1   Similarity

- Ratio of areas of two triangles
- Basic proportionality theorem
- Converse of basic proportionality theorem
- Tests of similarity of triangles
- Property of an angle bisector of a triangle
- Property of areas of similar triangles
- The ratio of the intercepts made on the transversals by three parallel lines

Let's recall.

We have studied Ratio and Proportion. The statement, 'the numbers a and b are in the ratio $\dfrac{m}{n}$' is also written as, 'the numbers a and b are in proportion $m{:}n$.'

For this concept we consider positive real numbers. We know that the lengths of line segments and area of any figure are positive real numbers.

We know the formula of area of a triangle.

Area of a triangle = $\dfrac{1}{2}$ Base × Height

Let's learn.

Ratio of areas of two triangles

Let's find the ratio of areas of any two triangles.

Ex. In $\triangle$ ABC, AD is the height and BC is the base.

In $\triangle$ PQR, PS is the height and QR is the base

$$\dfrac{A(\triangle ABC)}{A(\triangle PQR)} = \dfrac{\dfrac{1}{2} \times BC \times AD}{\dfrac{1}{2} \times QR \times PS}$$

Fig. 1.1   Fig. 1.2

$$\therefore \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{BC \times AD}{QR \times PS}$$

Hence the ratio of the areas of two triangles is equal to the ratio of the products of their bases and corrosponding heights.

Base of a triangle is b_1 and height is h_1. Base of another triangle is b_2 and height is h_2. Then the ratio of their areas $= \dfrac{b_1 \times h_1}{b_2 \times h_2}$

Suppose some conditions are imposed on these two triangles,

Condition 1 : If the heights of both triangles are equal then-
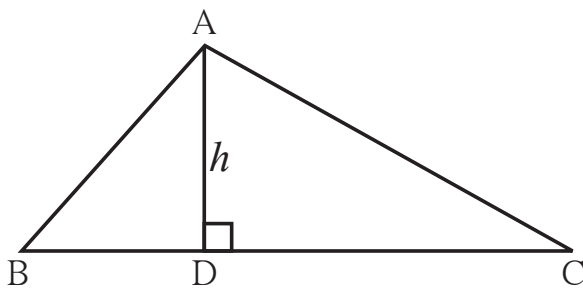
Fig. 1.3
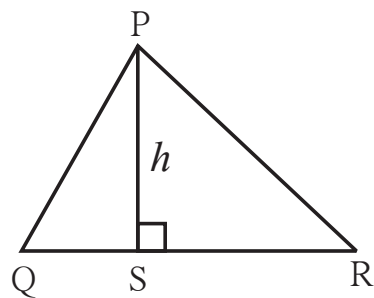
Fig. 1.4

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{BC \times h}{QR \times h} = \frac{BC}{QR}$$

$$\therefore \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{b_1}{b_2}$$

Property: The ratio of the areas of two triangles with equal heights is equal to the ratio of their corresponding bases.

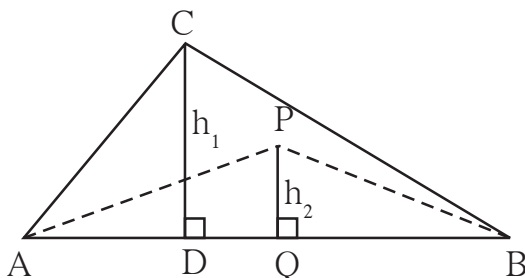**Condition 2 :** If the bases of both triangles are equal then -

Fig. 1.5

$$\frac{A(\triangle ABC)}{A(\triangle APB)} = \frac{AB \times h_1}{AB \times h_2}$$

$$\frac{A(\triangle ABC)}{A(\triangle APB)} = \frac{h_1}{h_2}$$

Property: The ratio of the areas of two triangles with equal bases is equal to the ratio of their corresponding heights.

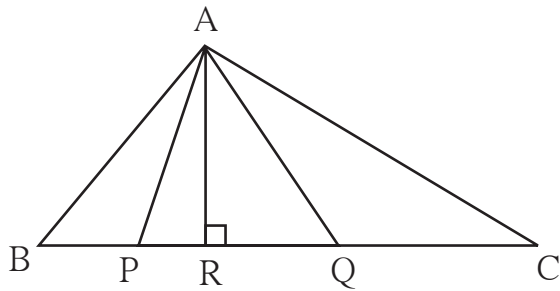**Activity :**

Fill in the blanks properly.

(i)

Fig. 1.6

$$\frac{A(\Delta ABC)}{A(\Delta APQ)} = \frac{\boxed{} \times \boxed{}}{\boxed{} \times \boxed{}} = \frac{\boxed{}}{\boxed{}}$$

(ii)
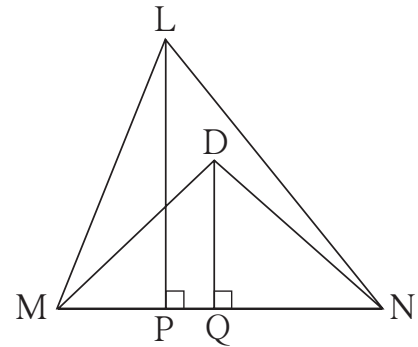
Fig.1.7

$$\frac{A(\Delta LMN)}{A(\Delta DMN)} = \frac{\boxed{} \times \boxed{}}{\boxed{} \times \boxed{}} = \frac{\boxed{}}{\boxed{}}$$

(iii) M is the midpoint of seg AB and seg CM is a median of Δ ABC

$$\therefore \frac{A(\Delta AMC)}{A(\Delta BMC)} = \frac{\boxed{}}{\boxed{}}$$

$$= \frac{\boxed{}}{\boxed{}} = \boxed{}$$

State the reason.
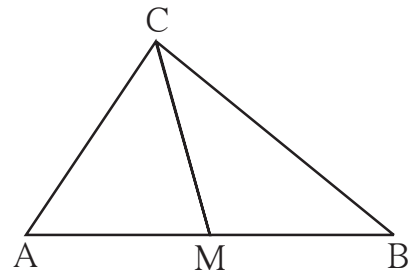

Fig. 1.8

🦋🦋🦋🦋🦋🦋🦋🦋🦋🦋🦋🦋🦋 **Solved Examples** 🦋🦋🦋🦋🦋🦋🦋🦋🦋🦋🦋🦋🦋
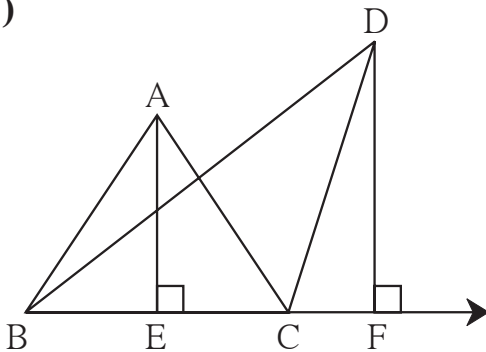
Ex. (1)

Fig.1.9

In adjoining figure

AE $\perp$ seg BC, seg DF $\perp$ line BC,

AE = 4, DF = 6 , then find $\dfrac{A(\Delta ABC)}{A(\Delta DBC)}$.

