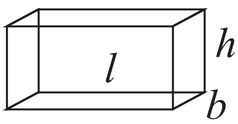
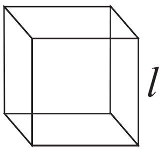
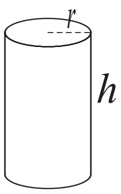
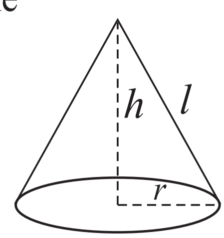
7 Mensuration

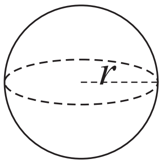
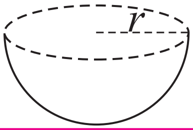
Let's study.

- Mixed examples on surface area and volume of different solid figures
- Arc of circle - length of arc
- Area of a sector
- Area of segment of a circle

Let's recall.

Last year we have studied surface area and volume of some three dimentional figures. Let us recall the formulae to find the surface areas and volumes.

No.	Three dimensional figure	Formulae
1 .	Cuboid 	Lateral surface area $= 2h\,(\,l + b\,)$ Total surface area $= 2\,(lb + bh + hl\,)$ Volume $= lbh$
2 .	Cube 	Lateral surface area $= 4l^2$ Total surface area $= 6l^2$ Volume $= l^3$
3 .	Cylinder 	Curved surface area $= 2\pi rh$ Total surface area $= 2\pi r\,(\,r + h\,)$ Volume $= \pi r^2 h$
4 .	Cone 	Slant height $(l) = \sqrt{h^2 + r^2}$ Curved surface area $= \pi rl$ Total surface area $= \pi r\,(r + l)$ Volume $= \frac{1}{3} \times \pi r^2 h$

No.	Three dimensional figure	Formulae
5.	Sphere 	Surface area = $4\ \pi r^2$ Volume = $\frac{4}{3}\ \pi r^3$
6.	Hemisphere 	Curved surface area = $2\pi r^2$ Total surface area of a solid hemisphere = $3\pi r^2$ Volume = $\frac{2}{3}\ \pi r^3$

Solve the following examples

Ex. (1)

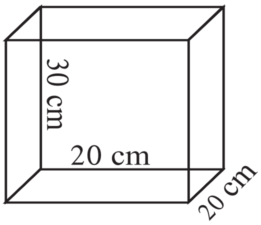


Fig 7.1

The length, breadth and height of an oil can are 20 cm, 20 cm and 30 cm respectively as shown in the adjacent figure.

How much oil will it contain ?

(1 litre = 1000 cm^3)

Ex. (2)

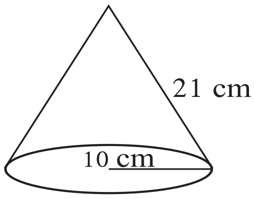


Fig 7.2

The adjoining figure shows the measures of a Joker's cap. How much cloth is needed to make such a cap ?

Let's think.

As shown in the adjacent figure, a sphere is placed in a cylinder. It touches the top, bottom and the curved surface of the cylinder. If radius of the base of the cylinder is 'r',

(1) What is the ratio of the radii of the sphere and the cylinder ?

(2) What is the ratio of the curved surface area of the cylinder and the surface area of the sphere ?

(3) What is the ratio of the volumes of the cylinder and the sphere ?

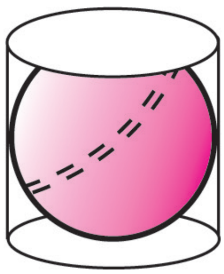


Fig. 7.3

Activity :

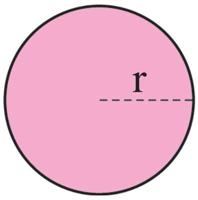


Fig. 7.4

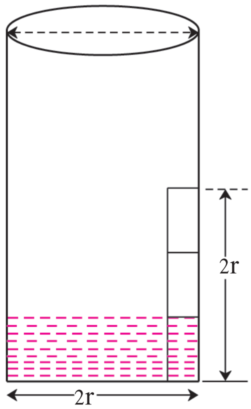


Fig. 7.5

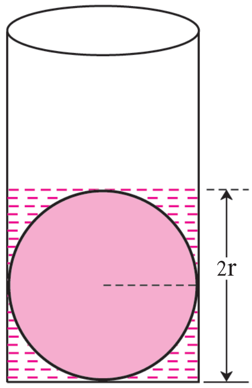


Fig. 7.6

As shown in the above figures, take a ball and a beaker of the same radius as that of the ball. Cut a strip of paper of length equal to the diameter of the beaker. Draw two lines on the strip dividing it into three equal parts. Stick it on the beaker straight up from the bottom. Fill water in the beaker upto the first mark of the strip from the bottom. Push the ball in the beaker slowly so that it touches its bottom. Observe how much the water level rises.

You will notice that the water level has risen exactly upto the total height of the strip. Try to understand how we get the formula for the volume of a the sphere. The shape of the beaker is cylindrical.

Therefore, the volume of the part of the beaker upto height 2r can be obtained by the formula of volume of a cylinder. Let us assume that the volume is V.

$$\therefore\ V = \pi r^2 h = \pi \times r^2 \times 2r = 2\pi r^3 \qquad (\because\ h = 2r)$$

But V = volume of the ball + volume of the water which was already in the beaker.

$$= \text{volume of the ball} + \frac{1}{3} \times 2\pi r^3$$

$$\therefore\ \text{volume of the ball} = V - \frac{1}{3} \times 2\pi r^3$$

$$= 2\pi r^3 - \frac{2}{3}\pi r^3$$

$$= \frac{6\pi r^3 - 2\pi r^3}{3} \qquad = \frac{4\pi r^3}{3}$$

Hence we get the formula of the volume of a sphere as $V = \frac{4}{3}\pi r^3$

(Now you can find the answer of the question number 3 relating to figure 7.3)

Solved Examples

Ex. (1) The radius and height of a cylindrical water reservoir is 2.8 m and 3.5 m respectively. How much maximum water can the tank hold ? A person needs 70 litre of water per day. For how many persons is the water sufficient for a day ? ($\pi = \frac{22}{7}$)

Solution : (r) = 2.8 m, (h) = 3.5 m, $\pi = \frac{22}{7}$

Capacity of the water reservoir = Volume of the cylindrical reservoir

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 2.8 \times 2.8 \times 3.5$$

$$= 86.24 \text{ m}^3$$

$$= 86.24 \times 1000 \quad (\because 1m^3 = 1000 \text{ litre})$$

$$= 86240.00 \text{ litre.}$$

$\therefore$ the reservoir can hold 86240 litre of water.

