

## CHAPTER - 3 : PARTITION VALUES

### LET'S RECALL

- Are you familiar with the word 'averages'?
- Can you tell the meaning of individual series, discrete data and continuous data.
- Name the positional averages that you have previously studied.

| Types of Average   | Individual Data  | Discrete Data  | Continuous Data   |
|--------------------|--|--|---|
| 1) Arithmetic Mean | $\bar{x} = \frac{\sum x}{n}$                                       | $\bar{x} = \frac{\sum f_i x_i}{n}$                                 | Direct method $\bar{x} = \frac{\sum f_i x_i}{n}$                        |
| 2) Mode            | Value repeated maximum number of times                             | The value which has maximum frequency                              | Mode = $l + \left[ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$ |
| 3) Median          | M = size of $\left( \frac{n+1}{2} \right)^{\text{th}}$ observation | M = size of $\left( \frac{n+1}{2} \right)^{\text{th}}$ observation | M = $l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h$            |

### Introduction :

The procedure of dividing the data into equal parts is called 'partitioning'. Values dividing the data into a required number of equal parts are called 'Partition Values'.

In Class X, you have already studied about the measures of central tendency i.e. averages such as Arithmetic Mean, Median and Mode. Median is the value of the middlemost observation in the data when the observations are arranged in increasing or decreasing order of values. 'Median', is a special type of partition value because there are equal number of observations above as well as below it. Like Median, Quartiles, Deciles and Percentiles are also partition values, since they divide the given set of observations into equal number of parts. In general, they are referred to as 'fractiles'. Partition values form a part of descriptive statistics.

In the forthcoming chapters such as Population, Unemployment and Poverty, students will get acquainted with the use of partition values in economic data analysis.

### Do you know?

**Statistics Day :** Prof. Prasanta Chandra Mahalanobis, an Indian Statistician was instrumental in formulating India's strategy for industrialization in the Second Five Year Plan (1956-61) which later came to be known as Mahalanobis Model.

Mahalanobis devised a measure of comparison between two data sets that is known as the Mahalanobis distance. He also devised a statistical method called 'fractile graphical analysis' which could be used to compare the socio-economic conditions of different groups of people. In recognition of the notable contributions made by P. C. Mahalanobis in the field of economic planning and statistical development, the Government of India has designated 29th of June every year, coinciding with his birth anniversary as 'Statistics Day', in the category of Special day to be celebrated at the national level.

### Need for Partition Values :

The data consists of extreme values on the lower side and also on the higher side in magnitude. Such values are known as 'outliers'. The average used for such data often misinterprets its representative value. To overcome this misinterpretation, generally partition values like median, quartiles, deciles and percentiles are used.

#### Always remember :

$$Q_2 = D_5 = P_{50} = \text{Median}$$

#### You should know :

##### Application of Quartiles, Deciles and Percentiles in Economics :

- Quartiles are used in the study of all types of financial information concerning economic data, income data, stock data, sales and survey data etc.
- Income quartiles is the most objective method of comparing changes among individual income groups caused by economic changes such as wage fluctuations and inflation.
- Deciles too have wide application in finance and economics. Government uses deciles to study the level of economic inequality, measurement of poverty line, drought conditions etc.
- Deciles are used in investments, particularly to assess the performance of a portfolio investment such as a group of mutual funds.
- Percentiles are used in the measurement of test scores, health indicators, household income, household wealth, percentile wages.
- Percentiles can be used for benchmarking and baseline purposes.

### LET'S LEARN :

#### Quartiles :

**Meaning :** 'Quartiles' are values of data which divide the whole set of observations into four equal parts. There are three Quartiles which divide the data into 4 equal parts. They are

known as  $Q_1$ ,  $Q_2$  and  $Q_3$  respectively. Second quartile is nothing but the median. To find out quartiles, generally the data is arranged in ascending order.

It is explained in the following example :



- a) In general, for individual and ungrouped data we get the formula for  $Q_1$ ,  $Q_2$  and  $Q_3$  as given below :

$$Q_i = \text{size of } i\left(\frac{n+1}{4}\right)^{\text{th Observation.}} \quad i = 1, 2, 3$$

- b) For grouped data or continuous data,

$$Q_i = l + \left(\frac{\frac{in}{4} - cf}{f}\right) \times h \quad i = 1, 2, 3$$

Where

$l$  = Lower limit of quartile class.

$f$  = Frequency of the quartile class

$cf$  = Cumulative frequency of the class preceding the quartile class.

