1. Gravitation

- Gravitation
- Circular motion and centripetal force
- Kepler's laws
- Newton's universal law of gravitation
- Acceleration due to the gravitational force of the Earth
- Free fall
- Escape velocity

Can you recall?

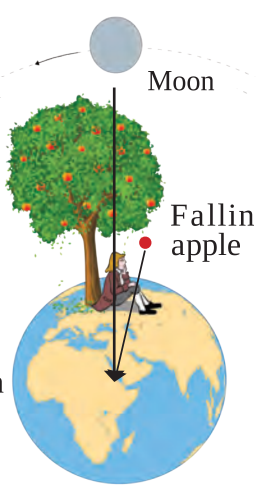
1. What are the effects of a force acting on an object?
2. What types of forces are you familiar with?
3. What do you know about the gravitational force?

We have seen in the previous standard that the gravitational force is a universal force and it acts not only between two objects on the earth but also between any two objects in the universe. Let us now learn how this force was discovered.

Gravitation

As we have learnt, the phenomenon of gravitation was discovered by Sir Isaac Newton. As the story goes, he discovered the force by seeing an apple fall from a tree on the ground. He wondered why all apples fall vertically downward and not at an angle to the vertical. Why do they not fly off in a horizontal direction?

After much thought, he came to the conclusion that the earth must be attracting the apple towards itself and this attractive force must be directed towards the center of the earth. The direction from the apple on the tree to the center of the earth is the vertical direction at the position of the apple and thus, the apple falls vertically downwards.



1.1 Concept of the gravitational force and the gravitational force between the earth and the moon.

Figure 1.1 on the left shows an apple tree on the earth. The force on an apple on the tree is towards the center of the earth i.e. along the perpendicular from the position of the apple to the surface of the earth. The Figure also shows the gravitational force between the earth and the moon. The distances in the figure are not according to scale.

Newton thought that if the force of gravitation acts on apples on the tree at different heights from the surface of the earth, can it also act on objects at even greater heights, much farther away from the earth, like for example, the moon? Can it act on even farther objects like the other planets and the Sun?

Use of ICT : Collect videos and ppts about the gravitational force of different planets.

Force and Motion

We have seen that a force is necessary to change the speed as well as the direction of motion of an object.

Can you recall? What are Newton's laws of motion?

Introduction to scientist

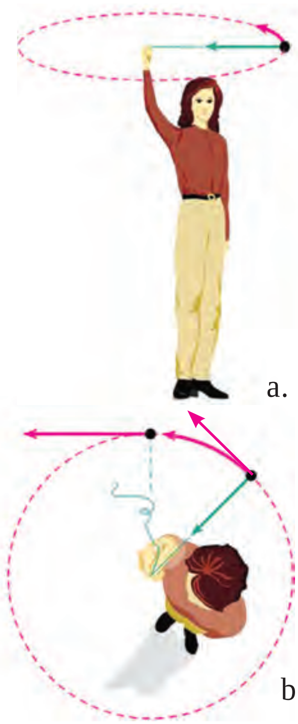
Great Scientists: Sir Isaac Newton (1642-1727) was one of the greatest scientists of recent times. He was born in England. He gave his laws of motion, equations of motion and theory of gravity in his book Principia. Before this book was written, Kepler had given three laws describing planetary motions. However, the reason why planets move in the way described by Kepler's laws was not known. Newton, with his theory of gravity, mathematically derived Kepler's laws.

In addition to this, Newton did ground breaking work in several areas including light, heat, sound and mathematics. He invented a new branch of mathematics. This is called calculus and has wide ranging applications in physics and mathematics. He was the first scientist to construct a reflecting telescope.

Circular motion and Centripetal force

Try this

Tie a stone to one end of a string. Take the other end in your hand and rotate the string so that the stone moves along a circle as shown in figure 1.2 a. Are you applying any force on the stone? In which direction is this force acting? How will you stop this force from acting? What will be the effect on the stone?



1.2 A stone tied to a string, moving along a circular path and its velocity in tangential direction

As long as we are holding the string, we are pulling the stone towards us i.e. towards the centre of the circle and are applying a force towards it. The force stops acting if we release the string. In this case, the stone will fly off along a straight line which is the tangent to the circle at the position of the stone when the string is released, because that is the direction of its velocity at that instant of time (Figure 1.2 b). You may recall that we have performed a similar activity previously in which a 5 rupee coin kept on a rotating circular disk flies off the disk along the tangent to the disk. Thus, a force acts on any object moving along a circle and it is directed towards the centre of the circle. This is called the **Centripetal force.** 'Centripetal' means centre seeking, i.e. the object tries to go towards the centre of the circle because of this force.

You know that the moon, which is the natural satellite of the earth, goes round it in a definite orbit. The direction of motion of the moon as well as its speed constantly changes during this motion. Do you think some force is constantly acting on the moon? What must be the direction of this force? How would its motion have been if no such force acted on it? Do the other planets in the solar system revolve around the Sun in a similar fashion? Is similar force acting on them? What must be its direction?