The daily requirement of water of a person is 70 litre.

$\therefore$ water in the tank is sufficient for $\frac{86240}{70}$ = 1232 persons.

Ex. (2) How many solid cylinders of radius 10 cm and height 6 cm can be made by melting a solid sphere of radius 30 cm ?

Solution : Radius of a sphere, r = 30 cm

Radius of the cylinder, R = 10 cm

Height of the cylinder, H = 6 cm

Let the number of cylinders be n.

Volume of the sphere = n × volume of a cylinder

$$\therefore \; n = \frac{\text{Volume of the sphere}}{\text{Volume of a cylinder}}$$

$$= \frac{\frac{4}{3}\pi(r)^3}{\pi(R)^2 H}$$

$$= \frac{\frac{4}{3}\times(30)^3}{10^2\times 6} = \frac{\frac{4}{3}\times 30\times 30\times 30}{10\times 10\times 6} = 60$$

$\therefore$ 60 cylinders can be made .

Ex. (3) A tent of a circus is such that its lower part is cylindrical and upper part is conical. The diameter of the base of the tent is 48 m and the height of the cylindrical part is 15 m. Total height of the tent is 33 m. Find area of canvas required to make the tent. Also find volume of air in the tent.

Solution : Total height of the tent = 33 m.

Let height of the cylindrical part be H

$\therefore$ H = 15 m.

Let the height of the conical part be h

$\therefore h = (33-15) = 18$ m.

Slant height of cone, $l = \sqrt{r^2 + (h)^2}$

$= \sqrt{24^2 + 18^2}$

$= \sqrt{576 + 324}$

$= \sqrt{900}$

= 30 m.

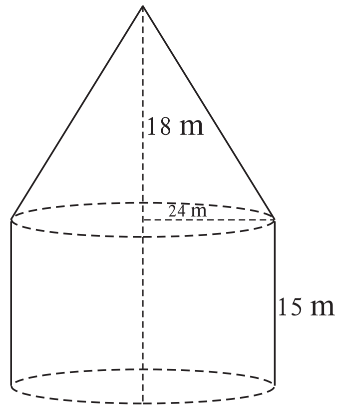


Fig 7.7

Canvas required for tent = Curved surface area of the cylindrical part + Curved surface area of conical part

$= 2\pi rH + \pi rl$

$= \pi r\,(2H + l)$

$= \frac{22}{7} \times 24\,(2 \times 15 + 30)$

$= \frac{22}{7} \times 24 \times 60$

$= 4525.71\ m^2$

Volume of air in the tent = volume of cylinder + volume of cone

$= \pi r^2 H + \frac{1}{3}\pi r^2 h$

$= \pi r^2 \left(H + \frac{1}{3}h\right)$

$= \frac{22}{7} \times 24^2\,(15 + \frac{1}{3} \times 18)$

$= \frac{22}{7} \times 576 \times 21$

$= 38{,}016\ m^3$

$\therefore$ canvas required for the tent = $4525.71\ m^2$

$\therefore$ volume of air in the tent = $38{,}016\ m^3$.

Practice set 7.1

1. Find the volume of a cone if the radius of its base is 1.5 cm and its perpendicular height is 5 cm.
2. Find the volume of a sphere of diameter 6 cm.
3. Find the total surface area of a cylinder if the radius of its base is 5 cm and height is 40 cm.
4. Find the surface area of a sphere of radius 7 cm.
5. The dimensions of a cuboid are 44 cm, 21 cm, 12 cm. It is melted and a cone of height 24 cm is made. Find the radius of its base.
6.

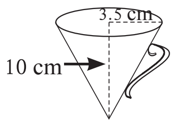


Fig 7.8
conical water jug

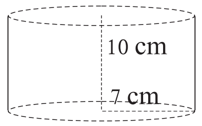


Fig 7.9
cylindrical water pot

Observe the measures of pots in figure 7.8 and 7.9. How many jugs of water can the cylindrical pot hold ?

7. A cylinder and a cone have equal bases. The height of the cylinder is 3 cm and the area of its base is 100 cm^2.The cone is placed upon the cylinder. Volume of the solid figure so formed is 500 cm^3. Find the total height of the figure.

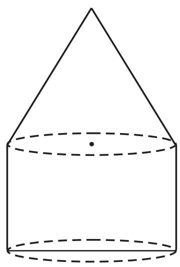


Fig 7.10

8. In figure 7.11, a toy made from a hemisphere, a cylinder and a cone is shown. Find the total area of the toy.

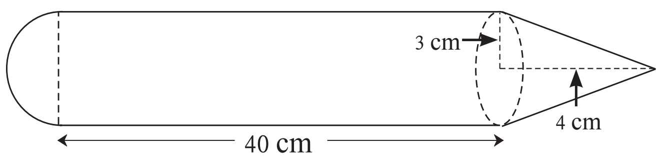


Fig. 7.11

9. In the figure 7.12, a cylindrical wrapper of flat tablets is shown. The radius of a tablet is 7 mm and its thickness is 5 mm. How many such tablets are wrapped in the wrapper ?

Fig. 7.12

10. Figure 7.13 shows a toy. Its lower part is a hemisphere and the upper part is a cone. Find the volume and the surface area of the toy from the measures shown in the figure.(π= 3.14)

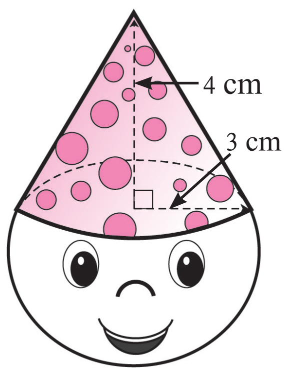


Fig. 7.13

11. Find the surface area and the volume of a beach ball shown in the figure.

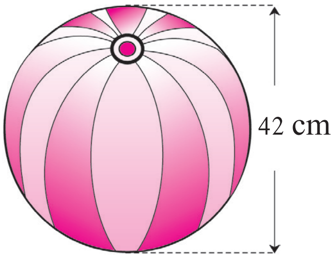


Fig. 7.14

12. As showm in the figure, a cylindrical glass contains water. A metal sphere of diameter 2 cm is immersed in it. Find the volume of the water.