$n$  = Total of frequency.

$h$  = Upper limit - lower limit of the quartile class.

### Calculation of Quartiles

#### Solved Examples

##### A) Individual Data :

1) Calculate  $Q_1$  and  $Q_3$  of the first semester examination marks scored by the students as given : 40, 85, 84, 83, 82, 69, 68, 65, 64, 55, 45

**Solution :** Arrange the series in ascending order i.e. 40, 45, 55, 64, 65, 68, 69, 82, 83, 84, 85

$n$  = Total number of observations

$$n = 11$$

$$Q_1 = \text{size of } \left(\frac{n+1}{4}\right)^{\text{th Observation.}}$$

$$Q_1 = \text{size of } \left(\frac{11+1}{4}\right)^{\text{th Observation}}$$

$$Q_1 = \text{size of } \left(\frac{12}{4}\right)^{\text{th Observation}}$$

$$Q_1 = \text{size of } 3^{\text{rd Observation.}}$$

$$Q_1 = \text{size of } 3^{\text{rd Observation.}} \text{ is } 55$$

$$\therefore Q_1 = 55$$

### Third Quartile

$$Q_3 = \text{size of } 3\left(\frac{n+1}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 3\left(\frac{11+1}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 3\left(\frac{12}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } (3 \times 3)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 9^{\text{th Observation}} \text{ is } 83$$

$$\therefore Q_3 = 83$$

|  |
|--|
| <b>Ans : <math>Q_1 = 55, Q_3 = 83</math></b> |
|--|

### 2) Calculate $Q_3$ for the given distribution.

20, 28, 31, 18, 19, 17, 32, 33, 22, 21

**Solution :** Arrange the data in ascending order.

17, 18, 19, 20, 21, 22, 28, 31, 32, 33

$$n = 10$$

$$Q_3 = \text{size of } 3\left(\frac{n+1}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 3\left(\frac{10+1}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } \left(3 \times \frac{11}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } \left(\frac{33}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 8.25^{\text{th Observation}}$$

$$Q_3 = \text{size of } 8^{\text{th observation}} + 0.25(9^{\text{th observation}} - 8^{\text{th observation}})$$

$$Q_3 = 31 + 0.25(32 - 31)$$

$$Q_3 = 31 + 0.25 \times 1$$

$$\therefore Q_3 = 31.25$$

|                                       |
|---------------------------------------|
| <b>Ans : <math>Q_3 = 31.25</math></b> |
|---------------------------------------|

**B) Discrete Data :** By arranging the observations in the data in ascending or descending order, we derive :

$$Q_i = \text{size of } i\left(\frac{n+1}{4}\right)^{\text{th Observation}} \quad \text{where } i = 1, 2, 3$$

1) Find out  $Q_1$  and  $Q_3$  from the following data.

|                        |   |   |    |    |    |   |    |
|------------------------|---|---|----|----|----|---|----|
| <b>Income (lakh ₹)</b> | 5 | 4 | 9  | 12 | 15 | 6 | 10 |
| <b>No. of Person</b>   | 8 | 6 | 12 | 8  | 6  | 9 | 10 |

**Solution :** Arrange the data in ascending order and find out the cumulative frequency.

| <b>Income (lakh ₹)</b><br><b>(x)</b> | <b>No. of Person</b><br><b>(f)</b> | <b>Cumulative</b><br><b>frequency (cf)</b> |
|--------------------------------------|------------------------------------|--|
| 4                                    | 6                                  | 6  |
| 5                                    | 8                                  | 14   |
| 6                                    | 9                                  | 23   |
| 9                                    | 12                                 | 35   |
| 10                                   | 10                                 | 45   |
| 12                                   | 8                                  | 53   |
| 15                                   | 6                                  | 59   |
|                                      | <b>n = 59</b>                      |  |

$$Q_1 = \text{size of } \left(\frac{n+1}{4}\right)^{\text{th Observation}}$$

$$Q_1 = \text{size of } \left(\frac{59+1}{4}\right)^{\text{th Observation}}$$

$$Q_1 = \text{size of } \left(\frac{60}{4}\right)^{\text{th Observation}}$$

$$Q_1 = \text{size of } 15^{\text{th Observation}}$$

Size of 15th observation lies in *cf* 23, hence quartile value = ₹ 6 lakhs

$$\therefore Q_1 = ₹ 6 \text{ lakhs}$$

$$Q_3 = \text{size of } 3\left(\frac{n+1}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 3\left(\frac{59+1}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 3\left(\frac{60}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } (3 \times 15)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 45^{\text{th Observation}}$$

Size of 45th observation lies in *cf* 45, hence quartile value = ₹ 10 lakhs

$$\therefore Q_3 = ₹ 10 \text{ lakhs}$$

|   |
|---|
| <b>Ans : <math>Q_1 = ₹ 6 \text{ lakhs}, Q_3 = ₹ 10 \text{ lakhs}</math></b> |
|---|

**C) Continuous data :**  $Q_1$  and  $Q_3$  for continuous frequency distribution are calculated by applying the following steps.