From the above activity, example and questions it is clear that for the moon to go around the earth, there must be a force which is exerted on the moon and this force must be exerted by the earth which attracts the moon towards itself. Similarly, the Sun must be attracting the planets, including the earth, towards itself.

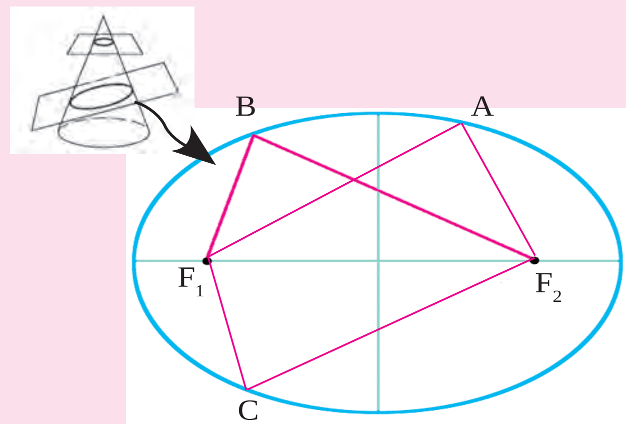
Kepler's Laws

Planetary motion had been observed by astronomers since ancient times. Before Galileo, all observations of the planet's positions were made with naked eyes. By the 16th century a lot of data were available about planetary positions and motion. Johannes Kepler, studied these data. He noticed that the motion of planets follows certain laws. He stated three laws describing planetary motion. These are known as Kepler's laws which are given below.

Do you know ?

An ellipse is the curve obtained when a cone is cut by an inclined plane. It has two focal points. The sum of the distances to the two focal points from every point on the curve is constant. F_1 and F_2 are two focal points of the ellipse shown in figure 1.3. If A, B and C are three points on the ellipse then,

$$AF_1 + AF_2 = BF_1 + BF_2 = CF_1 + CF_2$$



1.3 An ellipse

Kepler's first law :

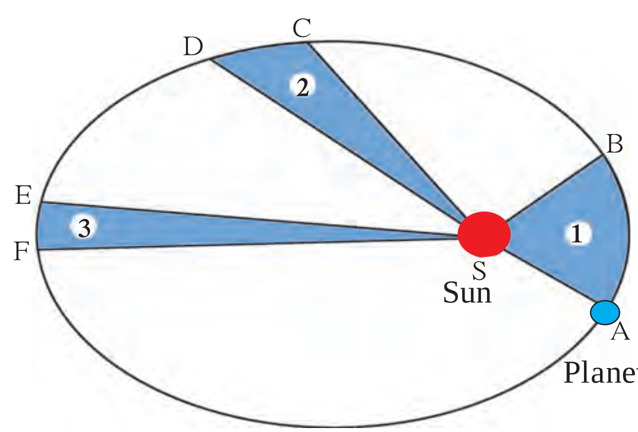
The orbit of a planet is an ellipse with the Sun at one of the foci.

Figure 1.4 shows the elliptical orbit of a planet revolving around the sun. The position of the Sun is indicated by S.

Kepler's second law :

The line joining the planet and the Sun sweeps equal areas in equal intervals of time.

AB and CD are distances covered by the planet in equal time i.e. after equal intervals of time, the positions of the planet starting from A and C are shown by B and D respectively.



1.4 The orbit of a planet moving around the Sun.

The straight lines AS and CS sweep equal area in equal interval of time i.e. area ASB and CSD are equal.

Kepler's third law :

The square of its period of revolution around the Sun is directly proportional to the cube of the mean distance of a planet from the Sun.

Thus, if r is the average distance of the planet from the Sun and T is its period of revolution then,

T^2 a r^3 i.e. $\frac{T^2}{r^3}$ = constant = K .... (1)

Kepler obtained these laws simply from the study of the positions of planets obtained by regular observations. He had no explanation as to why planets obey these laws. We will see below how these laws helped Newton in the formulation of his theory of gravitation.

Use your brain power

If the area ESF in figure 1.4 is equal to area ASB, what will you infer about EF?

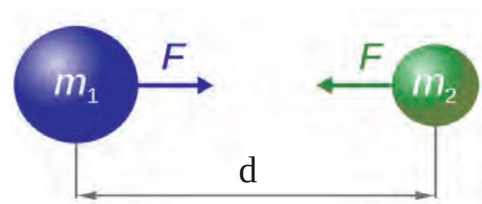
Newton's universal law of gravitation

All the above considerations including Kepler's laws led Newton to formulate his theory of Universal gravity. According to this theory, every object in the Universe attracts every other object with a definite force. This force is directly proportional to the product of the masses of the two objects and is inversely proportional to the square of the distance between them.

An introduction to scientists

Johannes Kepler (1571-1630) was a German astronomer and mathematician. He started working as a helper to the famous astronomer Tycho Brahe in Prague in 1600. After the sudden death of Brahe in 1601, Kepler was appointed as the Royal mathematician in his place. Kepler used the observations of planetary positions made by Brahe to discover the laws of planetary motion. He wrote several books. His work was later used by Newton in postulating his law of gravitation.