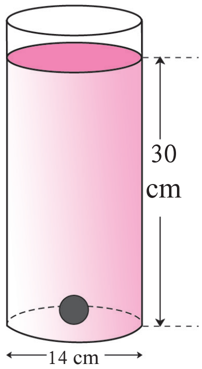


Fig. 7.15

Let's learn.

Frustum of a cone

The shape of glass used to drink water as well as the shape of water it contains, are examples of frustum of a cone.

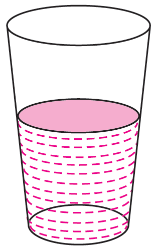


Fig. 7.16

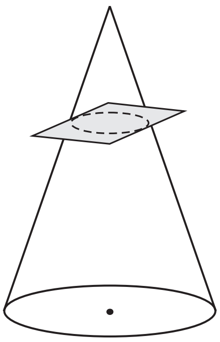


Fig. 7.17
A cone being cut

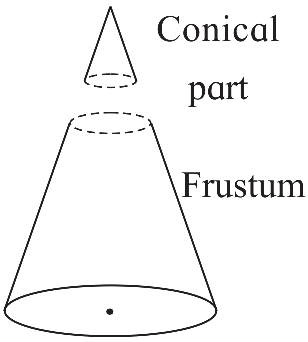


Fig. 7.18
Two parts of the cone

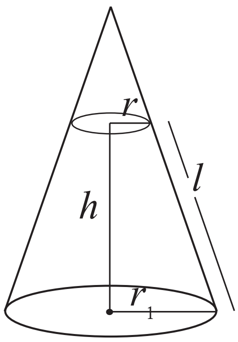


Fig. 7.19
Frustum

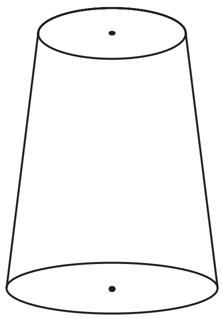


Fig. 7.20
A glass placed upside down

When a cone is cut parallel to its base we get two figures; one is a cone and the other is a frustum.

Volume and surface area of a frustum can be calculated by the formulae given below.

Remember this!

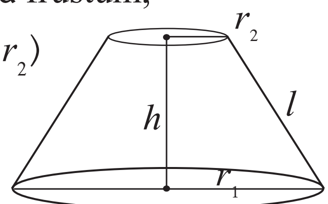


Fig. 7.21

h = height of a frustum, l = slant height height of a frustum,

r_1 and r_2 = radii of circular faces of a frustum ($r_1 > r_2$)

Slant height of a frustum $= l = \sqrt{h^2 + (r_1 - r_2)^2}$

Curved surface area of a frustum $= \pi l\,(r_1 + r_2)$

Total surface area of a frustum $= \pi l\,(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

Volume of a frustum $= \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 \times r_2)$

Solved Examples

Ex. (1) A bucket is frustum shaped. Its height is 28 cm. Radii of circular faces are 12 cm and 15 cm. Find the capacity of the bucket. ($\pi = \frac{22}{7}$)

Solution : r_1 = 15 cm, r_2 = 12 cm, h = 28 cm

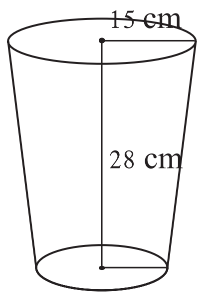


Fig. 7.22

Capacity of the bucket = Volume of frustum

$$= \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 \times r_2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 28\,(15^2 \times 12^2 + 15 \times 12)$$

$$= \frac{22 \times 4}{3} \times (225 + 144 + 180)$$

$$= \frac{22 \times 4}{3} \times 549$$

$$= 88 \times 183$$

$$= 16104\ \text{cm}^3 = 16.104\ \text{litre}$$

$\therefore$ capacity of the bucket is 16.104 litre.

Ex. (2) Radii of the top and the base of a frustum are 14 cm, 8 cm respectively. Its height is 8 cm. Find its

i) curved surface area   ii) total surface area   iii) volume.

Solution : r_1 = 14 cm , r_2 = 8 cm, h = 8 cm

Slant height of the frustum $= l = \sqrt{h^2 + (r_1 - r_2)^2}$

$$= \sqrt{8^2 + (14 - 8)^2}$$

$$= \sqrt{64 + 36} = 10\ \text{cm}$$

Curved surface area of the frustum = $\pi(r_1 + r_2)\, l$

$= 3.14 \times (14 + 8) \times 10$

$= 690.8 \text{ cm}^2$

Total surface area of frustum = $\pi\, l\, (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

$= 3.14 \times 10\,(14 + 8) + 3.14 \times 14^2 + 3.14 \times 8^2$

$= 690.8 + 615.44 + 200.96$

$= 690.8 + 816.4$

$= 1507.2 \text{ cm}^2$

Volume of the frustum = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 \times r_2)$

$= \frac{1}{3} \times 3.14 \times 8\,(14^2 + 8^2 + 14 \times 8)$

$= 3114.88 \text{cm}^3$

Practice set 7.2

1. The radii of two circular ends of frustum shape bucket are 14 cm and 7 cm. Height of the bucket is 30 cm. How many liters of water it can hold ?
(1 litre = 1000 cm^3)

2. The radii of ends of a frustum are 14 cm and 6 cm respectively and its height is 6 cm. Find its

i) curved surface area ii) total surface area. iii) volume ($\pi = 3.14$)

3. The circumferences of circular faces of a frustum are 132 cm and 88 cm and its height is 24 cm. To find the curved surface area of the frustum complete the following activity. ($\pi = \frac{22}{7}$).

circumference$_1$ = $2\pi r_1 = 132$

$r_1 = \frac{132}{2\pi} = \square$

circumference$_2$ = $2\pi r_2 = 88$

$r_2 = \frac{88}{2\pi} = \square$

slant height of frustum, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{\square^2 + \square^2}$

$= \square$ cm

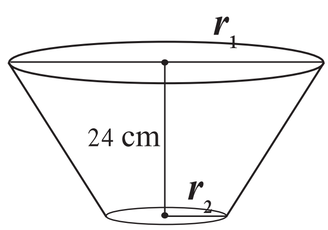


Fig. 7.23

curved surface area of the frustum = $\pi(r_1 + r_2)l$

$= \pi \times \square \times \square$

$= \square$ sq.cm.