- 1) Arrange the data in ascending or descending order.

- 2) Write respective frequencies of the class.
- 3) Find out cumulative frequency ( $cf$ )
- 4) Determine the quartile class.

**Formula :**

**Step - I :** First find the value of quartile

$$Q_1 = \text{size of } \left(\frac{n}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } \left(\frac{3n}{4}\right)^{\text{th Observation}}$$

**Step - II :**

$$Q_i = l + \left(\frac{\frac{in}{4} - cf}{f}\right) \times h \quad i = 1, 2, 3$$

Where

$l$  = Lower limit of quartile class.

$f$  = Frequency of the quartile class

$cf$  = Cumulative frequency of the class preceding the quartile class.

$n$  = Total of frequency.

$h$  = Upper limit - lower limit of the quartile class.

1) Find out  $Q_1$  and  $Q_3$  quartile for the following data.

|                          |       |       |       |       |
|--------------------------|-------|-------|-------|-------|
| <b>Rainfall (in cms)</b> | 20-30 | 30-40 | 40-50 | 50-60 |
| <b>No. of years</b>      | 7     | 20    | 17    | 6     |

| Rainfall (in cms) | No. of years ( $f$ ) | Cumulative frequency ( $cf$ ) |
|-------------------|----------------------|-------------------------------|
| 20-30             | 7                    | 7                             |
| 30-40             | 20                   | 27                            |
| 40-50             | 17                   | 44                            |
| 50-60             | 6                    | 50                            |
|                   | $n = 50$             |                               |

**Step I**

$$Q_1 = \text{size of } \left(\frac{n}{4}\right)^{\text{th Observation}}$$

$$Q_1 = \text{size of } \left(\frac{50}{4}\right)^{\text{th Observation}}$$

$$Q_1 = \text{size of } (12.5)^{\text{th Observation}}$$

12.5 lies in  $cf$  27, therefore first quartile class is 30 - 40

$$\therefore l = 30 \quad f = 20 \quad cf = 27 \quad n = 50 \quad h = 10$$

**Step II**

$$Q_1 = l + \left(\frac{\frac{n}{4} - cf}{f}\right) \times h$$

$$Q_1 = 30 + \left(\frac{\frac{50}{4} - 27}{20}\right) \times 10$$

$$Q_1 = 30 + \left(\frac{12.5 - 27}{20}\right) \times 10$$

$$Q_1 = 30 + \left(\frac{5.5}{20}\right) \times 10$$

$$Q_1 = 30 + \left(\frac{55}{20}\right)$$

$$Q_1 = 30 + 2.75$$

$$Q_1 = 32.75$$

$$\therefore Q_1 = 32.75$$

**Step I**

$$Q_3 = \text{size of } \left(\frac{3n}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } \left(\frac{3 \times 50}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } \left(\frac{150}{4}\right)^{\text{th Observation}}$$

$$Q_3 = \text{size of } 37.5^{\text{th Observation}}$$

37.5 lies in  $cf$  44 hence third quartile class is 40 - 50

$$\therefore l = 40 \quad f = 17 \quad cf = 27 \quad n = 50 \quad h = 10$$

**Step II**

$$Q_3 = l + \left(\frac{\frac{3n}{4} - cf}{f}\right) \times h$$

$$Q_3 = 40 + \left(\frac{\frac{3 \times 50}{4} - 27}{17}\right) \times 10$$

$$Q_3 = 40 + \left(\frac{37.5 - 27}{17}\right) \times 10$$

$$Q_3 = 40 + \left(\frac{10.5}{17}\right) \times 10$$

$$Q_3 = 40 + \left(\frac{105}{17}\right)$$

$$Q_3 = 40 + 6.18$$

$$\therefore Q_3 = 46.18$$

|  |
|--|
| <b>Ans : <math>Q_1 = 32.75, Q_3 = 46.18</math></b> |
|--|

## Deciles :

**Meaning :** 'Deciles' are values of data which divide the whole set of observations into ten equal parts. There are nine points i.e.  $D_1$ ,  $D_2$  to  $D_9$  which divide the data into 10 equal parts. To find out deciles, generally the data is arranged in ascending order.

