12. Magnetism

Can you recall?

1. What is a bar magnet?
2. What are the magnetic lines of force?
3. What are the rules concerning the lines of force?
4. If you freely hang a bar magnetic horizontally, in which direction will it become stable?

12.1 Introduction:

The history of magnetism dates back to earlier than 600 B.C., but it is only in the twentieth century that scientists began to understand it and developed technologies based on this understanding. William Gilbert (1544-1603) was the first to systematically investigate the phenomenon of magnetism using scientific method. He also discovered that Earth is a weak magnet. Danish physicist Hans Oersted (1777-1851) suggested a link between electricity and magnetism. James Clerk Maxwell (1831-1879) proved that electricity and magnetism represent different aspects of the same fundamental force field.

In electrostatics you have learnt about the relationship between the electric field and force due to electric charges and electric dipoles. Analogous concepts exist in magnetism except that magnetic poles do not exist in isolation, and we always have a magnetic dipole or a quadrupole. In this Chapter the main focus will be on elementary aspects of magnetism and terrestrial magnetism.

Do you know ?

Some commonly known facts about magnetism.

(i) Every magnet regardless of its size and shape has two poles called north pole and south pole.

(ii) If a magnet is broken into two or more pieces then each piece behaves like an independent magnet with somewhat weaker magnetic field.
Thus isolated magnetic monopoles do not exist. The search for magnetic monopoles is still going on.

(iii) Like magnetic poles repel each other, whereas unlike poles attract each other.

(iv) When a bar magnet/ magnetic needle is suspended freely or is pivoted, it aligns itself in geographically North-South direction.

12.2 Magnetic Lines of Force and Magnetic Field:

You have studied properties of electric lines of force earlier in the Chapter on electrostatics. In a similar manner, magnetic lines of force originate from the north pole and end at the south pole of a bar magnet. The magnetic lines of force of a magnet have the following properties:

i) The magnetic lines of force of a magnet or a solenoid form closed loops. This is in contrast to the case of an electric dipole, where the electric lines of force originate from the positive charge and end on the negative charge, without forming a complete loop (see Fig. 12.4).

ii) The direction of the net magnetic field $\vec{B}$ at a point is given by the tangent to the magnetic line of force at that point in the direction of line of force.

iii) The number of lines of force crossing per unit area decides the magnitude of the magnetic field $\vec{B}$.

iv) The magnetic lines of force do not intersect. This is because had they intersected, the direction of magnetic field would not be unique at that point.

Try this

You can take a bar magnet and a small compass needle. Place the bar magnet at a fixed position on a paper and place the needle at various positions. Noting the orientation of the needle, the magnetic field direction at various locations can be traced.

Density of lines of force i.e., the number of lines of force per unit area normal to the surface around a particular point determines the strength of the magnetic field at that point. The number of lines of force is called magnetic flux (ϕ). SI unit of magnetic flux (ϕ) is weber (Wb). For a specific case of uniform magnetic field which is normal to the finite area *A,* the magnitude of magnetic field strength *B* at a point in the area *A* is given by

$$\text{Magnetic Field} = \frac{\text{magnetic flux}}{\text{area}}$$

$$\text{i.e.} \quad B = \frac{\phi}{A} \qquad \text{--- (12.1)}$$

SI unit of magnetic field (*B*) is expressed as weber/m^2 or Tesla.

1 Tesla = 10^4 Gauss.

However, magnetic lines are only a crude way of representing magnetic field. It is a pictorial representation of the strength of the magnetic field (*B*). It is better defined in terms of Lorentz force law which you will learn in std XII.

12.3 The Bar magnet:

A bar magnet is said to have magnetic pole strength $+q_m$ and - q_m at the north and south poles, respectively. The separation of magnetic poles inside the magnet is 2*l*. As the bar magnet has two poles, with equal and opposite pole strength, it is called a magnetic dipole. This is analogous to an electric dipole. The magnetic dipole moment, therefore, becomes $\vec{m} = q_m.2\vec{l}$ ($2\vec{l}$ is a vector from south pole to north pole) in analogy with the electric dipole moment.

SI unit of pole strength (q_m) is A m.

SI unit of magnetic dipole moment *m* is A m^2.

Axis:- It is the line passing through both the poles of a bar magnet. Obviously, there is only one axis for a given bar magnet.