Solution : $\dfrac{A(\Delta ABC)}{A(\Delta DBC)} = \dfrac{AE}{DF}$ .... bases are equal, hence areas proportional to heights.
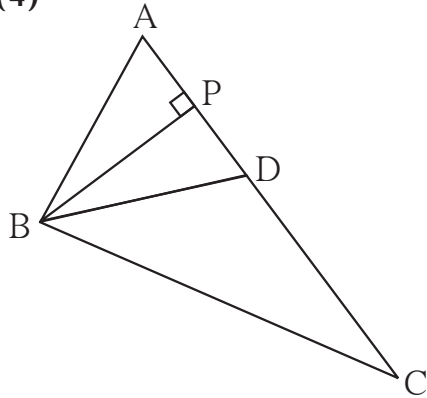
$$= \frac{4}{6} = \frac{2}{3}$$

Ex. (2) In $\triangle$ ABC point D on side BC is such that DC = 6, BC = 15. Find A($\triangle$ ABD) : A($\triangle$ ABC) and A($\triangle$ ABD) : A($\triangle$ ADC).

Solution : Point A is common vertex of $\triangle$ ABD, $\triangle$ ADC and $\triangle$ ABC and their bases are collinear. Hence, heights of these three triangles are equal

Fig. 1.10

BC = 15, DC = 6 $\therefore$ BD = BC − DC = 15 − 6 = 9

$$\frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{BD}{BC} \quad \ldots\ldots\ldots \text{ heights equal, hence areas proportional to bases.}$$

$$= \frac{9}{15} \quad = \quad \frac{3}{5}$$

$$\frac{A(\triangle ABD)}{A(\triangle ADC)} = \frac{BD}{DC} \quad \ldots\ldots\ldots \text{ heights equal, hence areas proportional to bases.}$$

$$= \frac{9}{6} \quad = \quad \frac{3}{2}$$

Ex. (3)

Fig. 1.11

$\square$ ABCD is a parallelogram. P is any point on side BC. Find two pairs of triangles with equal areas.

Solution : $\square$ ABCD is a parallelogram.

$\therefore$ AD $\parallel$ BC and AB $\parallel$ DC

Consider $\triangle$ ABC and $\triangle$ BDC.

Both the triangles are drawn in two parallel lines. Hence the distance between the two parallel lines is the height of both triangles.

In $\triangle$ ABC and $\triangle$ BDC, common base is BC and heights are equal.

Hence, A($\triangle$ ABC) = A($\triangle$ BDC)

In $\triangle$ ABC and $\triangle$ ABD, AB is common base and and heights are equal.

$\therefore$ A($\triangle$ ABC) = A($\triangle$ ABD)

Ex.(4)

Fig. 1.12

In adjoining figure in $\triangle$ ABC, point D is on side AC. If AC = 16, DC = 9 and BP $\perp$ AC, then find the following ratios.

(i) $\dfrac{A(\triangle ABD)}{A(\triangle ABC)}$ (ii) $\dfrac{A(\triangle BDC)}{A(\triangle ABC)}$

(iii) $\dfrac{A(\triangle ABD)}{A(\triangle BDC)}$

Solution : In $\triangle$ ABC point P and D are on side AC, hence B is common vertex of $\triangle$ ABD, $\triangle$ BDC, $\triangle$ ABC and $\triangle$ APB and their sides AD, DC, AC and AP are collinear. Heights of all the triangles are equal. Hence, areas of these triangles are proportinal to their bases. AC = 16, DC = 9

$\therefore$ AD = 16 − 9 = 7

$\therefore \dfrac{A(\triangle ABD)}{A(\triangle ABC)} = \dfrac{AD}{AC} = \dfrac{7}{16}$ . . . . triangles having equal heights

$\dfrac{A(\triangle BDC)}{A(\triangle ABC)} = \dfrac{DC}{AC} = \dfrac{9}{16}$ . . . . triangles having equal heights

$\dfrac{A(\triangle ABD)}{A(\triangle BDC)} = \dfrac{AD}{DC} = \dfrac{7}{9}$ . . . . triangles having equal heights

Remember this !

- Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.
- Areas of triangles with equal heights are proportional to their corresponding bases.
- Areas of triangles with equal bases are proportional to their corresponding heights.

Practice set 1.1

1. Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.

2. In figure 1.13 $BC \perp AB$, $AD \perp AB$,

BC = 4, AD = 8, then find $\dfrac{A(\Delta\ ABC)}{A(\Delta\ ADB)}$.

Fig. 1.13

Fig. 1.14

3. In adjoining figure 1.14

seg PS $\perp$ seg RQ seg QT $\perp$ seg PR.

If RQ = 6, PS = 6 and PR = 12,

then find QT.

4. In adjoining figure, AP $\perp$ BC,

AD $\parallel$ BC, then find

$A(\Delta\ ABC) : A(\Delta\ BCD)$.

Fig. 1.15

Fig. 1.16

5. In adjoining figure PQ $\perp$ BC,

AD $\perp$ BC   then find following ratios.

(i) $\dfrac{A(\Delta\ PQB)}{A(\Delta\ PBC)}$

(ii) $\dfrac{A(\Delta\ PBC)}{A(\Delta\ ABC)}$

(iii) $\dfrac{A(\Delta\ ABC)}{A(\Delta\ ADC)}$

(iv) $\dfrac{A(\Delta\ ADC)}{A(\Delta\ PQC)}$

Basic proportionality theorem

Theorem : **If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.**

Given : In Δ ABC line $l \parallel$ line BC and line l intersects AB and AC in point P and Q respectively

To prove : $\dfrac{AP}{PB} = \dfrac{AQ}{QC}$

Construction: Draw seg PC and seg BQ

Fig. 1.17

Proof : Δ APQ and Δ PQB have equal heights.

$\therefore \dfrac{A(\Delta\ APQ)}{A(\Delta\ PQB)} = \dfrac{AP}{PB}$ .... (I) (areas proportionate to bases)

and $\dfrac{A(\Delta\ APQ)}{A(\Delta\ PQC)} = \dfrac{AQ}{QC}$ .... (II) (areas proportionate to bases)

seg PQ is common base of Δ PQB and Δ PQC. seg PQ $\parallel$ seg BC, hence Δ PQB and Δ PQC have equal heights.

$A(\Delta\ PQB) = A(\Delta\ PQC)$ .... (III)

$\dfrac{A(\Delta\ APQ)}{A(\Delta\ PQB)} = \dfrac{A(\Delta\ APQ)}{A(\Delta\ PQC)}$ .... [from (I), (II) and (III)]

$\therefore \dfrac{AP}{PB} = \dfrac{AQ}{QC}$ .... [from (I) and (II)]

Converse of basic proportionality theorem

Theorem : **If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.**

In figure 1.18, line l interesects the side AB and side AC of Δ ABC in the points P and Q respectively and $\dfrac{AP}{PB} = \dfrac{AQ}{QC}$, hence line $l \parallel$ seg BC.

This theorem can be proved by indirect method.

Fig. 1.18

Activity :

- Draw a Δ ABC.
- Bisect $\angle$ B and name the point of intersection of AC and the angle bisector as D.
- Measure the sides.

  AB = ☐ cm   BC = ☐ cm

  AD = ☐ cm   DC = ☐ cm

- Find ratios $\dfrac{AB}{BC}$ and $\dfrac{AD}{DC}$.
- You will find that both the ratios are almost equal.
- Bisect remaining angles of the triangle and find the ratios as above. You can verify that the ratios are equal.

Fig. 1.19

Let's learn.

Property of an angle bisector of a triangle

Theorem : **The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.**

Given : In Δ ABC, bisector of $\angle$C interesects seg AB in the point E.

To prove : $\dfrac{AE}{EB} = \dfrac{CA}{CB}$

Construction : Draw a line parallel to ray CE, passing through the point B. Extend AC so as to intersect it at point D.