Let's recall.

Complete the following table with the help of figure 7.24.

Type of arc	Name of the arc	Measure of the arc
Minor arc	arc AXB	....
....	arc AYB	....

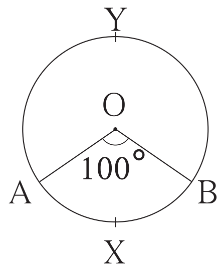


Fig. 7.24

Let's learn.

Sector of a circle

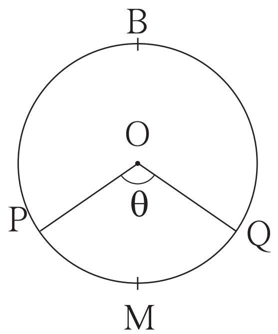


Fig. 7.25

In the adjacent figure, the central angle divides the circular region in two parts. Each of the parts is called a sector of the circle. Sector of a circle is the part enclosed by two radii of the circle and the arc joining their end points.

In the figure 7.25, O –PMQ and O–PBQ are two sectors of the circle.

Minor Sector :

Sector of a circle enclosed by two radii and their corresponding minor arc is called a 'minor sector'.

In the above figure O–PMQ is a minor sector.

Major Sector :

Sector of a circle that is enclosed by two radii and their corresponding major arc is called a 'major sector'.

In the above figure, O–PBQ is a major sector.

Area of a sector

Observe the figures below. Radii of all circles are equal. Observe the areas of the shaded regions and complete the following table.

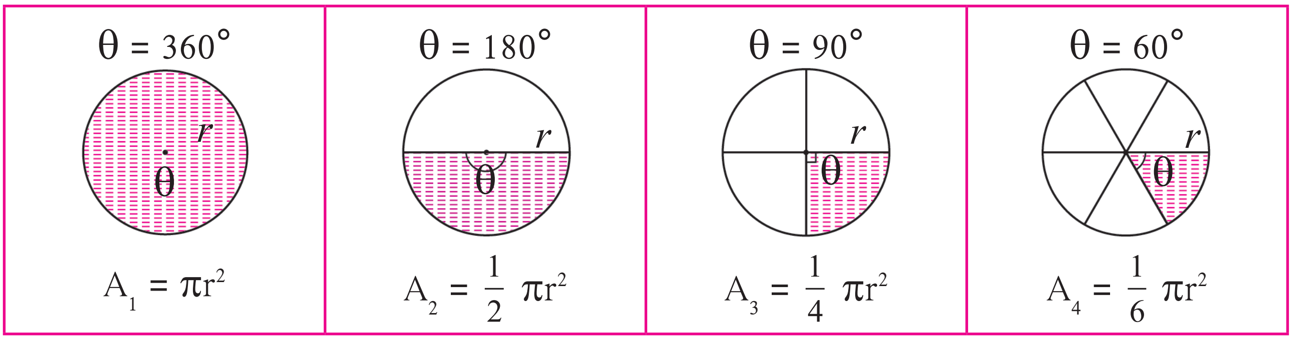


Fig. 7.26

Central angle of a circle is = 360° = complete angle

Central angle of a circle is = 360°, Area of a circle = πr^2			
Sector of circle	Measure of arc of the sector	$\frac{\theta}{360}$	Area of the sector A
A_1	360°	$\frac{360}{360}=1$	$1 \times \pi r^2$
A_2	180°	$\frac{1}{2}$	$\frac{1}{2} \times \pi r^2$
A_3	90°	$\frac{1}{4}$	$\frac{1}{4} \times \pi r^2$
A_4	60°	....	....
A	θ	$\frac{\theta}{360}$	$\frac{\theta}{360} \times \pi r^2$

From the above table we see that, if measure of an arc of a circle is θ, then the area of its corresponding sector is obtained by multiplying area of the circle by $\frac{\theta}{360}$.

$$\text{Area of sector (A)} = \frac{\theta}{360} \times \pi r^2$$

From the formula,

$$\frac{A}{\pi r^2} = \frac{\theta}{360} \text{ ; that is } \frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\theta}{360}$$

Length of an arc

In the following figures, radii of all circles are equal. Observe the length of arc in each figure and complete the table.

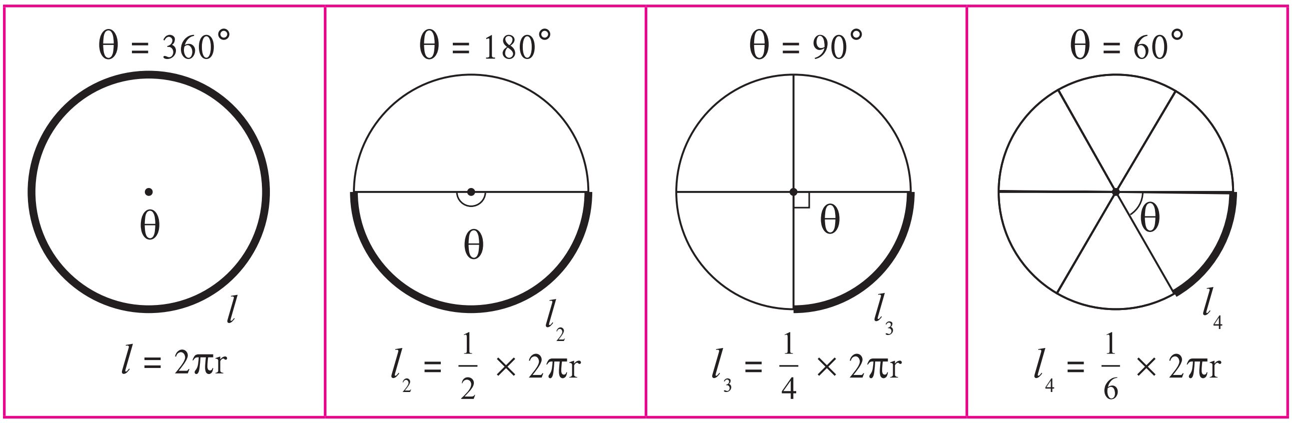


Fig. 7.27

Circumference of a circle = $2\pi r$			
Length of the arc	Measure of the arc (θ)	$\frac{\theta}{360}$	Length of the arc (l)
l_1	360°	$\frac{360}{360} = 1$	$1 \times 2\pi r$
l_2	180°	$\frac{180}{360} = \frac{1}{2}$	$\frac{1}{2} \times 2\pi r$
l_3	90°	$\frac{90}{360} = \frac{1}{4}$	$\frac{1}{4} \times 2\pi r$
l_4	60°	....	....
l	θ	$\frac{\theta}{360}$	$\frac{\theta}{360} \times 2\pi r$