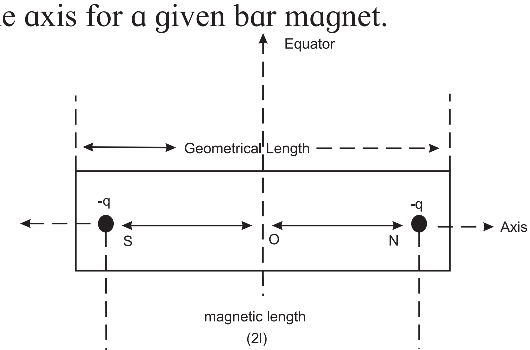


Fig. 12.1: Bar magnet

Equator:- A line passing through the centre of a magnet and perpendicular to its axis is called magnetic equator. The plane containing all equators is called the equatorial plane. The locus of points, on the equatorial plane, which are equidistant from the centre of the magnet is called the equatorial circle. The popularly known 'equator' in Geography is actually an 'equatorial circle'. Such a circle with any diameter is an equator.

Magnetic length (2*l*):- It is the distance between the two poles of a magnet.

$$\text{Magnetic length (2l)} = \frac{5}{6} \times \text{Geometric length} \qquad \text{--- (12.2)}$$

12.3.1 Magnetic field due to a bar magnet at a point along its axis and at a point along its equator:

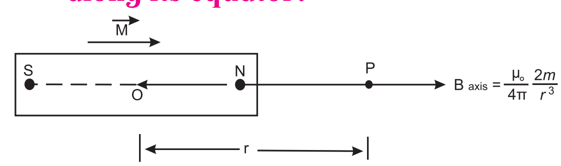


Fig. 12.2 (a): Magnetic field at a point along the axis of the magnet.

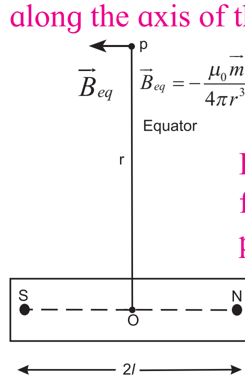


Fig. 12.2 (b): Magnetic field along the equatorial point.

Consider a bar magnet of dipole length 2*l* and magnetic dipole moment $\vec{m}$ as shown in Fig. 12.2 (a). We will now find magnetic field at a point P along the axis of the bar magnet.

Let r be the distance of point P from the centre O of the magnetic dipole.

OS = ON = *l*

$$\therefore \; NP = SP = \sqrt{\left(r^2 + l^2\right)}$$

We now use the electrostatic analogy to obtain the magnetic field due to a bar magnet at a large distance $r >> l$. Consider the electric field due to an electric dipole with a dipole moment *p*.

The Electrostatic Analogue:

As suggested by Maxwell, electricity and magnetism could be studied analogously. The pole strength (q_m) in magnetism can be compared with charge q in electrostatics. Accordingly, we can write the equivalent physical quantities in electrostatics and magnetism as shown in table 12.1.

Table 12.1: The Electrostatic Analogue

Quantity	Electrostatics	Magnetism
Basic physical quantity	Electrostatic charge	Magnetic pole
Field	Electric Field $\vec{E}$	Magnetic Field $\vec{B}$
Constant	$\frac{1}{4\pi\varepsilon_0}$	$\frac{\mu_0}{4\pi}$
Dipole moment	$\vec{p} = q\,(2\vec{l}\,)$ along (-ve) ⟶ (+ve) charge	$\vec{m} = q_m\,(2\vec{l}\,)$ (bar magnet) along S ⟶ N pole
Force	$\vec{F} = q\vec{E}$	$\vec{F} = q_m \vec{B}$
Energy (In external field) of a dipole	$U = -\ \vec{p}.\vec{E}$	$U = -\vec{m}.\vec{B}$
Coulomb's law	$F = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1q_2}{r^2}$	No analogous law as magnetic monopoles do not exist
Axial field for a short dipole	$\frac{2p}{4\pi\varepsilon_0 r^3}$ along $\vec{p}$	$\frac{\mu_0 2\vec{m}}{4\pi r^3}$
Equatorial field for a short dipole	$\frac{p}{4\pi\varepsilon_0 r^3}$ opposite to $\vec{p}$	$\frac{-\mu_0\vec{m}}{4\pi r^3}$