- a) For calculating  $D_1$  to  $D_9$  for individual and discrete data, use the following formula.

$$D_j = j \left( \frac{n+1}{10} \right)^{\text{th Observation}} \quad \text{where } j = 1, 2 \dots 9$$

- b) For grouped data or continuous data,

$$D_j = l + \left( \frac{\frac{jn}{10} - cf}{f} \right) \times h, \quad j = 1, 2 \dots 9$$

Where

$D$  = Decile

$l$  = Lower limit of decile class

$f$  = Frequency of decile class

$cf$  = Cumulative frequency of class preceding decile class

$h$  = Upper limit of the class - lower limit of the decile class.

## Calculation of Deciles

### Solved Examples

#### A) Individual Data :

- 1) Calculate  $D_4$  and  $D_8$  for the following data.

10, 15, 7, 8, 12, 13, 14, 11, 9

**Solution :** Arrange the data in ascending order.

7, 8, 9, 10, 11, 12, 13, 14, 15

$$D_4 = \text{size of } 4 \left( \frac{n+1}{10} \right)^{\text{th Observation}}$$

$$D_4 = \text{size of } 4 \left( \frac{9+1}{10} \right)^{\text{th Observation}}$$

$$D_4 = \text{size of } 4 \left( \frac{10}{10} \right)^{\text{th Observation}}$$

$$D_4 = \text{size of } (4 \times 1)^{\text{th Observation}}$$

$$D_4 = \text{size of } 4^{\text{th Observation}}$$

$$\therefore D_4 = 10$$

## Calculation of $D_8$

$$D_8 = \text{size of } 8 \left( \frac{n+1}{10} \right)^{\text{th Observation}}$$

$$D_8 = \text{size of } 8 \left( \frac{9+1}{10} \right)^{\text{th Observation}}$$

$$D_8 = \text{size of } 8 \left( \frac{10}{10} \right)^{\text{th Observation}}$$

$$D_8 = \text{size of } (8 \times 1)^{\text{th Observation}}$$

$$D_8 = \text{size of } 8^{\text{th Observation}}$$

$$\therefore D_8 = 14$$

$$\text{Ans : } D_4 = 10, D_8 = 14$$

- 2) Calculate  $D_8$  from the given data

14, 13, 12, 11, 15, 16, 18, 17, 19, 20

**Solution :** First arrange the data in ascending order.

11, 12, 13, 14, 15, 16, 17, 18, 19, 20

$n = 10$

$$D_8 = \text{size of } 8 \left( \frac{n+1}{10} \right)^{\text{th Observation}}$$

$$D_8 = \text{size of } 8 \left( \frac{10+1}{10} \right)^{\text{th Observation}}$$

$$D_8 = \text{size of } 8 \left( \frac{11}{10} \right)^{\text{th Observation}}$$

$$D_8 = \text{size of } (8 \times 1.1)^{\text{th Observation}}$$

$$D_8 = \text{size of } (8.8)^{\text{th Observation}}$$

$$D_8 = \text{size of } 8^{\text{th observation}} + 0.8(9^{\text{th observation}} - 8^{\text{th observation}})$$

$$D_8 = 18 + 0.8(19 - 18)$$

$$D_8 = 18 + (0.8 \times 1)$$

$$D_8 = 18 + 0.8$$

$$\therefore D_8 = 18.8$$

$$\text{Ans : } D_8 = 18.8$$

#### B) Discrete data :

- 1) Find out  $D_2$  and  $D_4$  for the following data.

| Marks           | 10 | 20 | 30 | 40 | 50 | 60 |
|-----------------|----|----|----|----|----|----|
| No. of Students | 5  | 6  | 4  | 5  | 10 | 9  |