The pattern in the above table shows that, if measure of an arc of a circle is θ, then its length is obtained by multiplying the circumference of the circle by $\frac{\theta}{360}$.

$$\text{Length of an arc } (l) = \frac{\theta}{360} \times 2\pi r$$

From the formula, $\frac{l}{2\pi r} = \frac{\theta}{360}$

that is, $\frac{\text{Length of an arc}}{\text{Circumference}} = \frac{\theta}{360}$

A relation between length of an arc and area of the sector

Area of a sector, (A) = $\frac{\theta}{360} \times \pi r^2$ I

Length of an arc, (l) = $\frac{\theta}{360} \times 2\pi r$

$\therefore \frac{\theta}{360} = \frac{l}{2\pi r}$ II

$\therefore A = \frac{l}{2\pi r} \times \pi r^2$ ....From I and II

$A = \frac{1}{2} lr = \frac{lr}{2}$

$\therefore$ Area of a sector = $\frac{\text{Length of the arc} \times \text{Radius}}{2}$

Similarly, $\frac{A}{\pi r^2} = \frac{l}{2\pi r} = \frac{\theta}{360}$

Solved Examples

Ex. (1) The measure of a central angle of a circle is 150 ° and radius of the circle is 21 cm. Find the length of the arc and area of the sector associated with the central angle.

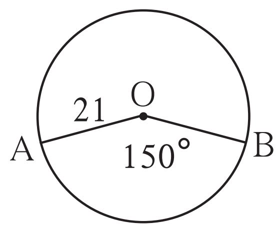


Fig. 7.28

Solution : r = 21 cm, θ = 150, $\pi = \frac{22}{7}$

Area of the sector, A $= \frac{\theta}{360} \times \pi r^2$

$= \frac{150}{360} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1155}{2} = 577.5 \text{ cm}^2$

Length of the arc, $l = \frac{\theta}{360} \times 2\pi r$

$= \frac{150}{360} \times 2 \times \frac{22}{7} \times 21$

= 55 cm

ex. (2) In figure 7.29, P is the centre of the circle of radius 6 cm. Seg QR is a tangent at Q. If PR = 12, find the area of the shaded region. ($\sqrt{3}$ = 1.73)

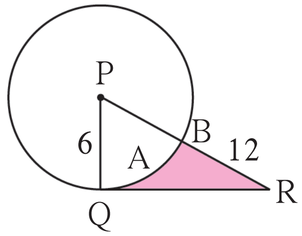


Fig. 7.29

Solution Radius joining point of contact of the tangent is perpendicular to the tangent.

$\therefore$ in Δ PQR, $\angle$ PQR = 90°, PQ = 6 cm, PR = 12 cm $\therefore$ PQ = $\frac{PR}{2}$

If one side of a right anglesd triangle is half the hypoteneus then angle opposite to, that side is of 30° measure

$\therefore \angle R = 30°$ and $\angle P = 60°$

$\therefore$ by 30°-60°-90° theorem, QR = $\frac{\sqrt{3}}{2} \times PR = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}$

$\therefore$ QR = $6\sqrt{3}$ cm

$\therefore A(\Delta PQR) = \frac{1}{2}$ QR × PQ

$= \frac{1}{2} \times 6\sqrt{3} \times 6$

$= 18\sqrt{3} = 18 \times 1.73$

$= 31.14$ cm^2

Area of a sector = $\frac{\theta}{360} \times \pi r^2$

A(P-QAB) = $\frac{60}{360} \times 3.14 \times 6^2$

$= \frac{1}{6} \times 3.14 \times 6 \times 6 = 3.14 \times 6$

$= 18.84$ cm^2

Area of shaded region = A(PQR) – A(P-QAB)

= 31.14 – 18.84

= 12.30 cm^2

Area of the shaded region = 12.30 cm^2

Activity In figure 7.30, side of square ABCD is 7 cm. With centre D and radius DA, sector D − AXC is drawn. Fill in the following boxes properly and find out the area of the shaded region.

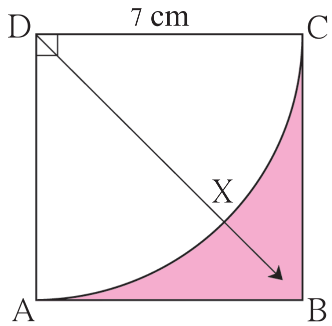


Fig. 7.30

Solution : Area of a square = $\square$ (Formula)

= $\square$

= 49 cm^2

Area of sector (D- AXC) = $\square$ (Formula)

$= \dfrac{\square}{360} \times \dfrac{22}{7} \times \square$

= 38.5 cm^2

A (shaded region) = A $\square$ − A $\square$

= $\square$ cm^2 − $\square$ cm^2

= $\square$ cm^2

Practice set 7.3

1. Radius of a circle is 10 cm. Measure of an arc of the crcleis 54°. Find the area of the sector associated with the arc. (π = 3.14)
2. Measure of an arc of a circle is 80 cm and its radius is 18 cm. Find the length of the arc. (π = 3.14)
3. Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector.
4. Radius of a circle is 10 cm. Area of a sector of the sector is 100 cm^2. Find the area of its corresponding major sector. (π = 3.14)
5. Area of a sector of a circle of radius 15 cm is 30 cm^2. Find the length of the arc of the sector.
6. In the figure 7.31, radius of the circle is 7 cm and m(arc MBN) = 60°, find (1) Area of the circle .
   (2) A(O - MBN) .
   (3) A(O - MCN) .