You have studied the electric field due to an electric dipole of length $2l$ (p=2ql) at a distance r along the dipolar axis (Eq. 10.24) which is given by,

$$\left|\vec{E}_a\right| = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}, \qquad r >> l$$

The electric field on the equator (Eq. 10.28) is antiparallel to $\vec{p}$ and is given by

$$\left|\vec{E}_{eq}\right| = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}, \qquad r >> l$$

Using the analogy given in Table 12.1, we can thus write the axial magnetic field of a bar magnet at a distance r, $r >> l$, $2l$ being the length of bar magnet,

$$\vec{B}_a = +\frac{\mu_0}{4\pi}\frac{2\vec{m}}{r^3} \qquad \text{--- (12.3)}$$

Similarly, the equatorial magnetic field

$$\vec{B}_{eq} = -\frac{\mu_0\vec{m}}{4\pi r^3} \qquad \text{--- (12.4)}$$

Negative sign shows that the direction of $\vec{B}_{eq}$ is opposite to $\vec{m}$.

For the same distance from centre O of a bar magnet,

$$B_{axis} = 2B_{eq} \qquad \text{--- (12.5)}$$

12.3.2 Magnetic field due to a bar magnet at an arbitrary point:

Fig. 12.3 Shows a bar magnet of magnetic moment $\vec{m}$ with centre at O. P is any point in its magnetic field. Magnetic moment $\vec{m}$ is resolved (*about the centre of the magnet*) into components along $\vec{r}$ and perpendicular to $\vec{r}$. For the component $m\cos\theta$ along $\vec{r}$, the point P is an axial point.

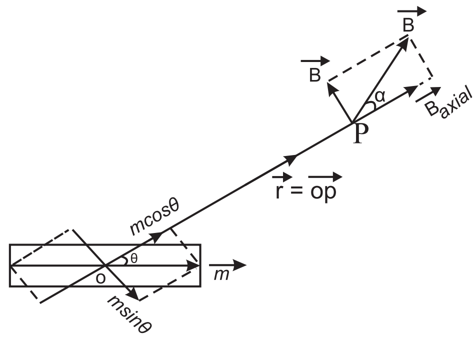


Fig. 12.3: Magnetic field at an arbitrary point.

Also, for the component $m\sin\theta$ perpendicular to $\vec{r}$, the point P is an equatorial point at the same distance $\vec{r}$. Using the results of axial and equatorial fields, we get

$$B_a = \frac{\mu_o}{4\pi}\frac{2m\cos\theta}{r^3} \quad \text{--- (12.6)}$$

directed along $m\cos\theta$ and

$$B_{eq} = \frac{\mu_o}{4\pi}\frac{m\sin\theta}{r^3} \quad \text{--- (12.7)}$$

directed opposite to $m\sin\theta$

Thus, the magnitude of the resultant magnetic field B, at point P is given by

$$B = \sqrt{B_a^{\ 2} + B_{eq}^{\ 2}}$$

$$\therefore\ B = \frac{\mu_o}{4\pi}\frac{m}{r^3}\sqrt{[2\cos\theta]^2 + [\sin\theta]^2}$$

$$\therefore B = \frac{\mu_o}{4\pi}\frac{m}{r^3}\sqrt{3\cos^2\theta + 1} \quad \text{--- (12.8)}$$

Let α be the angle made by the direction of $\vec{B}$ with $\vec{r}$. Then, by using eq (12.6) and eq (12.7),

$$\tan\alpha = \frac{B_{eq}}{B_a} = \frac{1}{2}(\tan\theta) \quad \text{--- (12.9)}$$

The angle between directions of $\vec{B}$ and $\vec{m}$ is then $(\theta + \alpha)$.