Fig. 1.20

Proof : ray CE $\parallel$ ray BD and AD is transversal,

$\therefore$ $\angle$ ACE = $\angle$ CDB   ………. (corresponding angles) …(I)

Now taking BC as transversal

$\angle$ ECB = $\angle$ CBD   ………. (alternate angle)   …(II)

But $\angle$ ACE $\cong$ $\angle$ ECB   ………. (given)   …(III)

$\therefore$ $\angle$ CBD $\cong$ $\angle$ CDB   ………. [from (I), (II) and (III)]

In $\triangle$ CBD, side CB $\cong$ side CD   …….(sides opposite to congruent angles)

$\therefore$ CB = CD   …(IV)

Now in $\triangle$ ABD, seg EC $\parallel$ seg BD ………. (construction)

$\therefore$ $\dfrac{AE}{EB} = \dfrac{AC}{CD}$ ….(Basic proportionality theorem)..(V)

$\therefore$ $\dfrac{AE}{EB} = \dfrac{AC}{CB}$ ………. [from (IV) and (V)]

For more information :

Write another proof of the theorem yourself.

Draw DM $\perp$ AB and DN $\perp$ AC. Use the following properties and write the proof.

(1) The areas of two triangles of equal heights are proportional to their bases.
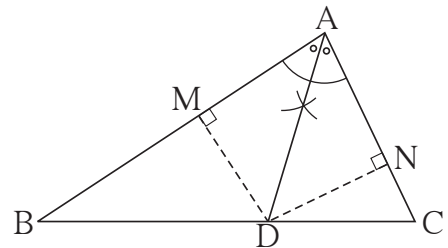
Fig. 1.21

(2) Every point on the bisector of an angle is equidistant from the sides of the angle.

Fig. 1.22

Converse of angle bisector theorem

If in $\triangle$ ABC, point D on side BC such that $\dfrac{AB}{AC} = \dfrac{BD}{DC}$, then ray AD bisects $\angle$ BAC.

Property of three parallel lines and their transversals

Activity:

- Draw three parallel lines.
- Label them as *l, m, n*.
- Draw transversals t_1 and t_2.
- AB and BC are intercepts on transversal t_1.
- PQ and QR are intercepts on transversal t_2.

Fig. 1.23

- Find ratios $\dfrac{AB}{BC}$ and $\dfrac{PQ}{QR}$. You will find that they are almost equal.

Theorem : The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corrosponding intercepts made on any other transversal by the same parallel lines.

Given : line *l* $\parallel$ line *m* $\parallel$ line *n*

t_1 and t_2 are transversals.
Transversal t_1 intersects the lines in points A, B, C and t_2 intersects the lines in points P, Q, R.

To prove : $\dfrac{AB}{BC} = \dfrac{PQ}{QR}$

Fig. 1.24

Proof : Draw seg PC , which intersects line *m* at point D.

In $\triangle$ ACP, BD $\parallel$ AP

$\therefore \dfrac{AB}{BC} = \dfrac{PD}{DC}$ . . . . . (I) (Basic proportionality theorem)

In $\triangle$ CPR, DQ $\parallel$ CR

$\therefore \dfrac{PD}{DC} = \dfrac{PQ}{QR}$ . . . . . (II) (Basic proportionality theorem)

$\therefore \dfrac{AB}{BC} = \dfrac{PD}{DC} = \dfrac{PQ}{QR}$ . . . . . . from (I) and (II). $\therefore \dfrac{AB}{BC} = \dfrac{PQ}{QR}$

A

P

B

Q   C

Fig. 1.25

(1) Basic proportionality theorem.

In $\triangle$ ABC, if seg PQ $\parallel$ seg AC

then $\dfrac{AP}{BP} = \dfrac{QC}{BQ}$

(2) Converse of basic proportionality theorem.

In $\triangle PQR$, if $\dfrac{PS}{SQ} = \dfrac{PT}{TR}$

then seg ST $\parallel$ seg QR.

P

S   T

Q   R

Fig. 1.26

A

D

B   C
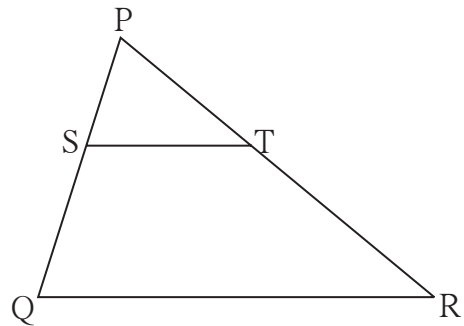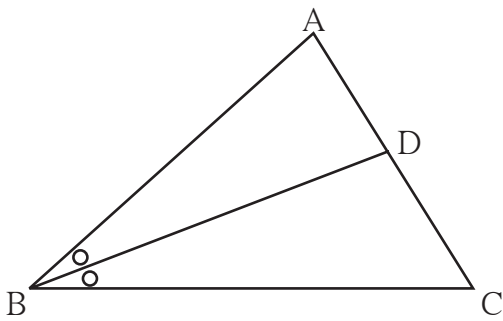
Fig. 1.27

(3) Theorem of bisector of an angle of a triangle.

If in $\triangle$ ABC, BD is bisector of $\angle$ ABC,

then $\dfrac{AB}{BC} = \dfrac{AD}{DC}$

(4) Property of three parallel lines and their transversals.

If line AX $\parallel$ line BY $\parallel$ line CZ

and line l and line m are their

transversals then $\dfrac{AB}{BC} = \dfrac{XY}{YZ}$

A   B   C l

X   Y   Z m
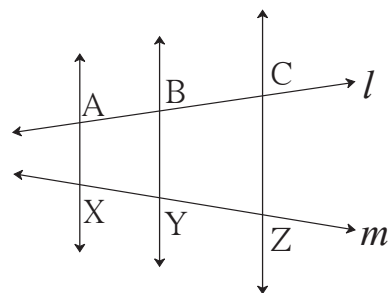
Fig. 1.28

Ex. (1) In Δ ABC, DE ‖ BC

If DB = 5.4 cm, AD = 1.8 cm

EC = 7.2 cm then find AE.
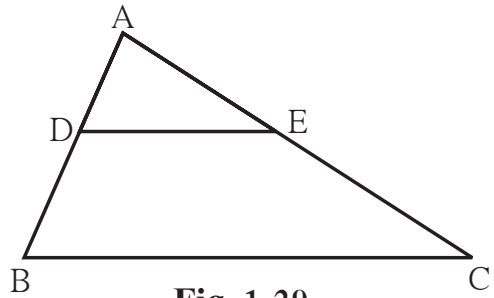
Solution : In Δ ABC, DE ‖ BC

∴ $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ ..... Basic proportionality theorem

∴ $\dfrac{1.8}{5.4} = \dfrac{AE}{7.2}$

∴ AE × 5.4 = 1.8 × 7.2

∴ AE = $\dfrac{1.8 \times 7.2}{5.4}$ = 2.4

AE = 2.4 cm

Fig. 1.29

Ex. (2) In Δ PQR, seg RS bisects ∠ R.

If PR = 15, RQ = 20 PS = 12

then find SQ.
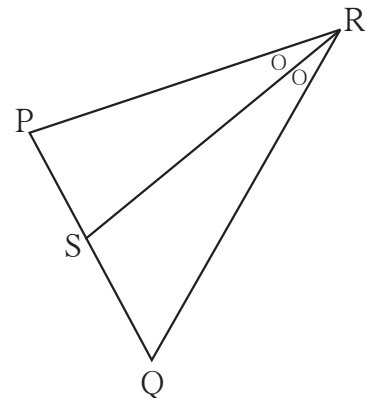
Solution : In Δ PRQ, seg RS bisects ∠ R.