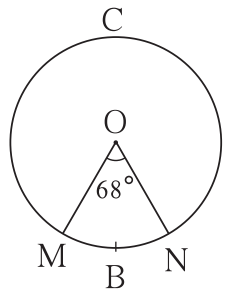


Fig. 7.31

7. In figure 7.32, radius of circle is 3.4 cm and perimeter of sector P-ABC is 12.8 cm. Find A(P-ABC).

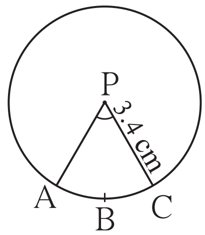


Fig. 7.32

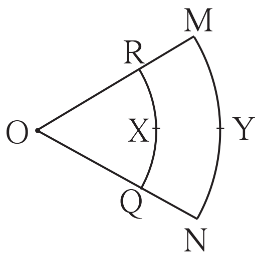


Fig. 7.33

8. In figure 7.33 O is the centre of the sector. $\angle$ ROQ = $\angle$ MON = 60°. OR = 7 cm, and OM = 21 cm. Find the lengths of arc RXQ and arc MYN. ($\pi = \frac{22}{7}$)

9. In figure 7.34, if A(P-ABC) = 154 cm^2 radius of the circle is 14 cm,

find (1) $\angle$ APC.

(2) l(arc ABC) .

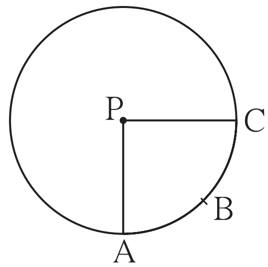


Fig. 7.34

10. Radius of a sector of a circle is 7 cm. If measure of arc of the sector is -

(1) 30° (2) 210° (3) three right angles;

find the area of the sector in each case.

11. The area of a minor sector of a circle is 3.85 cm^2 and the measure of its central angle is 36°. Find the radius of the circle.

12. In figure 7.35, □ PQRS is a rectangle. If PQ = 14 cm, QR = 21 cm, find the areas of the parts x, y and z .

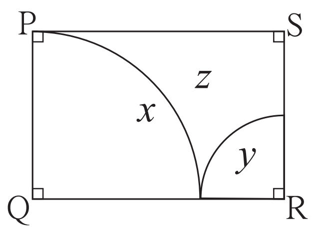


Fig. 7.35

13. Δ LMN is an equilateral triangle. LM = 14 cm. As shown in figure, three sectors are drawn with vertices as centres and radius7 cm.

Find,

(1) A (Δ LMN)

(2) Area of any one of the sectors.

(3) Total area of all the three sectors.

(4) Area of the shaded region.

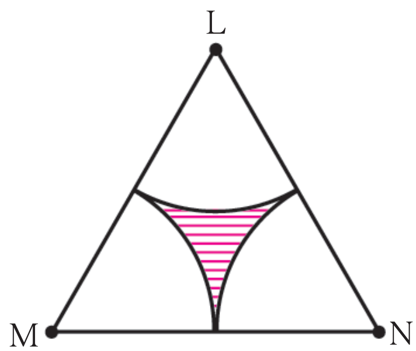


Fig. 7.36

Let's learn.

Segment of a circle

Segment of a circle is the region bounded by a chord and its corresponding arc of the circle.

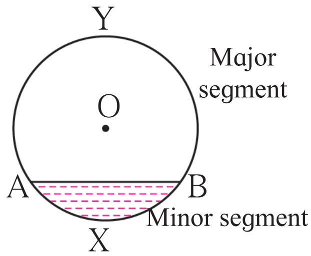


Fig. 7.37

Minor segment : The area enclosed by a chord and its corresponding minor arc is called a minor segment. In the figure, segment AXB is a minor segment.

Major segment : The area enclosed by a chord and its corresponding major arc is called a major segment. In the figure, seg AYB is a major segment.

Semicircular segment : A segment formed by a diameter of a circle is called a semicircular segment.

Area of a Segment

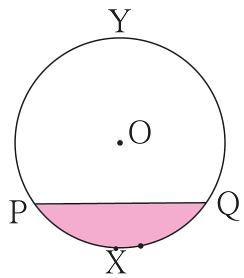


Fig. 7.38

In figure 7.38, PXQ is a minor segment and PYQ is a major segment.

How can we calculate the area of a minor segment?

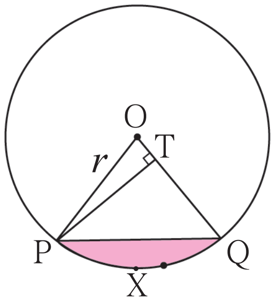


Fig. 7.39

In figure 7.39, draw radii OP and OQ. You know how to find the area of sector O−PXQ and Δ OPQ. We can get area of segment PXQ by substracting area of the triangle from the area of the sector.

A (segment PXQ) = A (O − PXQ) − A (Δ OPQ)

$$= \frac{\theta}{360} \times \pi r^2 - A(\Delta\ OPQ) \ \ldots\ldots\ldots\ldots\ (I)$$

In the figure, seg PT $\perp$ radius OQ.

Now, in Δ OTP $\sin\theta = \frac{PT}{OP}$

$\therefore$ PT = OP $\sin\theta$

$PT = r \times \sin\theta \qquad (\because OP = r)$

$A(\Delta OPQ) = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times OQ \times PT$

$= \frac{1}{2} \times r \times r \sin\theta$

$= \frac{1}{2} \times r^2 \sin\theta$ .... (II)

From (I) and (II) ,

$A(\text{segment } PXQ) = \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \times \sin\theta$

$= r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$

(Note that, we have studied the sine ratios of acute angles only. So we can use the above formula when $\theta \leq 90°$.)