Figure 1.5 shows two objects with masses m_1 and m_2 kept at a distance d from each other. Mathematically, the gravitational force of attraction between these two bodies can be written as



1.5 Gravitational force between two objects

$$F \; a \; \frac{m_1 m_2}{d^2} \quad \text{or} \quad F = G \frac{m_1 m_2}{d^2} \quad .... (2)$$

Here, G is the constant of proportionality and is called the Universal gravitational constant.

The above law means that if the mass of one object is doubled, the force between the two objects also doubles. Also, if the distance is doubled, the force decreases by a factor of 4. If the two bodies are spherical, the direction of the force is always along the line joining the centres of the two bodies and the distance between the centres is taken to be d. In case when the bodies are not spherical or have irregular shape, then the direction of force is along the line joining their centres of mass and d is taken to be the distance between the two centres of mass.

From equation (2), it can be seen that the value of G is the gravitational force acting between two unit masses kept at a unit distance away from each other. Thus, in SI units, the value of G is equal to the gravitational force between two masses of 1 kg kept 1 m apart.

Use your brain power

Show that in SI units, the unit of G is Newton m^2 kg^{-2}. The value of G was first experimentally measured by Henry Cavendish. In SI units its value is 6.673×10^{-11} N m^2 kg^{-2}.

The centre of mass of an object is the point inside or outside the object at which the total mass of the object can be assumed to be concentrated. The centre of mass of a spherical object having uniform density is at its geometrical centre. The centre of mass of any object having uniform density is at its centroid.

Why did Newton assume inverse square dependence on distance in his law of gravitation? He was helped by Kepler's third law in this as shown below.

Uniform circular motion / Magnitude of centripetal force

Consider an object moving in a circle with constant speed. We have seen earlier that such a motion is possible only when the object is constantly acted upon by a force directed towards the centre of the circle. This force is called the centripetal force. If m is the mass of the object, v is its speed and r is the radius of the circle, then it can be shown that this force is equal to $F = m v^2/r$.

If a planet is revolving around the Sun in a circular orbit in uniform circular motion, then the centripetal force acting on the planet towards the Sun must be $F = mv^2/r$, where, m is the mass of the planet, v is its speed and r is its distance from the Sun.

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

The speed of the planet can be expressed in terms of the period of revolution T as follows.

The distance travelled by the planet in one revolution =perimeter of the orbit $2\pi r$; r = distance of the planet from the Sun, Time taken = Period of revolution = T

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{2\pi r}{T}$$

$$F = \frac{mv^2}{r} = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4 m \pi^2 r}{T^2}$$, multiplying and dividing by r^2 we get,

$$F = \frac{4 m \pi^2}{r^2} \times \left(\frac{r^3}{T^2}\right)$$. According to Kepler's third law, $\frac{T^2}{r^3} = K$

$$F = \frac{4 m \pi^2}{r^2 K}$$, But $\frac{4 m \pi^2}{K} = \text{Constant}$ $\quad \therefore F = \text{constant} \times \frac{1}{r^2} \quad \therefore F \propto \frac{1}{r^2}$

Thus, Newton concluded that the centripetal force which is the force acting on the planet and is responsible for its circular motion, must be inversely proportional to the square of the distance between the planet and the Sun. Newton identified this force with the force of gravity and hence postulated the inverse square law of gravitation. The gravitational force is much weaker than other forces in nature but it controls the Universe and decides its future. This is possible because of the huge masses of planets, stars and other constituents of the Universe.

Use your brain power

Is there a gravitational force between two objects kept on a table or between you and your friend sitting next to you? If yes, why don't the two move towards each other?

Example 1 : Mahendra and Virat are sitting at a distance of 1 metre from each other. Their masses are 75 kg and 80 kg respectively. What is the gravitational force between them?

Given : $r = 1$ m, $m_1 = 75$ kg, $m_2 = 80$ kg and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

According to Newton's law

$$F = \frac{G m_1 m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 75 \times 80}{1^2} \text{ N}$$

$= 4.002 \times 10^{-7}$ N

The gravitational force between Mahendra and Virat is 4.002×10^{-7} N

This is a very small force. If the force of friction between Mahendra and the bench on which he is sitting is zero, then he will start moving towards Virat under the action of this force. We can calculate his acceleration and velocity by using Newton's laws of motion.

Example 2 : In the above example, assuming that the bench on which Mahendra is sitting is frictionless, starting with zero velocity, what will be Mahendra's velocity of motion towards Virat after 1 s ? Will this velocity change with time and how?