Example 12.1: A short magnetic dipole has magnetic moment 0.5 A m^2. Calculate its magnetic field at a distance of 20 cm from the centre of magnetic dipole on (i) the axis (ii) the equatorial line (Given $\mu_0 = 4\pi \times 10^{-7}$ SI units)

Solution :

$$m = 0.5\ \text{Am}^2,\ r = 20\ \text{cm} = 0.2\ \text{m}$$

$$B_a = \frac{\mu_0}{4\pi}\frac{2m}{r^3} = \frac{10^{-7}\times 2\times 0.5}{(0.2)^3} = \frac{1\times 10^{-7}}{8\times 10^{-3}}$$

$$= \frac{1}{8}\times 10^{-4} = 1.25\times 10^{-5}\ \text{Wb/m}^2$$

$$B_{eq} = \frac{\mu_0}{4\pi}\frac{m}{r^3}$$

$$= \frac{10^{-7}\times 0.5}{(0.2)^3} = \frac{5\times 10^{-8}}{8\times 10^{-3}} = 0.625\times 10^{-5}\ \text{Wb/m}^2$$

12.4 Gauss' Law of Magnetism:

The Gauss' law for electric field is known to you. It states that the net electric flux through a closed Gaussian surface is proportional to the net electric charge enclosed by the surface (Eq. (10.18)). The Gauss' law for magnetic fields states that the net magnetic flux Φ_B through a closed Gaussian surface is zero, i.e.,

$$\Phi_B = \oint \vec{B}\cdot d\vec{S} = 0 \text{ (Gauss' law for magnetic fields)}$$

The magnetic force lines of (a) bar magnet, (b) current carrying finite solenoid, and (c) electric dipole are shown in Fig.12.4(a), 12.4(b) and 12.4(c), respectively. The curves labelled (i) and (ii) are cross sections of three dimensional closed Gaussian surfaces.

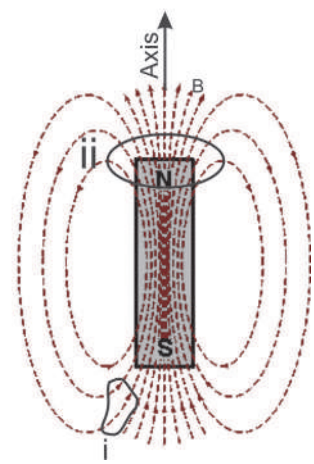


Fig. 12.4 (a): Bar magnet.

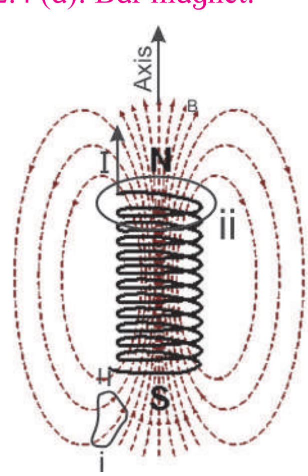


Fig. 12.4 (b): Current (I) carrying solenoid.

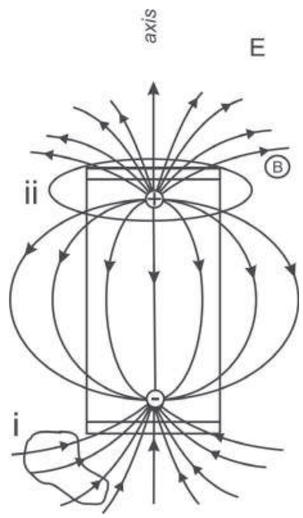


Fig. 12.4 (c): Electric dipole.

If we compare the number of lines of force entering in and leaving out of the surface (i), it is clearly seen that they are equal. The Gaussian surface does not include poles. It means that the flux associated with any closed surface is equal to zero. When we consider surface (ii), in Fig. 12.4 (b), we are enclosing the North pole. As even a thin slice of a bar magnet will have North and South poles associated with it, the closed Gaussian surface will also include a South pole. However in Fig. 12.4(c), for an electric dipole, the field lines begin from positive charge and end on negative charge. For a closed surface (ii), there is a net outward flux since it does include a net (positive) charge. According to the Gauss' law of electrostatics as studied earlier, $\Phi_E = \int \vec{E}\cdot\vec{dS} = \frac{q}{\varepsilon_0}$, where q is the positive charge enclosed. Thus, situation is entirely different from magnetic lines of force, which are shown in Fig. 12.4(a) and Fig. 12.4(b). Thus, Gauss' law of magnetism can be written as $\Phi_B = \int \vec{B}\cdot\vec{dS} = 0$.

From the above we conclude that for electrostatics, an isolated electric charge exists but an isolated magnetic pole does not exist. In short, only dipoles exist in case of magnetism.