Solved Examples

Ex. (1) In the figure 7.40, $\angle AOB = 30°$, OA = 12 cm . Find the area of the segment. ($\pi = 3.14$)

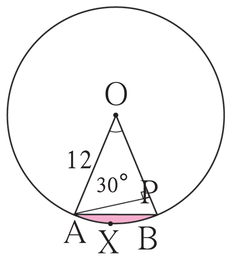


Fig. 7.40

Method I

$r = 12, \quad \theta = 30°, \quad \pi = 3.14$

$A(O\text{–}AXB) = \frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times 3.14 \times 12^2$

$= 3.14 \times 12$

$= 37.68 \text{ cm}^2$

$A(\Delta OAB) = \frac{1}{2} r^2 \times \sin\theta$

$= \frac{1}{2} \times 12^2 \times \sin 30$

$= \frac{1}{2} \times 144 \times \frac{1}{2}$

.....$(\because \sin 30° = \frac{1}{2})$

$= 36 \text{ cm}^2$

$$A(\text{segment AXB}) = A(O\text{-}AXB) - A(\Delta OAB)$$
$$= 37.68 - 36$$
$$= 1.68 \text{ cm}^2$$

Method II

$$A(\text{segment AXB}) = r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$$
$$= 12^2\left[\frac{3.14\times 30}{360} - \frac{\sin 30}{2}\right]$$
$$= 144\left[\frac{3.14}{12} - \frac{1}{2\times 2}\right]$$
$$= \frac{144}{4}\left[\frac{3.14}{3} - 1\right]$$
$$= 36\left[\frac{3.14-3}{3}\right]$$
$$= \frac{36}{3}\times 0.14$$
$$= 12 \times 0.14$$
$$= 1.68 \text{ cm}^2.$$

Ex. (2) The radius of a circle with centre P is 10 cm. If chord AB of the circle substends a right angle at P, find areas of the minor segment and the major segment. ($\pi = 3.14$)

Solution : $r = 10$ cm, $\theta = 90$, $\pi = 3.14$

$$A(P\text{-}AXB) = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{90}{360} \times 3.14 \times 10^2$$
$$= \frac{1}{4} \times 314$$
$$= 78.5 \text{ cm}^2$$

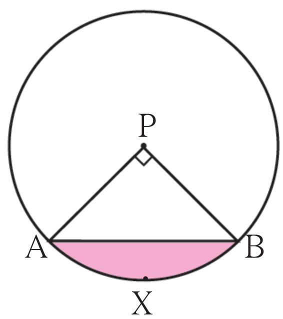


Fig. 7.41

$$A(\Delta APB) = \frac{1}{2} \text{ base} \times \text{height}$$
$$= \frac{1}{2} \times 10 \times 10$$
$$= 50 \text{ cm}^2$$

$$A(\text{minor segment}) = A(P\text{-}AXB) - A(\Delta PAB)$$
$$= 78.5 - 50$$
$$= 28.5 \text{ cm}^2$$

$$A(\text{major segment}) = A(\text{circle}) - A(\text{minor segment})$$
$$= 3.14 \times 10^2 - 28.5$$
$$= 314 - 28.5$$
$$= 285.5 \text{ cm}^2$$

Ex. (3) A regular hexagon is inscribed in a circle of radius 14 cm. Find the area of the region between the circle and the hexagon. $(\pi = \frac{22}{7}, \sqrt{3} = 1.732)$

Solution : side of the hexagon = 14 cm

$$A(\text{hexagon}) = 6 \times \frac{\sqrt{3}}{4} \times (\text{side})^2$$
$$= 6 \times \frac{\sqrt{3}}{4} \times 14^2$$
$$= 509.208 \text{ cm}^2$$

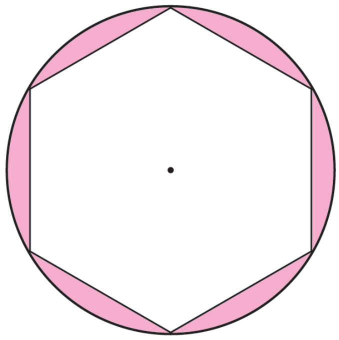


Fig. 7.42

$$A(\text{circle}) = \pi r^2$$
$$= \frac{22}{7} \times 14 \times 14$$
$$= 616 \text{ cm}^2$$

The area of the region between the circle and the hexagon

$$= A(\text{circle}) - A(\text{hexagon})$$
$$= 616 - 509.208$$
$$= 106.792 \text{ cm}^2$$

Practice set 7.4

1. In figure 7.43, A is the centre of the circle. $\angle ABC = 45°$ and $AC = 7\sqrt{2}$ cm. Find the area of segment BXC.

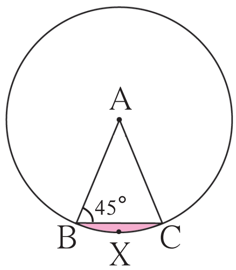


Fig. 7.43

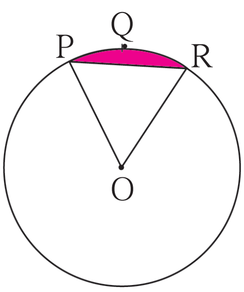


Fig. 7.44

2. In the figure 7.44, O is the centre of the circle. $m(\text{arc PQR}) = 60°$ OP = 10 cm. Find the area of the shaded region. $(\pi = 3.14, \sqrt{3} = 1.73)$

3. In the figure 7.45, if A is the centre of the circle. $\angle$ PAR = 30°, AP = 7.5, find the area of the segment PQR

(π = 3.14)

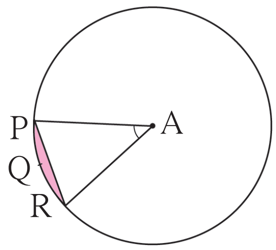


Fig. 7.45

4.

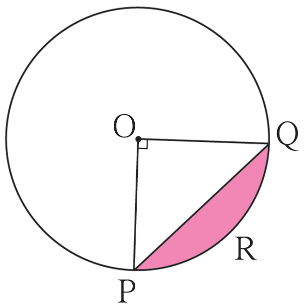


Fig. 7.46

In the figure 7.46, if O is the centre of the circle, PQ is a chord. $\angle$ POQ = 90°, area of shaded region is 114 cm^2, find the radius of the circle. (π = 3.14)

5. A chord PQ of a circle with radius 15 cm subtends an angle of 60° with the centre of the circle. Find the area of the minor as well as the major segment.

(π = 3.14, $\sqrt{3}$ = 1.73)

Problem set 7

1. Choose the correct alternative answer for each of the following questions.

(1) The ratio of circumference and area of a circle is 2:7. Find its circumference.