$\dfrac{PR}{RQ} = \dfrac{PS}{SQ}$ . . . . . . property of angle bisector

$\dfrac{15}{20} = \dfrac{12}{SQ}$

SQ = $\dfrac{12 \times 20}{15}$

∴ SQ = 16
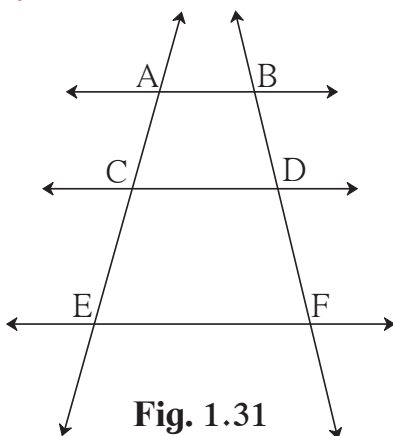
Fig. 1.30

Activity :

Fig. 1.31

In the figure 1.31, AB ‖ CD ‖ EF

If AC = 5.4, CE = 9, BD = 7.5

then find DF

Solution : AB ‖ CD ‖ EF

$\dfrac{AC}{\Box} = \dfrac{\Box}{DF}$ . . . . . (   )

$\dfrac{5.4}{9} = \dfrac{\Box}{DF}$ ∴ DF = $\Box$

Activity :
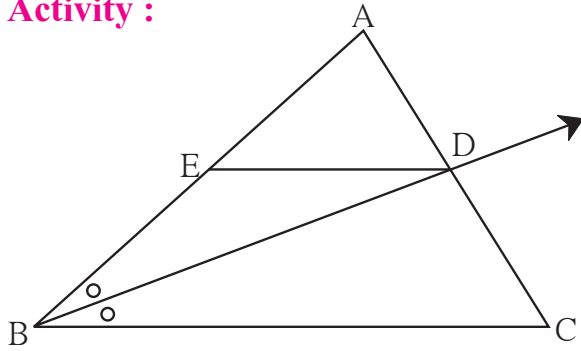
Fig. 1.32

In $\triangle ABC$, ray BD bisects $\angle ABC$. A-D-C, side DE || side BC, A-E-B then prove that, $\dfrac{AB}{BC} = \dfrac{AE}{EB}$

Proof : In $\triangle ABC$, ray BD bisects $\angle B$.

$\therefore \dfrac{AB}{BC} = \dfrac{AD}{DC}$ . . (I)(Angle bisector theorem)

In $\triangle ABC$, DE || BC

$\dfrac{AE}{EB} = \dfrac{AD}{DC}$ . . . . . (II) (. . . . .)

$\dfrac{AB}{\square} = \dfrac{\square}{EB}$ . . . . . . from (I) and (II)

Practice set 1.2

1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of $\angle$ QPR.

(1)

Fig. 1.33

(2)

Fig. 1.34

(3)
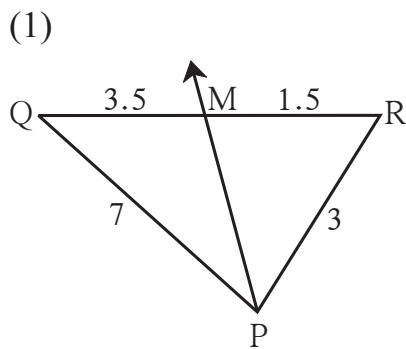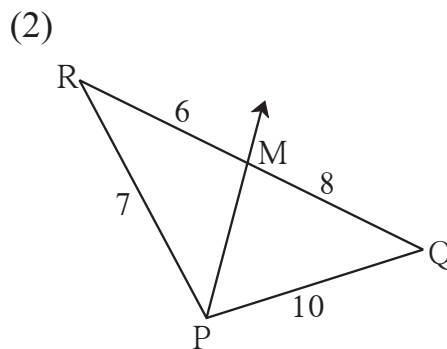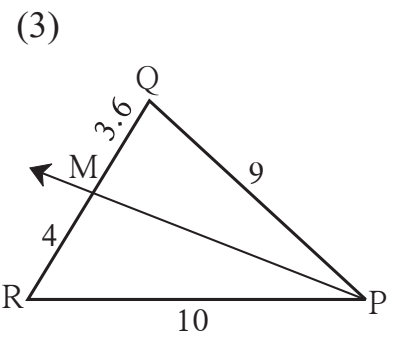
Fig. 1.35

2. In $\triangle$ PQR, PM = 15, PQ = 25 PR = 20, NR = 8. State whether line NM is parallel to side RQ. Give reason.
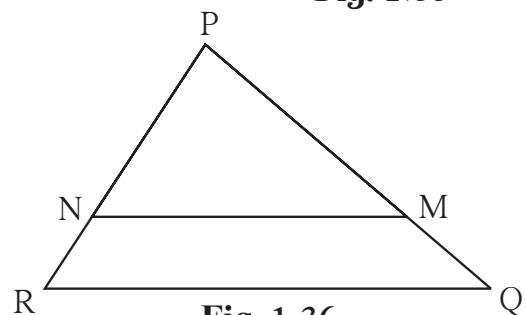
Fig. 1.36

3. In $\triangle$ MNP, NQ is a bisector of $\angle$ N. If MN = 5, PN = 7 MQ = 2.5 then find QP .
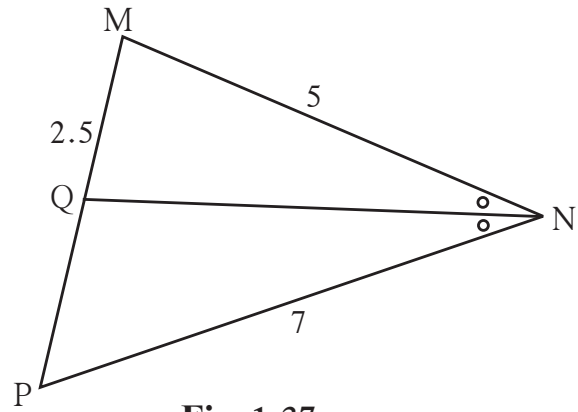
Fig. 1.37
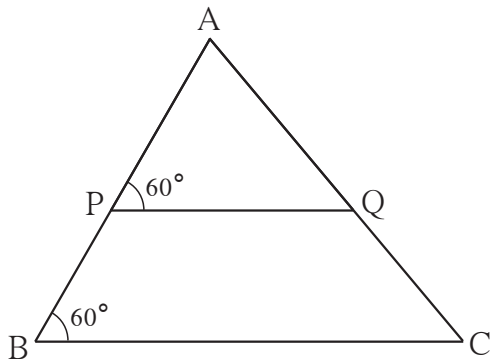
Fig. 1.38