Given: Force on Mahendra = $F = 4.002 \times 10^{-7}$ N, Mahendra's mass = m = 75 kg

According to Newton's second law, the acceleration produced by the force on Mahendra = m = 75 kg.

$$a = \frac{F}{m} = \frac{4.002 \times 10^{-7}}{75} = 5.34 \times 10^{-9} \text{ m/s}^2$$

Using Newton's first equation, we can calculate Mahendra's velocity after 1s, Newton's first equation of motion is

$v = u + a t$;

As Mahendra is sitting on the bench, his initial velocity is zero (u=0)

Assuming the bench to be frictionless,

$v = 0 + 5.34 \times 10^{-9} \times 1$ m/s

$= 5.34 \times 10^{-9}$ m/s

Mahendra's velocity after 1 s will be 5.34×10^{-9} m/s .

This is an extremely small velocity. The velocity will increase with time because of the acceleration. The acceleration will also not remain constant because as Mahendra moves towards Virat, the distance between them will decrease, causing an increase in the gravitational force, thereby increasing the acceleration as per Newton's second law of motion.

Use your brain power !

Assuming the acceleration in Example 2 above remains constant, how long will Mahendra take to move 1 cm towards Virat?

Do you know ?

You must be knowing about the high and low tides that occur regularly in the sea. The level of sea water at any given location along sea shore increases and decreases twice a day at regular intervals. High and low tides occur at different times at different places. The level of water in the sea changes because of the gravitational force exerted by the moon. Water directly under the moon gets pulled towards the moon and the level of water there goes up causing high tide at that place. At two places on the earth at 90^o from the place of high tide, the level of water is minimum and low tides occur there as shown in figure 1.6

1.6 Low and high tides

Collect information about high and low tides from geography books. Observe the timing of high and low tides at one place when you go for a picnic to be a beach. Take pictures and hold an exhibition.

Earth' gravitational force

Will the velocity of a stone thrown vertically upwards remain constant or will it change with time? How will it change? Why doesn't the stone move up all the time? Why does it fall down after reaching a certain height? What does its maximum height depend on ?

The earth attracts every object near it towards itself because of the gravitational force. The centre of mass of the earth is situated at its centre, so the gravitational force on any object due to the earth is always directed towards the centre of the earth. Because of this force, an object falls vertically downwards on the earth.

Similarly, when we throw a stone vertically upwards, this force tries to pull it down and reduces its velocity. Due to this constant downward pull, the velocity becomes zero after a while. The pull continues to be exerted and the stone starts moving vertically downward towards the centre of the earth under its influence.

Solved Examples

Example 1: Calculate the gravitational force due to the earth on Mahendra in the earlier example.

Given: Mass of the earth = $m_1 = 6 \times 10^{24}$ kg

Radius of the earth = $R = 6.4 \times 10^6$ m

Mahendra's mass = $m_2 = 75$ kg

$G = 6.67 \times 10^{-11}$ Nm^2/kg^2

Using the force law, the gravitational force on Mahendra due to earth is given by

This force is 1.83×10^9 times larger than the gravitational force between Mahendra and Virat.

$$F = \frac{G\, m_1 m_2}{R^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 75 \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} \text{ N} = 733 \text{ N}$$

Example 2: Starting from rest, what will be Mahendra's velocity after one second if he is falling down due to the gravitational force of the earth?

Given: u = 0, F = 733 N,

Mahendra's mass = m = 75 kg

time t = 1 s

Mahendra's acceleration

$$a = \frac{F}{m} = \frac{733}{75} \text{ m/s}^2$$

According to Newton's first equation of motion,

$v = u + a t$

Mahendra's velocity after 1 second

$v = 0 + 9.77 \times 1$ m/s

$v = 9.77$ m/s

This is 1.83×10^9 times Mahendra's velocity in example 2, on page 6.

Use your brain power !

According to Newton's law of gravitation, every object attracts every other object.

Thus, if the earth attracts an apple towards itself, the apple also attracts the earth towards itself with the same force. Why then does the apple fall towards the earth, but the earth does not move towards the apple?

The gravitational force due to the earth also acts on the moon because of which it revolves around the earth. Similar situation exists for the artificial satellites orbiting the earth. The moon and the artificial satellites orbit the earth. The earth attracts them towards itself but unlike the falling apple, they do not fall on the earth, why? This is because of the velocity of the moon and the satellites along their orbits. If this velocity was not there, they would have fallen on the earth.

Earth's gravitational acceleration

The earth exerts gravitational force on objects near it. According to Newton's second law of motion, a force acting on a body results in its acceleration. Thus, the gravitational force due to the earth on a body results in its acceleration. This is called acceleration due to gravity and is denoted by 'g'. Acceleration is a vector. As the gravitational force on any object due to the earth is directed towards the centre of the earth, the direction of the acceleration due to gravity is also directed towards the centre of the earth i.e. vertically downwards.