**Solution :**

| Marks | No. of students ( <i>f</i> ) | <i>cf</i> |
|-------|------------------------------|-----------|
| 10    | 5                            | 5         |
| 20    | 6                            | 11        |
| 30    | 4                            | 15        |
| 40    | 5                            | 20        |
| 50    | 10                           | 30        |
| 60    | 9                            | 39        |
|       | <i>n</i> = 39                |           |

$$D_2 = \text{size of } 2\left(\frac{n+1}{10}\right)^{\text{th Observation}}$$

$$D_2 = \text{size of } 2\left(\frac{39+1}{10}\right)^{\text{th Observation}}$$

$$D_2 = \text{size of } 2\left(\frac{40}{10}\right)^{\text{th Observation}}$$

$$D_2 = \text{size of } (2 \times 4)^{\text{th Observation}}$$

$$D_2 = \text{size of } (8)^{\text{th Observation}}$$

Size of 8<sup>th</sup> Observation lies in *cf* 11

Hence  $D_2 = 20$  marks

$$\therefore D_2 = 20$$

#### Calculation of $D_4$

$$D_4 = \text{size of } 4\left(\frac{n+1}{10}\right)^{\text{th Observation}}$$

$$D_4 = \text{size of } 4\left(\frac{39+1}{10}\right)^{\text{th Observation}}$$

$$D_4 = \text{size of } 4\left(\frac{40}{10}\right)^{\text{th Observation}}$$

$$D_4 = \text{size of } (4 \times 4)^{\text{th Observation}}$$

$$D_4 = \text{size of } 16^{\text{th Observation}}$$

Size of 16<sup>th</sup> Observation lies in *cf* 20

Hence  $D_4 = 40$  marks

$$\therefore D_4 = 40$$

**Ans :  $D_2 = 20, D_4 = 40$**

### C) Continuous Data :

Apply the steps as mentioned in Qualities of continuous data.

1) Find out  $D_5$  and  $D_7$  for the following data of marks of 100 students in a class test.

| Marks           | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|-----------------|------|-------|-------|-------|-------|
| No. of Students | 10   | 10    | 40    | 20    | 20    |

| Marks | No. of students ( <i>f</i> ) | <i>cf</i> |
|-------|------------------------------|-----------|
| 0-10  | 10                           | 10        |
| 10-20 | 10                           | 20        |
| 20-30 | 40                           | 60        |
| 30-40 | 20                           | 80        |
| 40-50 | 20                           | 100       |
|       | <i>n</i> = 100               |           |

#### Calculation of $D_5$

##### Step I

$$D_5 = \text{size of } \left(\frac{5n}{10}\right)^{\text{th Observation}}$$

$$D_5 = \text{size of } \left(\frac{5 \times 100}{10}\right)^{\text{th Observation}}$$

$$D_5 = \text{size of } \left(\frac{500}{10}\right)^{\text{th Observation}}$$

$$D_5 = \text{size of } 50^{\text{th Observation}}$$

Size of 50<sup>th</sup> Observation lies in *cf* 60

Hence Decile class = 20-30

$$\therefore l = 20 \quad f = 40 \quad cf = 20 \quad n = 100 \quad h = 10$$

##### Step II

$$D_5 = l + \left(\frac{\frac{5n}{10} - cf}{f}\right) \times h$$

$$D_5 = 20 + \left(\frac{\frac{5 \times 100}{10} - 20}{40}\right) \times 10$$

$$D_5 = 20 + \left(\frac{\frac{500}{10} - 20}{40}\right) \times 10$$

$$D_5 = 20 + \left(\frac{50 - 20}{40}\right) \times 10$$

$$D_5 = 20 + \left(\frac{30}{40}\right) \times 10$$

$$D_5 = 20 + \frac{300}{40}$$

$$D_5 = 20 + 7.5$$

$$D_5 = 27.5 \text{ marks}$$

$$\therefore D_5 = 27.5$$

#### Calculation of $D_7$

##### Step I

$$D_7 = \text{size of } \left(\frac{7n}{10}\right)^{\text{th Observation}}$$

$$D_7 = \text{size of } \left( \frac{7 \times 100}{10} \right)^{\text{th Observation}}$$

