i) Change in dimensions is known as…..

(A) deformation (B) formation

(C) contraction (D) strain.

ii) The point on stress-strain curve at which strain begins to increase even without increase in stress is called….

(A) elastic point (B) yield point

(C) breaking point (D) neck point

iii) Strain energy of a stretched wire is 18×10^{-3} J and strain energy per unit volume of the same wire and same cross section is 6×10^{-3} J/m^3. Its volume will be....

(A) 3cm^3 (B) 3 m^3

(C) 6 m^3 (D) 6 cm^3

iv) ----- is the property of a material which enables it to resist plastic deformation.

(A) elasticity (B) plasticity

(C) hardness (D) ductility

v) The ability of a material to resist fracturing when a force is applied to it, is called……

(A) toughness (B) hardness

(C) elasticity (D) plasticity.

2. Answer in one sentence:

i) Define elasticity.

ii) What do you mean by deformation?

iii) State the SI unit and dimensions of stress.

iv) Define strain.

v) What is Young's modulus of a rigid body?

vi) Why bridges are unsafe after a very long use?

vii) How should be a force applied on a body to produce shearing stress?

viii) State the conditions under which Hooke's law holds good.

ix) Define Poisson's ratio.

x) What is an elastomer?

xi) What do you mean by elastic hysteresis?

xii) State the names of the hardest material and the softest material.

xiii) Define friction.

xiv) Why force of static friction is known as 'self-adjusting force'?

xv) Name two factors on which the co-efficient of friction depends.

3. Answer in short:

i) Distinguish between elasticity and plasticity.

ii) State any four methods to reduce friction.

iii) What is rolling friction? How does it arise?

iv) Explain how lubricants help in reducing friction?

v) State the laws of static friction.

vi) State the laws of kinetic friction.

vii) State advantages of friction.

viii) State disadvantages of friction.

ix) What do you mean by a brittle substance? Give any two examples.

4. Long answer type questions:

i) Distinguish between Young's modulus, bulk modulus and modulus of rigidity.

ii) Define stress and strain. What are their different types?

iii) What is Young's modulus? Describe an experiment to find out Young's modulus of material in the form of a long straight wire.

iv) Derive an expression for strain energy per unit volume of the material of a wire.

v) What is friction? Define coefficient of static friction and coefficient of kinetic friction. Give the necessary formula for each.

vi) State Hooke's law. Draw a labeled graph of tensile stress against tensile strain for a metal wire up to the breaking point. In this graph show the region in which Hooke's law is obeyed.

5. Answer the following

i) Calculate the coefficient of static friction for an object of mass 50 kg placed on horizontal table pulled by attaching a spring balance. The force is increased gradually it is observed that the object just moves when spring balance shows 50N.

[Ans: $\mu_s = 0.102$]

ii) A block of mass 37 kg rests on a rough horizontal plane having coefficient of static friction 0.3. Find out the least force required to just move the block horizontally.

[Ans: $F_s = 108.8$N]

iii) A body of mass 37 kg rests on a rough horizontal surface. The minimum horizontal force required to just start the motion is 68.5 N. In order to keep the body moving with constant velocity, a force of 43 N is needed. What is the value of a) coefficient of static friction? and b) coefficient of kinetic friction?

[Ans: a) $\mu_{s=}$ 0.188
b) $\mu_k = 0.118$]

iv) A wire gets stretched by 4mm due to a certain load. If the same load is applied to a wire of same material with half the length and double the diameter of the first wire. What will be the change in its length?

[Ans: 0.5mm]

v) Calculate the work done in stretching a steel wire of length 2m and cross sectional area 0.0225mm^2 when a load of 100 N is slowly applied to its free end. [Young's modulus of steel= 2×10^{11} N/m^2]

[Ans: 2.222J]

vi) A solid metal sphere of volume 0.31m^3 is dropped in an ocean where water pressure is 2×10^7 N/m^2. Calculate change in volume of the sphere if bulk modulus of the metal is 6.1×10^{10} N/m^2

[Ans: 10^{-4} m^3]

vii) A wire of mild steel has initial length 1.5 m and diameter 0.60 mm is extended by 6.3 mm when a certain force is applied to it. If Young's modulus of mild steel is 2.1 x 10^{11} N/m^2, calculate the force applied.

[Ans: 250 N]

viii) A composite wire is prepared by joining a tungsten wire and steel wire end to end. Both the wires are of the same length and the same area of cross section. If this composite wire is suspended to a rigid support and a force is applied to its free end, it gets extended by 3.25mm. Calculate the increase in length of tungsten wire and steel wire separately.

[Given: $Y_{steel} = 2 \times 10^{11}$N/m^2,
$Y_{Tungsten} = 3.40 \times 10^8$ N/m^2]

[Ans: extension in tungsten wire = 3.244 mm,
extension in steel wire = 0.0052 mm]

ix) A steel wire having cross sectional area 1.2 mm^2 is stretched by a force of 120 N. If a lateral strain of 1.455 mm is produced in the wire, calculate the Poisson's ratio.

[Given: $Y_{steel} = 2 \times 10^{11}$N/m^2]

[Ans: 0.291]

x) A telephone wire 125m long and 1mm in radius is stretched to a length 125.25m when a force of 800N is applied. What is the value of Young's modulus for material of wire?

[Ans: 1.27×10^{11}N/m^2]

xi) A rubber band originally 30cm long is stretched to a length of 32cm by certain load. What is the strain produced?

[Ans: 6.667×10^{-2}]

xii) What is the stress in a wire which is 50m long and 0.01cm^2 in cross section, if the wire bears a load of 100kg?

[Ans: 9.8×10^8 N/m^2]

xiii) What is the strain in a cable of original length 50m whose length increases by 2.5cm when a load is lifted?

[Ans: 5×10^{-4}]

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