Think about it

1. What would happen if there were no gravity?
2. What would happen if the value of G was twice as large?

Value of g on the surface of the earth

We can calculate the value of g by using Newton's universal law of gravitation for an object of mass m situated at a distance r from the centre of the earth. The law of gravitation gives

$$F = \frac{G\,M\,m}{r^2} \quad \text{............. (3)}$$

M is the mass of the earth.

$$F = m\,g \quad \text{................... (4)}$$

From (3) and (4), $mg = \frac{G\,M\,m}{r^2}$

$$g = \frac{G\,M}{r^2} \quad \text{............ (5)}$$

If the object is situated on the surface of the earth, r = R = Radius of the earth. Thus, the value of g on the surface of the earth is.

$$g = \frac{G\,M}{R^2} \quad \text{......... (6)}$$

The unit of g in SI units is m/s^2. The mass and radius of the earth are 6 $\times 10^{24}$ kg and 6.4$\times 10^6$ m, respectively. Using these in (6)

$$g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.77\ m/s^2 \quad \text{......................... (7)}$$

This acceleration depends only on the mass M and radius R of the earth and so the acceleration due to gravity at a given point on the earth is the same for all objects. It does not depend on the properties of the object.

Can you tell?

What would be the value of g on the surface of the earth if its mass was twice as large and its radius half of what it is now?

Variation in value of g

A. Change along the surface of the earth : Will the value of g be the same everywhere on the surface of the earth? The answer is no. The reason is that the shape of the earth is not exactly spherical and so the distance of a point on the surface of the earth from its centre differs somewhat from place to place. Due to its rotation, the earth bulges at the equator and is flatter at the poles. Its radius is largest at the equator and smallest at the poles. The value of g is thus highest (9.832 m/s^2) at the poles and decreases slowly with decreasing latitude. It is lowest (9.78 m/s^2) at the equator.

B. Change with height : As we go above the earth's surface, the value of r in equation (5) increases and the value of g decreases. However, the decrease is rather small for heights which are small in comparison to the earth's radius. For example, remember that the radius of the earth is 6400 km. If an aeroplane is flying at a height 10 km above the surface of the earth, its distance from the earth's surface changes from 6400 km to 6410 km and the change in the value of g due to it is negligible. On the other hand, when we consider an artificial satellite orbiting the earth, we have to take into account the change in the value of g due to the large change in the distance of the satellite from the centre of the earth. Some typical heights and the values of g at these heights are given in the following table.

Place	Height (km)	g (m/s^2)
Surface of the earth (average)	0	9.8
Mount Everest	8.8	9.8
Maximum height reached by man-made balloon	36.6	9.77
Height of a typical weather satellite	400	8.7
Height of communication satellite	35700	0.225

1.7 Table showing change of g with height above the earth's surface

C. Change with depth : The value of g also changes if we go inside the earth. The value of r in equation (5) decreases and one would think that the value of g should increase as per the formula. However, the part of the earth which contributes towards the gravitational force felt by the object also decreases. Which means that the value of M to be used in equation (5) also decreases. As a combined result of change in r and M, the value of g decreases as we go deep inside the earth.

Think about it

1. Will the direction of the gravitational force change as we go inside the earth?
2. What will be the value of g at the centre of the earth?

Every planet and satellite has different mass and radius. Hence, according to equation (6), the values of g on their surfaces are different. On the moon it is about $1/6^{th}$ of the value on the earth. As a result, using the same amount of force, we can jump 6 times higher on the moon as compared to that on the earth.

Mass and Weight

Mass : Mass is the amount of matter present in the object. The SI unit of mass is kg. Mass is a scalar quantity. Its value is same everywhere. Its value does not change even when we go to another planet. According to Newton's first law, it is the measure of the inertia of an object. Higher the mass, higher is the inertia.

Weight : The weight of an object is defined as the force with which the earth attracts the object. The force (F) on an object of mass m on the surface of the earth can be written using equation (4)

$$\therefore \text{Weight, } W = F = m\,g \quad .... \quad \left(g = \frac{G\,M}{R^2} \right)$$

Weight being a force, its SI unit is Newton. Also, the weight, being a force, is a vector quantity and its direction is towards the centre of the earth. As the value of g is not same everywhere, the weight of an object changes from place to place, though its mass is constant everywhere.

Colloquially we use weight for both mass and weight and measure the weight in kilograms which is the unit of mass. But in scientific language when we say that Rajeev's weight is 75 kg, we are talking about Rajeev's mass. What we mean is that Rajeev's weight is equal to the gravitational force on 75 kg mass. As Rajeev's mass is 75 kg, his weight on earth is F = mg = 75 x 9.8 = 735 N. The weight of 1 kg mass is 1 x 9.8 = 9.8 N. Our weighing machines tell us the mass. The two scale balances in shops compare two weights i.e. two masses.

Use your brain power !

1. Will your weight remain constant as you go above the surface of the earth?
2. Suppose you are standing on a tall ladder. If your distance from the centre of the earth is 2R, what will be your weight?

Solved Examples

Example 1: If a person weighs 750 N on earth, how much would be his weight on the Moon given that moon's mass is $\frac{1}{81}$ of that of the earth and its radius is $\frac{1}{3.7}$ of that of the earth ?