$$D_7 = \text{size of } \left( \frac{700}{10} \right)^{\text{th Observation}}$$

$$D_7 = \text{size of } 70^{\text{th Observation}}$$

$$D_7 = \text{size of } 70^{\text{th Observation}} \text{ lies in } cf \ 80$$

Hence Decile class = 30-40

$$\therefore l = 30 \quad f = 20 \quad cf = 60 \quad n = 100 \quad h = 10$$

### Step II

$$D_7 = l + \left( \frac{\frac{7n}{10} - cf}{f} \right) \times h$$

$$D_7 = 30 + \left( \frac{\frac{7 \times 100}{10} - 60}{20} \right) \times 10$$

$$D_7 = 30 + \left( \frac{\frac{700}{10} - 60}{20} \right) \times 10$$

$$D_7 = 30 + \left( \frac{70 - 60}{20} \right) \times 10$$

$$D_7 = 30 + \left( \frac{10}{20} \right) \times 10$$

$$D_7 = 30 + \left( \frac{100}{20} \right)$$

$$D_7 = 30 + 5$$

$$D_7 = 35 \text{ marks}$$

$$\therefore D_7 = 35$$

**Ans :  $D_5 = 27.5$ ,  $D_7 = 35$**

### Percentiles :

**Meaning :** 'Percentiles' are values of data which divide the whole set of observations into 100 equal parts. There are 99 percentiles giving ninety nine dividing points. Symbolically, value of percentiles are denoted by  $P_1, P_2, \dots, P_{99}$ . To find out percentiles, generally the data is arranged in ascending order.

- a) For calculating  $P_1$  to  $P_{99}$  for individual and discrete data we use following formula.

$$P_k = \text{size of } k \left( \frac{n+1}{100} \right)^{\text{th Observation}} \quad k = 1, 2, \dots, 99$$

- b) For grouped data or continuous data,

$$P_k = l + \left( \frac{\frac{kn}{100} - cf}{f} \right) \times h \quad k = 1, 2, \dots, 99$$

Where

**P** = Percentile

**l** = Lower limit of percentile class

**f** = Frequency of percentile class

**cf** = Cumulative frequency of class preceding percentile class

**h** = Upper limit of the class - lower limit of the percentile class.

### Calculation of Percentiles

#### Solved Examples

#### A) Individual Data :

- 1) Find  $P_{40}$  for the following data.

10, 15, 8, 16, 19, 11, 12, 14, 9

**Solution :** Arrange the data in ascending order

i.e. 8, 9, 10, 11, 12, 14, 15, 16, 19

$$n = 9$$

$$P_{40} = \text{size of } 40 \left( \frac{n+1}{100} \right)^{\text{th Observation}}$$

$$P_{40} = \text{size of } 40 \left( \frac{9+1}{100} \right)^{\text{th Observation}}$$

$$P_{40} = \text{size of } 40 \times \left( \frac{10}{100} \right)^{\text{th Observation}}$$

$$P_{40} = \text{size of } \left( \frac{40 \times 10}{100} \right)^{\text{th Observation}}$$

$$P_{40} = \text{size of } \left( \frac{400}{100} \right)^{\text{th Observation}}$$

$$P_{40} = \text{size of } 4^{\text{th Observation}}$$

$$P_{40} = \text{size of } 4^{\text{th Observation}} \text{ is } 11$$

$$\therefore P_{40} = 11$$

**Ans :  $P_{40} = 11$**

- 2) Calculate  $P_{85}$  from the following data.

79, 82, 36, 38, 51, 72, 68, 70, 64, 63

**Solution :** Arrange the data in ascending order

i.e. 36, 38, 51, 63, 64, 68, 70, 72, 79, 82

$$n = 10$$

$$P_{85} = \text{size of } 85 \left( \frac{n+1}{100} \right)^{\text{th Observation}}$$

$$P_{85} = \text{size of } 85 \left( \frac{10+1}{100} \right)^{\text{th Observation}}$$

$$P_{85} = \text{size of } 85 \left( \frac{11}{100} \right)^{\text{th Observation}}$$

$$P_{85} = \text{size of } (85 \times 0.11)^{\text{th Observation}}$$

$$P_{85} = \text{size of } (9.35)^{\text{th Observation}}$$

$$P_{85} = \text{size of } 9^{\text{th observation}} + 0.35(10^{\text{th observation}} - 9^{\text{th observation}})$$

$$P_{85} = 79 + 0.35(82 - 79)$$

$$P_{85} = 79 + 0.35 \times 3$$

$$P_{85} = 79 + 1.05$$

$$\therefore P_{85} = 80.05$$

**Ans :  $P_{85} = 80.05$**

### B) Discrete Data :

1) Find out  $P_{20}$  and  $P_{60}$  for the following data :

| Height (in inches) | 58 | 59 | 60 | 61 | 62 | 63 | 64 |
|--------------------|----|----|----|----|----|----|----|
| No. of persons     | 4  | 5  | 6  | 10 | 12 | 2  | 1  |