12.5 Earth's Magnetism:

It is common experience that a bar magnet or a magnetic needle suspended freely in air always aligns itself along geographic N-S direction. If it has a freedom to rotate about horizontal axis, it inclines with some angle with the horizontal in the vertical N-S plane.

This fact clearly indicates that there is some magnetic field present everywhere on the Earth . This is called Terrestrial Magnetism. It is extremely useful during navigation.

Magnetic parameters of the Earth are described below. The magnetic lines of force enter the Earth's surface at the north pole and emerge from the south pole.

Unless and otherwise stated, the directions mentioned (South, North, etc.) are always, Geographic.

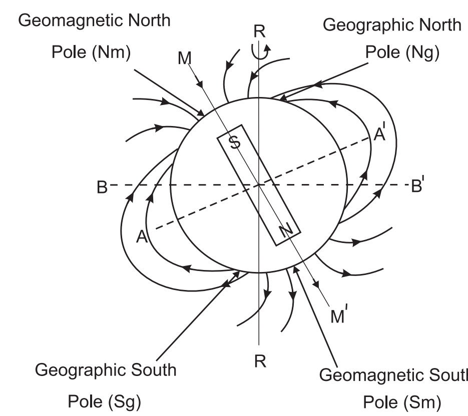


Fig. 12.5: Earth's magnetism.

Magnetic Axis :- The Earth is considered to be a huge magnet. Magnetic north pole (N) of the Earth is located below Antarctica while the south pole (S) is below north Canada. The straight line NS joining these two poles is called the magnetic axis, MM'.

Magnetic equator :- A great circle in the plane perpendicular to magnetic axis is magnetic equatorial circle, AA'. It happens to pass through India near Thiruvananthapuram.

Geographic Meridian:- A plane perpendicular to the surface of the Earth (vertical plane) perpendicular to geographic axis is geographic meridian. (Fig.12.6)

Magnetic Meridian:- A plane perpendicular to surface of the Earth (Vertical plane) and passing through the magnetic axis is magnetic meridian. Direction of resultant magnetic field of the Earth is always along or parallel to magnetic meridian. (Fig.12.6)

Magnetic declination:- Angle between the geographic and the magnetic meridian at a place is called 'magnetic declination' (α). The declination is small in India. It is $0°\ 58'$ west at Mumbai and $0^0 41'$ east at Delhi. Thus, at both these places, magnetic needle shows true North accurately (Fig.12.6).

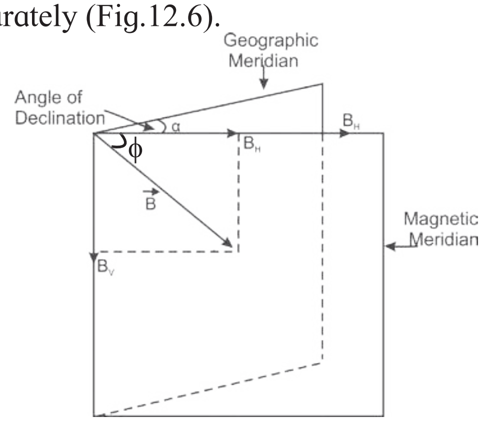


Fig. 12.6: Magnetic declination.

Magnetic inclination or angle of dip (ϕ)**:-** Angle made by the direction of resultant magnetic field with the horizontal at a place is inclination or angle of dip at the place (Fig. 12.7).

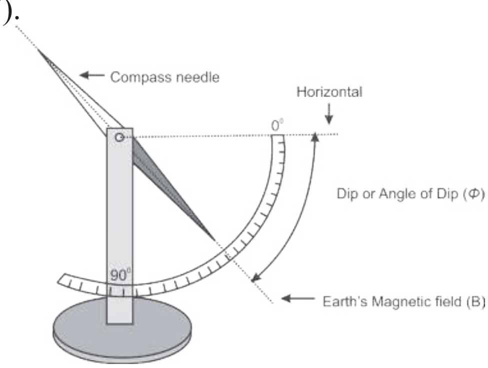


Fig. 12.7: Magnetic inclination.