Given: Weight on earth = 750 N,

Ratio of mass of the earth (M_E) to mass of the moon (M_M) = $\frac{M_E}{M_M} = 81$

Ratio of radius of earth (R_E) to radius of moon (R_M) = $\frac{R_E}{R_M} = 3.7$

Let the mass of the person be m kg

Weight on the earth = m g = 750 = $\frac{m\,G\,M_E}{R_E^{\,2}}$ $\quad \therefore m = \frac{750\,R_E^{\,2}}{(G\,M_E)}$ .... (i)

Weight on Moon $= \frac{m\,G\,M_M}{R_M^{\,2}}$ using (i)

$$= \frac{750\,R_E^{\,2}}{(G\,M_E)} \times \frac{G\,M_M}{R_M^{\,2}} = 750\,\frac{R_E^{\,2}}{R_M^{\,2}} \times \frac{M_M}{M_E} = 750 \times (3.7)^2 \times \frac{1}{81} = 126.8 \text{ N}$$

The weight on the moon is nearly 1/6th of the weight on the earth. We can write the weight on moon as mg_m (g_m is the acceleration due to gravity on the moon). Thus g_m is 1/6th of the g on the earth.

Do you know ?

Gravitational waves

Waves are created on the surface of water when we drop a stone into it. Similarly you must have seen the waves generated on a string when both its ends are held in hand and it is shaken. Light is also a type of wave called the electromagnetic wave. Gamma rays, X-rays, ultraviolet rays, infrared rays, microwave and radio waves are all different types of electromagnetic waves. Astronomical objects emit these waves and we receive them using our instruments. All our knowledge about the universe has been obtained through these waves.

Gravitational waves are a very different type of waves. They have been called the waves on the fabric of space-time. Einstein predicted their existence in 1916. These waves are very weak and it is very difficult to detect them. Scientists have constructed extremely sensitive instruments to detect the gravitational waves emitted by astronomical sources. Among these, LIGO (Laser Interferometric Gravitational Wave Observatory) is the prominent one. Exactly after hundred years of their prediction, scientists detected these waves coming from an astronomical source. Indian scientists have contributed significantly in this discovery. This discovery has opened a new path to obtain information about the Universe.

Free fall

Try this

Take a small stone. Hold it in your hand. Which forces are acting on the stone? Now release the stone. What do you observe? What are the forces acting on the stone after you release it?

We know that the force of gravity due to the earth acts on each and every object. When we were holding the stone in our hand, the stone was experiencing this force, but it was balanced by a force that we were applying on it in the opposite direction. As a result, the stone remained at rest. Once we release the stone from our hands, the only force that acts on it is the gravitational force of the earth and the stone falls down under its influence. Whenever an object moves under the influence of the force of gravity alone, it is said to be falling freely. Thus the released stone is in a free fall. In free fall, the initial velocity of the object is zero and goes on increasing due to the acceleration due to gravity of the earth. During free fall, the frictional force due to air opposes the motion of the object and a buoyant force also acts on the object. Thus, true free fall is possible only in vacuum.

For a freely falling object, the velocity on reaching the earth and the time taken for it can be calculated by using Newton's equations of motion. For free fall, the initial velocity u = 0 and the acceleration a = g. Thus we can write the equations as

$$v = g\,t$$

$$s = \frac{1}{2}\,g\,t^2$$

$$v^2 = 2\,g\,s$$

For calculating the motion of an object thrown upwards, acceleration is negative, i.e. in a direction opposite to the velocity and is taken to be $-g$. The magnitude of g is the same but the velocity of the object decreases because of this -ve acceleration.

The moon and the artificial satellites are moving only under the influence of the gravitational field of the earth. Thus they are in free fall.

Do you know ?

The value of g is the same for all objects at a given place on the earth. Thus, any two objects, irrespective of their masses or any other properties, when dropped from the same height and falling freely will reach the earth at the same time. Galileo is said to have performed an experiment around 1590 in the Italian city of Pisa. He dropped two spheres of different masses from the leaning tower of Pisa to demonstrate that both spheres reached the ground at the same time.

When we drop a feather and a heavy stone at the same time from a height, they do not reach the earth at the same time. The feather experiences a buoyant force and a frictional force due to air and therefore floats and reaches the ground slowly, later than the heavy stone. The buoyant and frictional forces on the stone are much less than the weight of the stone and does not affect the speed of the stone much. Recently, scientists performed this experiment in vacuum and showed that the feather and stone indeed reach the earth at the same time.

https://www.youtube.com/watch?v=eRNC5kcvINA

Solved Examples

Example 1. An iron ball of mass 3 kg is released from a height of 125 m and falls freely to the ground. Assuming that the value of g is 10 m/s^2, calculate

(i) time taken by the ball to reach the ground

(ii) velocity of the ball on reaching the ground

(iii) the height of the ball at half the time it takes to reach the ground.

Given: m = 3 kg, distance travelled by the ball s = 125 m, initial velocity of the ball = u = o and acceleration a = g = 10 m/s^2.