**Solution :** Arrange the data in ascending order.

| Height (in inches) | No. of persons ( $f$ ) | $cf$ |
|--------------------|------------------------|------|
| 58                 | 4                      | 4    |
| 59                 | 5                      | 9    |
| 60                 | 6                      | 15   |
| 61                 | 10                     | 25   |
| 62                 | 12                     | 37   |
| 63                 | 2                      | 39   |
| 64                 | 1                      | 40   |
|                    | $n = 40$               |      |

$$P_{20} = \text{size of } 20 \left( \frac{n+1}{100} \right)^{\text{th Observation}}$$

$$P_{20} = \text{size of } 20 \left( \frac{40+1}{100} \right)^{\text{th Observation}}$$

$$P_{20} = \text{size of } 20 \left( \frac{41}{100} \right)^{\text{th Observation}}$$

$$P_{20} = \text{size of } \left( \frac{20 \times 41}{100} \right)^{\text{th Observation}}$$

$$P_{20} = \text{size of } \left( \frac{820}{100} \right)^{\text{th Observation}}$$

$$P_{60} = \text{size of } 8.2^{\text{th Observation}}$$

$$P_{60} = \text{size of } 8.2^{\text{th Observation}} \text{ lies in } cf \ 9$$

$$\text{Hence } P_{20} = 59$$

$$\therefore P_{20} = 59$$

### Calculation of $P_{60}$

$$P_{60} = \text{size of } 60 \left( \frac{n+1}{100} \right)^{\text{th Observation}}$$

$$P_{60} = \text{size of } 60 \left( \frac{40+1}{100} \right)^{\text{th Observation}}$$

$$P_{60} = \text{size of } 60 \left( \frac{41}{100} \right)^{\text{th Observation}}$$

$$P_{60} = \text{size of } \left( \frac{60 \times 41}{100} \right)^{\text{th Observation}}$$

$$P_{60} = \text{size of } \left( \frac{2460}{100} \right)^{\text{th Observation}}$$

$$P_{60} = \text{size of } (24.6)^{\text{th Observation}}$$

$$P_{60} = \text{size of } 24.6^{\text{th Observation}} \text{ lies in } cf \ 25$$

$$\text{Hence } P_{60} = 61$$

$$\therefore P_{60} = 61$$

**Ans :  $P_{20} = 59, P_{60} = 61$**

### C) Continuous data :

Apply the steps as mentioned in Quartiles of continuous data.

1) Find  $P_{65}$  from the following data

| Marks           | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 |
|-----------------|-----|------|-------|-------|-------|
| No. of Students | 3   | 7    | 20    | 12    | 8     |

**Solution :**

| Marks | No. of students ( $f$ ) | $cf$ |
|-------|-------------------------|------|
| 0-5   | 3                       | 3    |
| 5-10  | 7                       | 10   |
| 10-15 | 20                      | 30   |
| 15-20 | 12                      | 42   |
| 20-25 | 8                       | 50   |
|       | $n = 50$                |      |



**Step I**

$$P_{65} = \text{size of } \left( \frac{65n}{100} \right)^{\text{th Observation}}$$

$$P_{65} = \text{size of } \left( \frac{65 \times 50}{100} \right)^{\text{th Observation}}$$

$$P_{65} = \text{size of } \left( \frac{3250}{100} \right)^{\text{th Observation}}$$

$$P_{65} = \text{size of } 32.5^{\text{th Observation}}$$

$P_{65}$  = size of 32.5<sup>th</sup> Observation lies in cf 42

Hence percentile class = 15-20

$$l=15 \quad f=12 \quad cf=30 \quad n=50 \quad h=5$$

**Step II**

$$P_{65} = l + \left( \frac{\frac{65n}{100} - cf}{f} \right) \times h$$

$$P_{65} = 15 + \left( \frac{\frac{65 \times 50}{100} - 30}{12} \right) \times 5$$

$$P_{65} = 15 + \left( \frac{\frac{3250}{100} - 30}{12} \right) \times 5$$

$$P_{65} = 15 + \left( \frac{32.5 - 30}{12} \right) \times 5$$

$$P_{65} = 15 + \left( \frac{2.5}{12} \right) \times 5$$

$$P_{65} = 15 + \left( \frac{2.5 \times 5}{12} \right)$$

$$P_{65} = 15 + \left( \frac{12.5}{12} \right)$$

$$P_{65} = 15 + 1.04$$

$$\therefore P_{65} = 16.04$$

$\text{Ans : } P_{65} = 16.04$

**EXERCISE**

**Q. 1. Choose the correct option :**

1) **Statements that do not apply to Quartiles.**

- a) First arrange the values in ascending or descending order.
- b) Observation can be divided into 4 parts.
- c) They are represented as  $Q_1$ ,  $Q_2$  and  $Q_3$ .
- d)  $Q_2$  is also known as median.