4. Measures of some angles in the figure are given. Prove that

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

5. In trapezium ABCD, side AB $\|$ side PQ $\|$ side DC, AP = 15, PD = 12, QC = 14, find BQ.
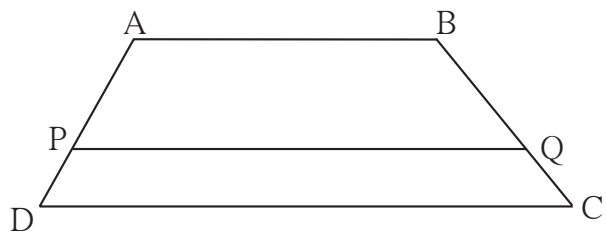
Fig. 1.39
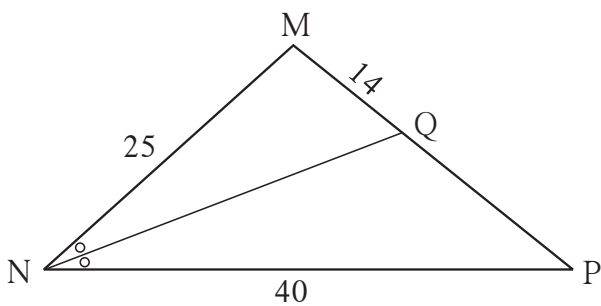
Fig. 1.40

6. Find QP using given information in the figure.

7. In figure 1.41, if AB $\|$ CD $\|$ FE then find x and AE.

Fig. 1.41

Fig. 1.42

8. In △ LMN, ray MT bisects ∠ LMN
If LM = 6, MN = 10, TN = 8,
then find LT.
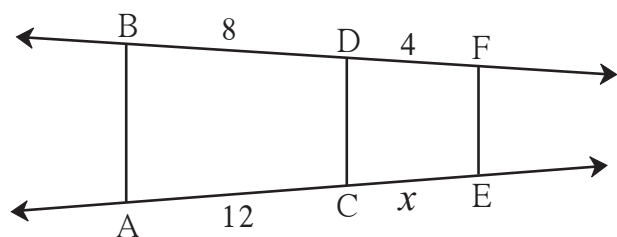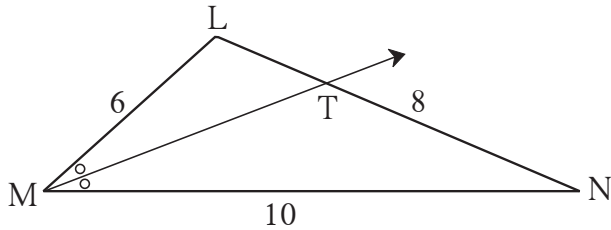
9. In △ ABC, seg BD bisects ∠ ABC.
If AB = x, BC = $x + 5$,
AD = $x - 2$, DC = $x + 2$, then find
the value of x.
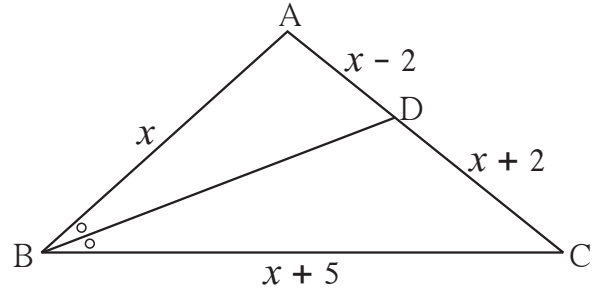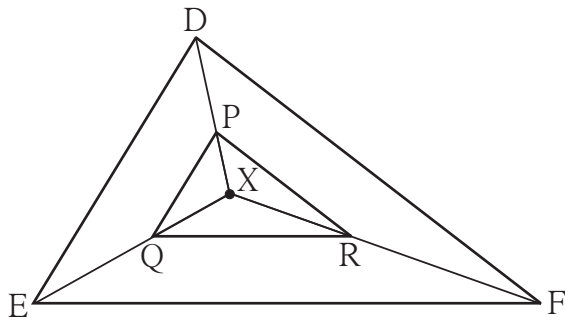
Fig. 1.43

Fig. 1.44

10. In the figure 1.44, X is any point in the interior of triangle. Point X is joined to vertices of triangle. Seg PQ ∥ seg DE, seg QR ∥ seg EF. Fill in the blanks to prove that, seg PR ∥ seg DF.

Proof : In △ XDE, PQ ∥ DE   .... ⬜

$$\therefore \frac{XP}{\boxed{}} = \frac{\boxed{}}{QE}$$ .... (I) (Basic proportionality theorem)

In △ XEF, QR ∥ EF   .... ⬜

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$ ....(II) ⬜

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$ .... from (I) and (II)

∴ seg PR ∥ seg DE   .... (converse of basic proportionality theorem)

11. In △ ABC, ray BD bisects ∠ ABC and ray CE bisects ∠ ACB. If seg AB ≅ seg AC then prove that ED ∥ BC.

Similar triangles
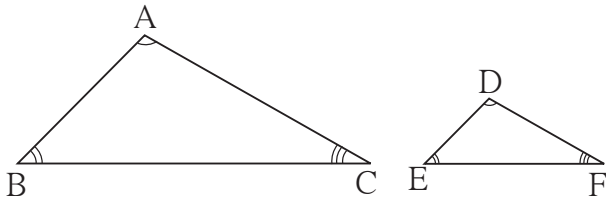
Fig. 1.45

In Δ ABC and Δ DEF, if ∠ A ≅ ∠ D, ∠ B ≅ ∠ E, ∠ C ≅ ∠ F

and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

then ΔABC and ΔDEF are similar triangles.

'Δ ABC and Δ DEF are similar' is expressed as 'Δ ABC ∼ Δ DEF'

Tests of similarity of triangles

For similarity of two triangles, the necessary conditions are that their corresponding sides are in same proportion and their corresponding angles are congruent. Out of these conditions; when three specifc conditions are fulfilled, the remaining conditions are automatically fulfilled. This means for similarity of two triangles, only three specific conditions are sufficient. Similarity of two triangles can be confirmed by testing these three conditions. The groups of such sufficient conditions are called tests of similarity, which we shall use.

AAA test for similarity of triangles

For a given correspondence of vertices, when corresponding angles of two triangles are congruent, then the two triangles are similar.

In Δ ABC and Δ PQR, in the correspondence ABC ↔ PQR if ∠ A ≅ ∠ P, ∠ B ≅ ∠ Q and ∠ C ≅ ∠ R then Δ ABC ∼ Δ PQR.

Fig. 1.46

Proof of AAA test

Fig. 1.47

Given : In Δ ABC and Δ PQR,
$\angle$A $\cong$ $\angle$P, $\angle$B $\cong$ $\angle$Q,
$\angle$C $\cong$ $\angle$R.

To prove : Δ ABC $\sim$ Δ PQR

Let us assume that Δ ABC is bigger than Δ PQR. Mark point M on AB, and point N on AC such that AM = PQ and AN = PR.

Show that Δ AMN $\cong$ Δ PQR. Hence show that MN $\parallel$ BC.

Now using basic proportionality theorem, $\dfrac{AM}{MB} = \dfrac{AN}{NC}$

That is $\dfrac{MB}{AM} = \dfrac{NC}{AN}$ .... (by invertendo)

$\dfrac{MB+AM}{AM} = \dfrac{NC+AN}{AN}$ .... (by componendo)

$\therefore \dfrac{AB}{AM} = \dfrac{AC}{AN}$

$\therefore \dfrac{AB}{PQ} = \dfrac{AC}{PR}$

Similarly it can be shown that $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$ $\qquad \therefore \Delta$ ABC $\sim \Delta$ PQR

A A test for similarity of triangles:

We know that for a given correspondence of vertices, when two angles of a triangle are congruent to two corresponding angles of another triangle, then remaining angle of first triangle is congruent to the remaining angle of the second triangle.

This means, when two angles of one triangle are congruent to two corresponding angles of another triangle then this condition is sufficient for similarity of two triangles. This condition is called AA test of similarity.