(i) Newton's second equation of motion gives

$$s = u\,t + \frac{1}{2}\,a\,t^2$$

$$\therefore\ 125 = 0\,t + \frac{1}{2} \times 10 \times t^2 = 5\,t^2$$

$$t^2 = \frac{125}{5} = 25 \qquad ,\ t = 5\ s$$

The ball takes 5 seconds to reach the ground.

(ii) According to Newton's first equation of motion final velocity = v = u + a t

= 0 + 10 x 5

= 50 m/s

The velocity of the ball on reaching the ground is 50 m/s

(iii) Half time $= t = \frac{5}{2} = 2.5$ s

Ball's height at this time = s

According to Newton's second equation

$$s = u\,t + \frac{1}{2}\,a\,t^2$$

$$s = 0 + \frac{1}{2}\,10 \times (2.5)^2 = 31.25\ m.$$

Thus the height of the ball at half time

= 125-31.25 = 93.75 m

Example 2. A tennis ball is thrown up and reaches a height of 4.05 m before coming down. What was its initial velocity? How much total time will it take to come down? Assume g = 10 m/s^2

Given: For the upward motion of the ball,

the final velocity of the ball = v = 0

Distance travelled by the ball = 4.05 m

acceleration a= - g = -10 m/s^2

Using Newton's third equation of motion

$$v^2 = u^2 + 2\,a\,s$$

$$0 = u^2 + 2\,(-10) \times 4.05$$

$$\therefore\ u^2 = 81$$

u = 9 m/s The initial velocity of the ball is 9 m/s

Now let us consider the downward motion of the ball. Suppose the ball takes t seconds to come down. Now the initial velocity of the ball is zero, u = 0. Distance travelled by the ball on reaching the ground = 4.05 m. As the velocity and acceleration are in the same direction,

a = g=10 m/s

According to Newton's second equation of motion

$$s = u\,t + \frac{1}{2}\,a\,t^2$$

$$4.05 = 0 + \frac{1}{2}\,10\,t^2$$

$$t^2 = \frac{4.05}{5} = 0.81\ , \qquad t = 0.9\ s$$

The ball will take 0.9 s to reach the ground. It will take the same time to go up. Thus, the total time taken = 2 x 0.9 = 1.8 s

Use your brain power !

According to Newton's law of gravitation, earth's gravitational force is higher on an object of larger mass. Why doesn't that object fall down with higher velocity as compared to an object with lower mass?

Gravitational potential energy

We have studied potential energy in last standard. The energy stored in an object because of its position or state is called potential energy. This energy is relative and increases as we go to greater heights from the surface of the earth. We had assumed that the potential energy of an object of mass m, at a height h from the ground is mgh and on the ground it is zero. When h is small compared to the radius R of the earth, we can assume g to be constant and can use the above formula (mgh). But for large values of h, the value of g decreases with increase in h. For an object at infinite distance from the earth, the value of g is zero and earth's gravitational force does not act on the object. So it is more appropriate to assume the value of potential energy to be zero there. Thus, for smaller distances, i.e. heights, the potential energy is less than zero, i.e. it is negative.

When an object is at a height h from the surface of the earth, its potential energy is $-\frac{GMm}{R+h}$

here, M and R are earth's mass and radius respectively.

Escape velocity

We have seen than when a ball is thrown upwards, its velocity decreases because of the gravitation of the earth. The velocity becomes zero after reaching a certain height and after that the ball starts falling down. Its maximum height depends on its initial velocity. According to Newton's third equation of motion is

$v^2 = u^2+2as$,

v = the final velocity of the ball $= 0$ and $a = -g$

$\therefore\ 0 = u^2 + 2(-g)s$ and maximum height of the ball $= s = -\frac{u^2}{2g}$

Thus, higher the initial velocity u, the larger is the height reached by the ball. The reason for this is that the higher the initial velocity, the ball will oppose the gravity of the earth more and larger will be the height to which it can reach.

We have seen above that the value of g keeps decreasing as we go higher above the surface of the earth. Thus, the force pulling the ball downward, decreases as the ball goes up. If we keep increasing the initial velocity of the ball, it will reach larger and larger heights and above a particular value of initial velocity of the ball, the ball is able to overcome the downward pull by the earth and can escape the earth forever and will not fall back on the earth. This velocity is called escape velocity. We can determine its value by using the law of conservation of energy as follows.

An object going vertically upwards from the surface of the earth, having an initial velocity equal to the escape velocity, escapes the gravitational force of the earth. The force of gravity, being inversely proportional to the square of the distance, becomes zero only at infinite distance from the earth. This means that for the object to be free from the gravity of the earth, it has to reach infinite distance from the earth. i.e. the object will come to rest at infinite distance and will stay there.