- Options :** 1) a    2) b and c  
                                 3) a, b and c                                      4) None of these

2)  **$D_7$  from the given data.**

Data - 4, 5, 6, 7, 8, 9, 10, 11, 12

- Options :** 1) 7    2) 9    3) 10    4) 12

3) **Statements related to partition values that are correct.**

- a) Exact divisions of percentiles into 100 parts gives 99 points
- b) Deciles have total 9 parts
- c) Quartiles are shown by  $Q_1$ ,  $Q_2$  and  $Q_3$
- d) Symbolically, Percentiles and Deciles are shown by P and D

- Options :** 1) a and c    2) a and b  
                                 3) a, b and c                                      4) a, c and d

**Q. 2. Identify the correct pairs from the given options :**

**Group A**

1) Quartiles

2) Deciles

3) Percentiles

**Group B**

a)  $D_j = \text{size of } j \left( \frac{n+1}{10} \right)^{\text{th Observation}}$

b)  $P_k = l + \left( \frac{\frac{kn}{100} - cf}{f} \right) \times h$

c)  $Q_i = l + \left( \frac{\frac{in}{4} - cf}{f} \right) \times h$

- Options :** 1) 1-b, 2-c, 3-a    2) 1- c, 2-a, 3-b  
                                 3) 1- c, 2-b, 3-a                                      4) 1- a, 2-b, 3-c

**Q. 3. Give economic terms :**

- 1) Procedure for dividing the data into equal parts.
- 2) Value that divides the series into ten equal parts.
- 3) Value that divides the whole set of observations in to four equal parts.

**Q. 4. Solve the following :**

- 1) Calculate  $Q_1$ ,  $D_4$  and  $P_{26}$  for the following data.

$18, 24, 45, 29, 4, 7, 28, 49, 16, 26, 25, 12, 10, 9, 8$

2) Calculate of  $Q_3$ ,  $D_5$ , and  $P_{35}$  for the given data.

|                            |   |   |    |    |    |    |
|----------------------------|---|---|----|----|----|----|
| <b>Income (in lakhs ₹)</b> | 1 | 2 | 3  | 4  | 5  | 6  |
| <b>No. of family</b>       | 2 | 5 | 20 | 25 | 15 | 12 |

3) Find out  $P_{50}$  for the following data.

| <b>Wages (in ₹) (x)</b> | <b>Number of workers</b> |
|-------------------------|--------------------------|
| 0-20                    | 4                        |
| 20-40                   | 6                        |
| 40-60                   | 10                       |
| 60-80                   | 25                       |
| 80-100                  | 15                       |

4) Calculate  $Q_3$  for the following data.

|                           |       |       |       |       |       |       |
|---------------------------|-------|-------|-------|-------|-------|-------|
| <b>Sales (in lakhs ₹)</b> | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
| <b>No. of firms</b>       | 20    | 30    | 70    | 48    | 32    | 50    |

5) Calculate  $D_7$  for the following data.

|                             |       |       |       |       |       |       |
|-----------------------------|-------|-------|-------|-------|-------|-------|
| <b>Profit (in crores ₹)</b> | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
| <b>No. of firms</b>         | 20    | 30    | 70    | 48    | 32    | 50    |

6) Calculate  $P_{15}$  for the following data.

|                                |      |       |       |       |       |       |
|--------------------------------|------|-------|-------|-------|-------|-------|
| <b>Investment (₹ in lakhs)</b> | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
| <b>No. of firms</b>            | 5    | 10    | 25    | 30    | 20    | 10    |

**Q. 5. State with reasons whether you agree or disagree with the following statements :**

- 1) Partition values have application only in theory but not in practice.
- 2) Average can misinterpret the representative value.
- 3) Median is also known as second quartile.

**Q. 6. Answer the following questions on the basis of the given table :**

|                        |    |    |    |    |    |
|------------------------|----|----|----|----|----|
| <b>Marks</b>           | 30 | 10 | 20 | 40 | 50 |
| <b>No. of Students</b> | 13 | 4  | 7  | 8  | 6  |

- 1) Write the formula of  $Q_1$  and  $Q_3$ .
- 2) Find out the cumulative frequency of the last value in the above data.
- 3) Find out the value of 'n' in the above data.