SAS test of similarity of triangles

For a given correspondence of vertices of two triangles, if two pairs of corresponding sides are in the same proportion and the angles between them are congruent, then the two triangles are similar.
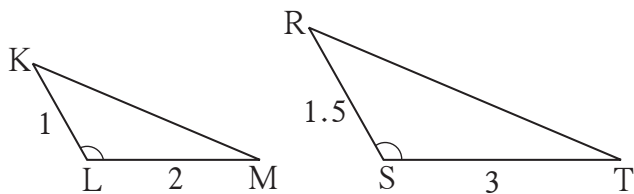
Fig. 1.48

For example, if in Δ KLM and Δ RST,

$$\angle \text{KLM} \cong \angle \text{RST}$$

$$\frac{\text{KL}}{\text{RS}} = \frac{\text{LM}}{\text{ST}} = \frac{2}{3}$$

Therefore, Δ KLM $\sim$ Δ RST

SSS test for similarity of triangles

For a given correspondence of vertices of two triangles, when three sides of a triangle are in proportion to corresponding three sides of another triangle, then the two triangles are similar.
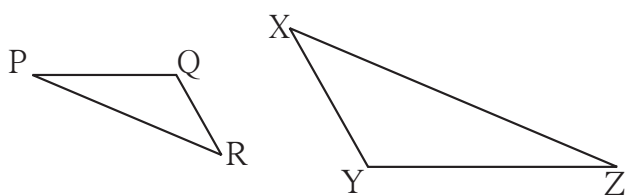
Fig. 1.49

For example, **if in** Δ PQR and Δ XYZ,

If $\dfrac{\text{PQ}}{\text{YZ}} = \dfrac{\text{QR}}{\text{XY}} = \dfrac{\text{PR}}{\text{XZ}}$

then Δ PQR $\sim$ Δ ZYX

Properties of similar triangles :

(1) Δ ABC $\sim$ Δ ABC – Reflexivity

(2) If Δ ABC $\sim$ Δ DEF then Δ DEF $\sim$ Δ ABC – Symmetry

(3) If Δ ABC $\sim$ Δ DEF and Δ DEF $\sim$ Δ GHI, then Δ ABC $\sim$ Δ GHI – Transitivity

ಌಌಌಌಌಌಌಌಌಌಌಌಌಌ **Solved Examples** ಌಌಌಌಌಌಌಌಌಌಌಌಌಌ

Ex. (1)   In Δ XYZ,
$\angle$ Y = 100°, $\angle$ Z = 30°,
In Δ LMN,
$\angle$ M = 100°, $\angle$ N = 30°,
Are Δ XYZ and Δ LMN
similar? If yes, by which test?

Fig. 1.50

Solution : In Δ XYZ and Δ LMN,

$\angle$ Y = 100°, $\angle$ M = 100°, $\therefore$ $\angle$ Y $\cong$ $\angle$ M

$\angle$ Z = 30°, $\angle$ N = 30°, $\therefore$ $\angle$ Z $\cong$ $\angle$ N

$\therefore$ Δ XYZ ~ Δ LMN   .....   by AA test.

Ex. (2) Are two triangles in figure 1.51 similar, according to the information given? If yes, by which test?

Fig. 1.51

Solution : In Δ PMN and Δ UVW

$\dfrac{PM}{UV} = \dfrac{6}{3} = \dfrac{2}{1}$, $\dfrac{MN}{VW} = \dfrac{10}{5} = \dfrac{2}{1}$

$\therefore$ $\dfrac{PM}{UV} = \dfrac{MN}{VW}$

and $\angle$ M $\cong$ $\angle$ V   .... Given

ΔPMN ~ ΔUVW   .... SAS test of similarity

Ex. (3) Can we say that the two triangles in figure 1.52 similar, according to information given? If yes, by which test ?

Fig. 1.52

Solution : Δ XYZ and Δ MNP,

$\dfrac{XY}{MN} = \dfrac{14}{21} = \dfrac{2}{3}$,

$\dfrac{YZ}{NP} = \dfrac{20}{30} = \dfrac{2}{3}$

It is given that $\angle$ Z $\cong$ $\angle$ P.

But $\angle$ Z and $\angle$ P are not included angles by sides which are in proportion.

$\therefore$ Δ XYZ and Δ MNP can not be said to be similar.

Ex. (4)

Fig. 1.53

In the adjoining figure $BP \perp AC$, $CQ \perp AB$, $A - P - C$, $A - Q - B$, then prove that Δ APB and Δ AQC are similar.

Solution : In Δ APB and Δ AQC

$$\angle APB = \boxed{}^{\circ} \text{ (I)}$$

$$\angle AQC = \boxed{}^{\circ} \text{ (II)}$$

$$\therefore \angle APB \cong \angle AQC \text{ ....from(I) and (II)}$$

$$\angle PAB \cong \angle QAC \text{ .... } (\boxed{})$$

$$\therefore \Delta APB \sim \Delta AQC \text{ ..... AA test}$$

Ex. (5) Diagonals of a quadrilateral ABCD intersect in point Q. If $2QA = QC$, $2QB = QD$, then prove that $DC = 2AB$.
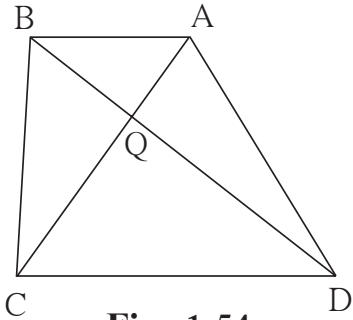
Fig. 1.54

Given : $2QA = QC$

$$2QB = QD$$

To prove : $CD = 2AB$

Proof : $2QA = QC \quad \therefore \dfrac{QA}{QC} = \dfrac{1}{2}$ .... (I)

$2QB = QD \quad \therefore \dfrac{QB}{QD} = \dfrac{1}{2}$ .... (II)

$\therefore \dfrac{QA}{QC} = \dfrac{QB}{QD}$ ....from (I) and (II)

In Δ AQB and Δ CQD,

$\dfrac{QA}{QC} = \dfrac{QB}{QD}$ .... proved

$\angle AQB \cong \angle DQC$ .... opposite angles

$\therefore \Delta$ AQB $\sim \Delta$ CQD .... (SAS test of similarity)

$\therefore \dfrac{AQ}{CQ} = \dfrac{QB}{QD} = \dfrac{AB}{CD}$ .... correponding sides are proportional

But $\dfrac{AQ}{CQ} = \dfrac{1}{2} \quad \therefore \dfrac{AB}{CD} = \dfrac{1}{2}$

$\therefore 2AB = CD$

1. In figure1.55, $\angle$ ABC=75°, $\angle$ EDC=75° state which two triangles are similar and by which test? Also write the similarity of these two triangles by a proper one to one correspondence.
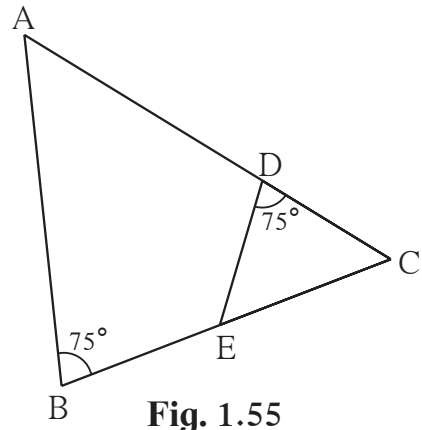
Fig. 1.55
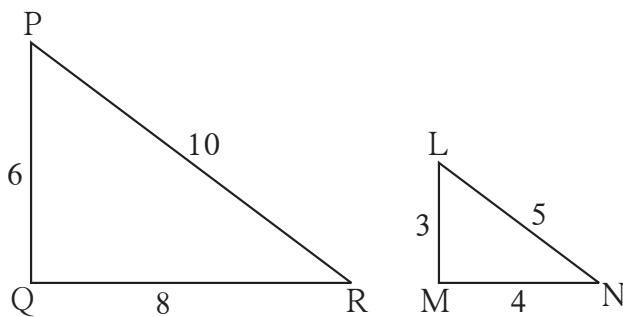
Fig. 1.56