For an object of mass m

on the surface of earth

A. Kinetic energy $= \frac{1}{2} mv^2_{esc}$

B. Potential energy $= -\frac{GMm}{R}$

C. Total energy $= E_1 =$ Kinetic energy + Potential energy

$= \frac{1}{2} mv^2_{esc} - \frac{GMm}{R}$

at infinite distance from the earth

A. Kinetic energy $= 0$

B. Potential energy $= -\frac{GMm}{\infty} = 0$

C. Total energy $= E_2 =$ Kinetic energy + potential energy

$= 0$

From the principle of conservation of energy

$E_1 = E_2$

$$\frac{1}{2} mv^2_{esc} - \frac{GMm}{R} = 0$$

$$v^2_{esc} = \frac{2\,GM}{R}$$

$$v_{esc} = \sqrt{\frac{2\,GM}{R}}$$

$$= \sqrt{2\,g\,R}$$

$$= \sqrt{(2 \times 9.8 \times 6.4 \times 10^6)} = 11.2 \text{ km/s}$$

The spacecrafts which are sent to the moon or other planets have to have their initial velocity larger than the escape velocity so that they can overcome earth's gravitational attraction and can travel to these objects.

Solved Examples

Example 1. Calculate the escape velocity on the surface of the moon given the mass and radius of the moon to be 7.34×10^{22} kg and 1.74×10^6 m respectively.

Given: $G = 6.67 \times 10^{-11}$ N m^2/kg^2, mass of the moon = M = 7.34×10^{22} kg and radius of the moon = R= 1.74×10^6 m.

Escape velocity = $v_{esc} = \sqrt{\frac{2\,GM}{R}}$

$$\sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{1.74 \times 10^6}}$$

= 2.37 km/s

Escape velocity on the moon 2.37 km/s.

Do you know ?

Weightlessness in space

Space travellers as well as objects in the spacecraft appear to be floating. Why does this happen? Though the spacecraft is at a height from the surface of the earth, the value of g there is not zero. In the space station the value of g is only 11% less than its value on the surface of the earth. Thus, the height of a spacecraft is not the reason for their weightlessness. Their weightlessness is caused by their being in the state of free fall. Though the spacecraft is not falling on the earth because of its velocity along the orbit, the only force acting on it is the gravitational force of the earth and therefore it is in a free fall. As the velocity of free fall does not depend on the properties of an object, the velocity of free fall is the same for the spacecraft, the travelers and the objects in the craft. Thus, if a traveller releases an object from her hand, it will remain stationary with respect to her and will appear to be weightless.

Exercise

1. Study the entries in the following table and rewrite them putting the connected items in a single row.

I	II	III
Mass	m/s^2	Zero at the centre
Weight	kg	Measure of inertia
Acceleration due to gravity	Nm^2/kg^2	Same in the entire universe
Gravitational constant	N	Depends on height

2. Answer the following questions.

a. What is the difference between mass and weight of an object. Will the mass and weight of an object on the earth be same as their values on Mars? Why?

b What are (i) free fall, (ii) acceleration due to gravity (iii) escape velocity (iv) centripetal force ?

c. Write the three laws given by Kepler. How did they help Newton to arrive at the inverse square law of gravity?

d. A stone thrown vertically upwards with initial velocity u reaches a height 'h' before coming down. Show that the time taken to go up is same as the time taken to come down.

e. If the value of g suddenly becomes twice its value, it will become two times more difficult to pull a heavy object along the floor. Why?

3. Explain why the value of g is zero at the centre of the earth.

4. Let the period of revolution of a planet at a distance R from a star be T. Prove that if it was at a distance of 2R from the star, its period of revolution will be $\sqrt{8}$ T.

5. Solve the following examples.

a. An object takes 5 s to reach the ground from a height of 5 m on a planet. What is the value of g on the planet?

Ans: 0.4 m/s^2

b. The radius of planet A is half the radius of planet B. If the mass of A is M_A, what must be the mass of B so that the value of g on B is half that of its value on A?

Ans: 2 M_A

c. The mass and weight of an object on earth are 5 kg and 49 N respectively. What will be their values on the moon? Assume that the acceleration due to gravity on the moon is 1/6th of that on the earth.

Ans: 5 kg and 8.17 N

d. An object thrown vertically upwards reaches a height of 500 m. What was its initial velocity? How long will the object take to come back to the earth? Assume g = 10 m/s^2

Ans: 100 m/s and 20 s

e. A ball falls off a table and reaches the ground in 1 s. Assuming g = 10 m/s^2, calculate its speed on reaching the ground and the height of the table.

Ans. 10 m/s and 5 m

f. The masses of the earth and moon are 6 x 10^{24} kg and 7.4x10^{22} kg, respectively. The distance between them is 3.84 x 10^5 km. Calculate the gravitational force of attraction between the two?

Use G = 6.7 x 10^{-11} N m^2 kg^{-2}

Ans: 2 x 10^{20} N

g. The mass of the earth is 6 x 10^{24} kg. The distance between the earth and the Sun is 1.5x 10^{11} m. If the gravitational force between the two is 3.5 x 10^{22} N, what is the mass of the Sun?

Use G = 6.7 x 10^{-11} N m^2 kg^{-2}

Ans: 1.96 x 10^{30} kg

Project:

Take weights of five of your friends. Find out what their weights will be on the moon and the Mars.

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