Earth's magnetic field:- Magnetic force experienced per unit pole strength is magnetic field $\vec{B}$ at that place. It can be resolved in components along the horizontal, $\vec{B}_H$ and along vertical, $\vec{B}_V$. The vertical component can be conveniently determined. The two components can be related with the angle of dip (ϕ) as,

$B_H = B\cos\phi$, $B_V = B\sin\phi$

$$\frac{B_V}{B_H} = \tan\phi \qquad \text{--- (12.10)}$$

$$B^2 = B_V{}^2 + B_H{}^2$$

$$\therefore B = \sqrt{B_V{}^2 + B_H{}^2} \qquad \text{--- (12.11)}$$

Special cases

1) At the magnetic North pole, $\vec{B} = \vec{B}_V$, directed upward, $\vec{B}_H = 0$ and $\phi = 90^0$.

2) At the magnetic south pole, $\vec{B} = \vec{B}_V$,- directed downward, $\vec{B}_H = 0$ and $\phi = 270^0$.

3) Anywhere on the magnetic great circle (magnetic equator) $B = B_H$ along South to North, $B_V = 0$ and $\phi = 0$

Magnetic maps of the Earth:-

Magnetic elements of the Earth (B_H, α and ϕ) vary from place to place and also with time. The maps providing these values at different locations are called magnetic maps. These are extremely useful for navigation. Magnetic maps drawn by joining places with the same value of a particular element are called Iso-magnetic charts.

Lines joining the places of equal horizontal components (B_H) are known as 'Isodynamic lines'

Lines joining the places of equal declination (α) are called Isogonic lines.

Lines joining the places of equal inclination or dip (ϕ) are called Aclinic lines.

Example 12.2: Earth's magnetic field at the equator is approximately 4×10^{-5} T. Calculate Earth's dipole moment. (Radius of Earth = 6.4×10^6 m, $\mu_0 = 4\pi\times10^{-7}$ SI units)

Solution: Given

$B_{eq} = 4\times10^{-5}$ T

$r = 6.4\times10^6$ m

Assume that Earth is a bar magnet with N and S poles being the geographical South and North poles, respectively. The equatorial magnetic field due to Earth's dipole can be written as

$$B_{eq} = \frac{\mu_0 m}{4\pi r^3}$$

$$m = 4\pi B_{eq} \times r^3 / \mu_0$$

$$= 4\times10^{-5}\times(6.4\times10^6)^3\times10^7$$

$$= 1.05\times10^{20}\ \text{A m}^2$$

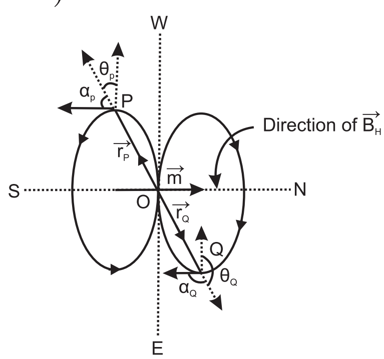
Example 12.3: At a given place on the Earth, a bar magnet of magnetic moment $\vec{m}$ is kept horizontal in the East-West direction. P and Q are the two neutral points due to magnetic field of this magnet and $\vec{B}_H$ is the horizontal component of the Earth's magnetic field.

(A) Calculate the angles between position vectors of P and Q with the direction of $\vec{m}$.

(B) Points P and Q are 1 m from the centre of the bar magnet and $B_H = 3.5\times10^{-5}$ T. Calculate magnetic dipole moment of the bar magnet.

Neutral point is that point where the resultant magnetic field is zero.

Solution: (A) As seen from the figure, the direction of magnetic field $\vec{B}$ due to the bar magnet is opposite to $\vec{B}_H$ at the points P and Q. Also, $(\theta+\alpha) = 90^0$ at P and it is 270^0 at Q.



$$\tan\alpha = \frac{1}{2}\tan\theta$$

$$\therefore \tan\theta = 2\tan\alpha$$

$$= 2\tan(90-\theta) \text{ and } 2\tan(270-\theta)$$

$$\therefore \tan\theta = \pm 2\cot\theta$$

$$\therefore \tan^2\theta = 2$$

$$\therefore \tan\theta = \pm\sqrt{2}$$

$$\therefore \theta = \tan^{-1}\left(\pm\sqrt{2}\right)$$

$$\therefore \theta = 54^0 44' \quad \text{and} \quad 180^0 - 54^0 44^0 = 116^0 4'$$