2. Are the triangles in figure 1.56 similar? If yes, by which test ?

3. As shown in figure 1.57, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time ?
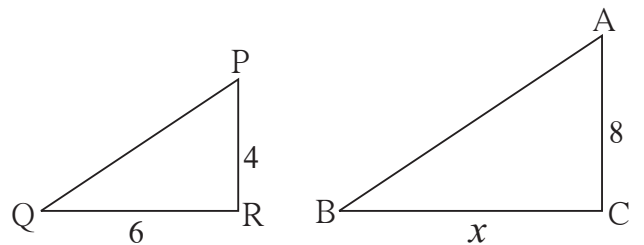
Fig. 1.57
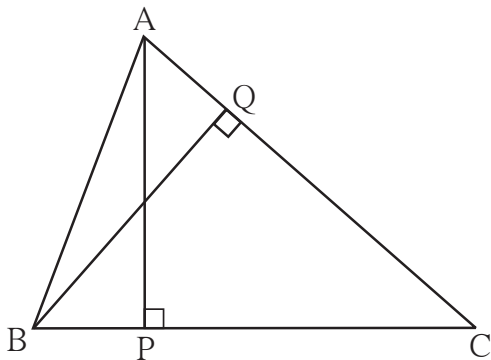
Fig. 1.58

4. In Δ ABC, AP $\perp$ BC, BQ $\perp$ AC B- P-C, A-Q - C then prove that, Δ CPA $\sim$ Δ CQB.

   If AP = 7, BQ=8, BC=12 then find AC.

5. **Given :** In trapezium PQRS, side PQ ∥ side SR, AR = 5AP, AS = 5AQ then prove that, SR = 5PQ
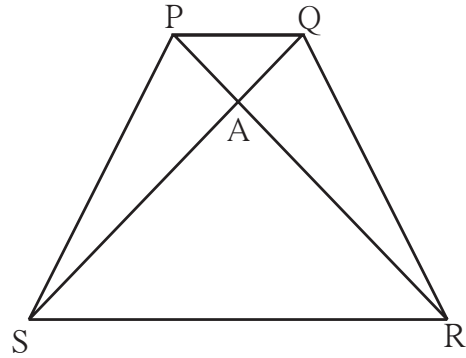
Fig. 1.59

6. In trapezium ABCD, (Figure 1.60) side AB ∥ side DC, diagonals AC and BD intersect in point O. If AB = 20, DC = 6, OB = 15 then find OD.
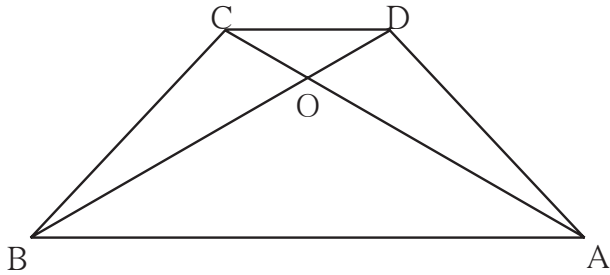

Fig. 1.60

7. ▱ABCD is a parallelogram point E is on side BC. Line DE intersects ray AB in point T. Prove that
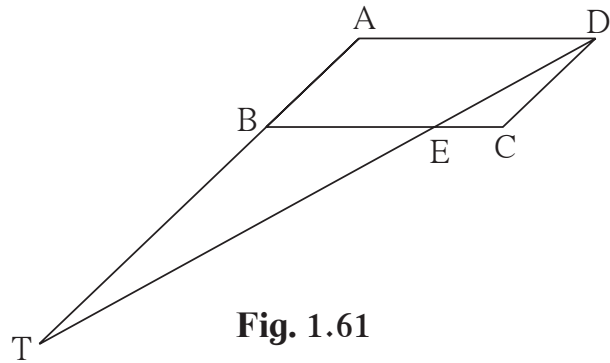
DE × BE = CE × TE.

Fig. 1.61

Fig. 1.62

8. In the figure, seg AC and seg BD intersect each other in point P and $\dfrac{AP}{CP} = \dfrac{BP}{DP}$. Prove that,
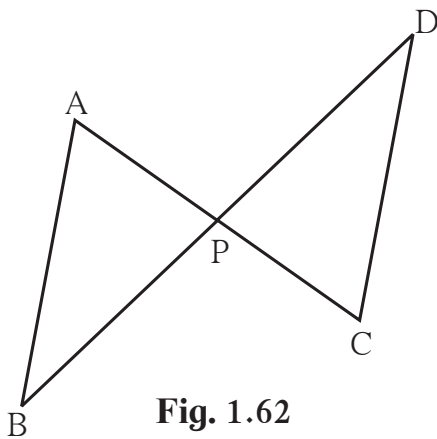
Δ ABP $\sim$ Δ CDP

9. In the figure, in Δ ABC, point D on side BC is such that,

$\angle$ BAC = $\angle$ ADC.
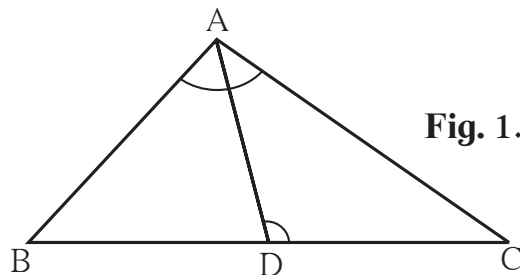
Prove that, CA² = CB × CD

Fig. 1.63

22

Theorem of areas of similar triangles

Theorem : When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.
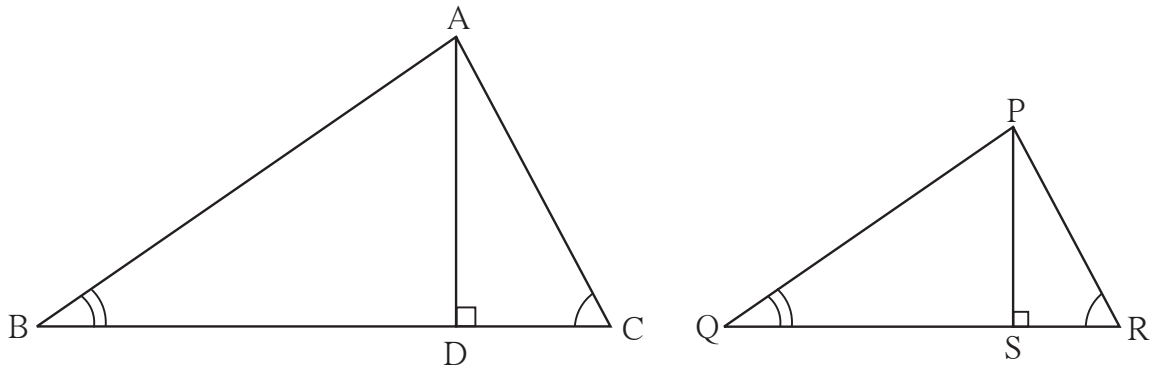