(A) 14π (B) $\frac{7}{\pi}$ (C) 7π (D) $\frac{14}{\pi}$

(2) If measure of an arc of a circle is 160° and its length is 44 cm, find the circumference of the circle.

(A) 66 cm (B) 44 cm (C) 160 cm (D) 99 cm

(3) Find the perimeter of a sector of a circle if its measure is 90 ° and radius is 7 cm.

(A) 44 cm (B) 25 cm (C) 36 cm (D) 56 cm

(4) Find the curved surface area of a cone of radius 7 cm and height 24 cm.

(A) 440 cm^2 (B) 550 cm^2 (C) 330 cm^2 (D) 110 cm^2

(5) The curved surface area of a cylinder is 440 cm^2 and its radius is 5 cm. Find its height.

(A) $\frac{44}{\pi}$ cm (B) 22π cm (C) 44π cm (D) $\frac{22}{\pi}$ cm

(6) A cone was melted and cast into a cylinder of the same radius as that of the base of the cone. If the height of the cylinder is 5 cm, find the height of the cone.

(A) 15 cm (B) 10 cm (C) 18 cm (D) 5 cm

(7) Find the volume of a cube of side 0.01 cm.

(A) 1 cm^3 (B) 0.001 cm^3 (C) 0.0001 cm^3 (D) 0.000001 cm^3

(8) Find the side of a cube of volume 1 m^3.

(A) 1 cm (B) 10 cm (C) 100 cm (D)1000 cm

2. A washing tub in the shape of a frustum of a cone has height 21 cm. The radii of the circular top and bottom are 20 cm and 15 cm respectively. What is the capacity of the tub ? ($\pi = \frac{22}{7}$)

3 ★. Some plastic balls of radius 1 cm were melted and cast into a tube. The thickness, length and outer radius of the tube were 2 cm, 90 cm and 30 cm respectively. How many balls were melted to make the tube ?

4. A metal parallelopiped of measures 16 cm × 11cm × 10 cm was melted to make coins. How many coins were made if the thickness and diameter of each coin was 2 mm and 2 cm respectively ?

5. The diameter and length of a roller is 120 cm and 84 cm respectively. To level the ground, 200 rotations of the roller are required. Find the expenditure to level the ground at the rate of Rs. 10 per sq.m.

6. The diameter and thickness of a hollow metals sphere are 12 cm and 0.01 m respectively. The density of the metal is 8.88 gm per cm^3. Find the outer surface area and mass of the sphere.

7. A cylindrical bucket of diameter 28 cm and height 20 cm was full of sand. When the sand in the bucket was poured on the ground, the sand got converted into a shape of a cone. If the height of the cone was 14 cm, what was the base area of the cone ?

8. The radius of a metallic sphere is 9 cm. It was melted to make a wire of diameter 4 mm. Find the length of the wire.

9. The area of a sector of a circle of 6 cm radius is 15π sq.cm. Find the measure of the arc and length of the arc corresponding to the sector.

10.

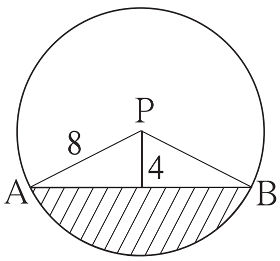


Fig.7.47

In the figure 7.47, seg AB is a chord of a circle with centre P. If PA = 8 cm and distance of chord AB from the centre P is 4 cm, find the area of the shaded portion. ($\pi = 3.14$, $\sqrt{3} = 1.73$)

11. In the figure 7.48, square ABCD is inscribed in the sector A-PCQ. The radius of sector C-BXD is 20 cm. Complete the following activity to find the area of shaded region.

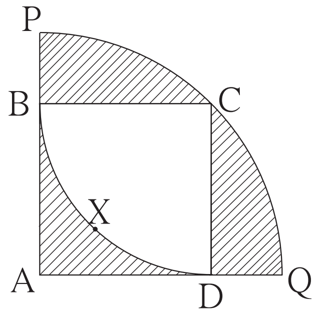


Fig. 7.48

Solution : Side of square ABCD = radius of sector C-BXD = $\square$ cm

Area of square = $(\text{side})^2 = \square^2 = \square$ ..... (I)

Area of shaded region inside the square

$= $ Area of square ABCD $-$ Area of sector C-BXD

$$= \square - \frac{\theta}{360} \times \pi r^2$$

$$= \square - \frac{90}{360} \times \frac{3.14}{1} \times \frac{400}{1}$$

$$= \square - 314$$

$$= \square$$

Radius of bigger sector = Length of diagonal of square ABCD

$= 20\sqrt{2}$

Area of the shaded regions outside the square

$= $ Area of sector A-PCQ $-$ Area of square ABCD

$= A(A\text{-}PCQ) - A(\square ABCD)$

$$= \left(\frac{\theta}{360} \times \pi \times r^2\right) - \square^2$$

$$= \frac{90}{360} \times 3.14\ (20\sqrt{2})^2 - (20)^2$$

$$= \square - \square$$

$$= \square$$

$\therefore$ total area of the shaded region = 86 + 228 = 314 sq.cm.

12.

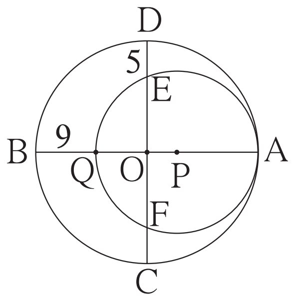


Fig. 7.49

In the figure 7.49, two circles with centres O and P are touching internally at point A. If BQ = 9, DE = 5, complete the following activity to find the radii of the circles.

Solution : Let the radius of the bigger circle be R and that of smaller circle be r.

OA, OB, OC and OD are the radii of the bigger circle

$\therefore$ OA = OB = OC = OD = R

PQ = PA = r

OQ = OB −BQ = $\square$

OE = OD − DE = $\square$

As the chords QA and EF of the circle with centre P intersect in the interior of the circle, so by the property of internal division of two chords of a circle,

$OQ \times OA = OE \times OF$

$\square \times R = \square \times \square$ .... ($\because$ OE = OF)

$R^2 - 9R = R^2 - 10R + 25$

R = $\square$

AQ = 2r = AB − BQ

2r = 50 − 9 = 41

r = $\square$ = $\square$