(B)

$$\tan^2\theta = 2$$

$$\therefore \sec^2\theta = 1+\tan^2\theta = 1+2 = 3$$

$$\therefore \cos^2\theta = \frac{1}{3}$$

$r = 1\text{m}$ and $B = B_H = 3.5\times10^{-5}$ T (Given)

we have,

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^3}\sqrt{3\cos^2\theta+1}$$

$$\therefore m = \frac{B_H \times r^3}{\left(\frac{\mu_0}{4\pi}\right)\sqrt{3\cos^2\theta+1}}$$

$$= \frac{3.5\times10^{-5}\times1^3}{10^{-7}\times\sqrt{3\frac{1}{3}+1}}$$

$$\therefore m = \frac{350}{\sqrt{2}} = 247.5 \text{ A m}^2$$

Always remember:
In this Chapter we have used *B* as a symbol for magnetic field. Calling it magnetic induction is unreasonable. We have used the words **magnetic field** which are used in spoken language.

Exercises

1. Choose the correct option.

i) Let r be the distance of a point on the axis of a bar magnet from its center. The magnetic field at r is always proportional to

(A) $1/r^2$ (B) $1/r^3$ (C) $1/r$
(D) not necessarily $1/r^3$ at all points

ii) Magnetic meridian is the plane

(A) perpendicular to the magnetic axis of Earth
(B) perpendicular to geographic axis of Earth
(C) passing through the magnetic axis of Earth
(D) passing through the geographic axis Earth

iii) The horizontal and vertical component of magnetic field of Earth are same at some place on the surface of Earth. The magnetic dip angle at this place will be

(A) 30° (B) 45°
(C) 0° (D) 90°

iv) Inside a bar magnet, the magnetic field lines

(A) are not present
(B) are parallel to the cross sectional area of the magnet
(C) are in the direction from N pole to S pole
(D) are in the direction from S pole to N pole

v) A place where the vertical components of Earth's magnetic field is zero has the angle of dip equal to

(A) 0° (B) 45°

(C) 60° (D) 90°

vi) A place where the horizontal component of Earth's magnetic field is zero lies at

(A) geographic equator

(B) geomagnetic equator

(C) one of the geographic poles

(D) one of the geomagnetic poles

vii) A magnetic needle kept nonparallel to the magnetic field in a nonuniform magnetic field experiences

(A) a force but not a torque

(B) a torque but not a force

(C) both a force and a torque

(D) neither force nor a torque

2. Answer the following questions in brief.

i) What happens if a bar magnet is cut into two pieces transverse to its length/ along its length?

ii) What could be the equation for Gauss' law of magnetism, if a monopole of pole strength p is enclosed by a surface?

3. Answer the following questions in detail.

i) Explain the Gauss' law for magnetic fields.

ii) What is a geographic meridian. How does the declination vary with latitude? Where is it minimum?

iii) Define the Angle of Dip. What happens to angle of dip as we move towards magnetic pole from magnetic equator?

4. Solve the following Problems.

i) A magnetic pole of bar magnet with pole strength of 100 A m is 20 cm away from the centre of a bar magnet. Bar magnet has pole strength of 200 A m and has a length 5 cm. If the magnetic pole is on the axis of the bar magnet, find the force on the magnetic pole.

[Ans: 2.5×10^{-2}N]

ii) A magnet makes an angle of 45° with the horizontal in a plane making an angle of 30° with the magnetic meridian. Find the true value of the dip angle at the place.

[Ans: $\tan^{-1}(0.866)$]

iii) Two small and similar bar magnets have magnetic dipole moment of 1.0 Am^2 each. They are kept in a plane in such a way that their axes are perpendicular to each other. A line drawn through the axis of one magnet passes through the center of other magnet. If the distance between their centers is 2 m, find the magnitude of magnetic field at the mid point of the line joining their centers.

[Ans: $\sqrt{5}\times10^{-7}$ T]

iv) A circular magnet is made with its north pole at the centre, separated from the surrounding circular south pole by an air a gap. Draw the magnetic field lines in the gap. [The magnet is hypothetical magnet].

Draw a diagram to illustrate the magnetic lines of force between the south poles of two such magnets.

v) Two bar magnets are placed on a straight line with their north poles facing each other on a horizontal surface. Draw magnetic lines around them. Mark the position of any neutral points (points where there is no resultant magnetic field) on your diagram.

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