Fig. 1.64

Given : $\triangle ABC \sim \triangle PQR$, $AD \perp BC$, $PS \perp QR$

To prove: $\dfrac{A(\triangle ABC)}{A(\triangle PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$

Proof : $\dfrac{A(\triangle ABC)}{A(\triangle PQR)} = \dfrac{BC \times AD}{QR \times PS} = \dfrac{BC}{QR} \times \dfrac{AD}{PS}$ .... (I)

In $\triangle ABD$ and $\triangle PQS$,

$\angle B = \angle Q$ .... given

$\angle ADB = \angle PSQ = 90°$

$\therefore$ According to AA test $\triangle ABD \sim \triangle PQS$

$\therefore \dfrac{AD}{PS} = \dfrac{AB}{PQ}$ .... (II)

But $\triangle ABC \sim \triangle PQR$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR}$ .... (III)

From (I), (II) and (III)

$\dfrac{A(\triangle ABC)}{A(\triangle PQR)} = \dfrac{BC}{QR} \times \dfrac{AD}{PS} = \dfrac{BC}{QR} \times \dfrac{BC}{QR} = \dfrac{BC^2}{QR^2} = \dfrac{AB^2}{PQ^2} = \dfrac{AC^2}{PR^2}$

Ex. (1) : $\triangle ABC \sim \triangle PQR$, A ($\triangle ABC$) = 16 , A ($\triangle PQP$) = 25, then find the value of ratio $\dfrac{AB}{PQ}$.

Solution : $\triangle ABC \sim \triangle PQR$

$$\therefore \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AB^2}{PQ^2} \qquad \dots\dots\dots \text{ theorem of areas of similar triangles}$$

$$\therefore \frac{16}{25} = \frac{AB^2}{PQ^2} \qquad \therefore \frac{AB}{PQ} = \frac{4}{5} \qquad \dots\dots\dots \text{ taking square roots}$$

Ex. (2) Ratio of corresponging sides of two similar triangles is 2:5, If the area of the small triangle is 64 sq.cm. then what is the area of the bigger triangle ?

Solution : Assume that $\triangle ABC \sim \triangle PQR$.

$\triangle ABC$ is smaller and $\triangle PQR$ is bigger triangle.

$$\therefore \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{(2)^2}{(5)^2} = \frac{4}{25} \qquad \dots\dots\dots \text{ ratio of areas of similar triangles}$$

$$\therefore \frac{64}{A(\triangle PQR)} = \frac{4}{25}$$

$$4 \times A(\triangle PQR) = 64 \times 25$$

$$A(\triangle PQR) = \frac{64 \times 25}{4} = 400$$

$\therefore$ area of bigger triangle = 400 sq.cm.

Ex. (3) In trapezium ABCD, side AB $\parallel$ side CD, diagonal AC and BD intersect each other at point P. Then prove that $\dfrac{A(\triangle ABP)}{A(\triangle CPD)} = \dfrac{AB^2}{CD^2}$.

Solution : In trapezium ABCD side AB $\parallel$ side CD

In $\triangle APB$ and $\triangle CPD$

$\angle PAB \cong \angle PCD$ ...... alternate angles

$\angle APB \cong \angle CPD$ .... opposite angles

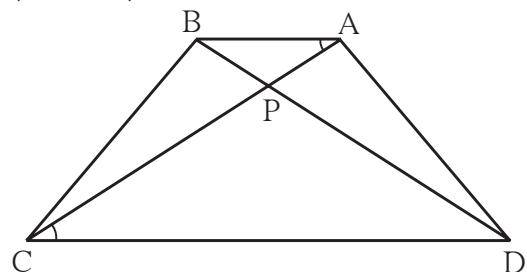$\therefore \triangle APB \sim \triangle CPD$ .... AA test of similarity

Fig. 1.65

$$\frac{A(\triangle APB)}{A(\triangle CPD)} = \frac{AB^2}{CD^2} \qquad \dots\dots \text{ theorem of areas of similar triangles}$$

1. The ratio of corresponding sides of similar triangles is 3 : 5; then find the ratio of their areas .

2. If $\triangle ABC \sim \triangle PQR$ and AB: PQ = 2:3, then fill in the blanks.

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AB^2}{\boxed{}} = \frac{2^2}{3^2} = \frac{\boxed{}}{\boxed{}}$$

3. If $\triangle ABC \sim \triangle PQR$, $A(\triangle ABC) = 80$, $A(\triangle PQR) = 125$, then fill in the blanks.

$$\frac{A(\triangle ABC)}{A(\triangle \ldots)} = \frac{80}{125} \qquad \therefore \frac{AB}{PQ} = \frac{\boxed{}}{\boxed{}}$$

4. $\triangle LMN \sim \triangle PQR$, $9 \times A(\triangle PQR) = 16 \times A(\triangle LMN)$. If QR = 20 then find MN.

5. Areas of two similar triangles are 225 sq.cm. 81 sq.cm. If a side of the smaller triangle is 12 cm, then find corresponding side of the bigger triangle .

6. $\triangle ABC$ and $\triangle DEF$ are equilateral triangles. If $A(\triangle ABC) : A(\triangle DEF) = 1 : 2$ and AB = 4, find DE.

7. In figure 1.66, seg PQ $\parallel$ seg DE, $A(\triangle PQF) = 20$ units, PF = 2 DP, then find $A(\square DPQE)$ by completing the following activity.

$A(\triangle PQF) = 20$ units, PF = 2 DP, Let us assume DP = x. $\therefore$ PF = $2x$

$$DF = DP + \boxed{} = \boxed{} + \boxed{} = 3x$$

In $\triangle FDE$ and $\triangle FPQ$,

$\angle FDE \cong \angle \ldots\ldots$ corresponding angles

$\angle FED \cong \angle \ldots\ldots$ corresponding angles

$\therefore \triangle FDE \sim \triangle FPQ \ldots\ldots$ AA test

$$\therefore \frac{A(\triangle FDE)}{A(\triangle FPQ)} = \frac{\boxed{}}{\boxed{}} = \frac{(3x)^2}{(2x)^2} = \frac{9}{4}$$

$$A(\triangle FDE) = \frac{9}{4} A(\triangle FPQ) = \frac{9}{4} \times \boxed{} = \boxed{}$$

$$A(\square DPQE) = A(\triangle FDE) - A(\triangle FPQ)$$

$$= \boxed{} - \boxed{}$$
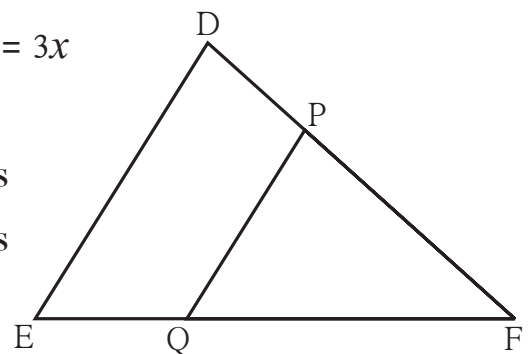
$$= \boxed{}$$

Fig. 1.66

1. Select the appropriate alternative.

(1) In Δ ABC and Δ PQR, in a one to one correspondence
$$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$$ then

(A) Δ PQR ∼ Δ ABC

(B) Δ PQR ∼ Δ CAB

(C) Δ CBA ∼ Δ PQR

(D) Δ BCA ∼ Δ PQR
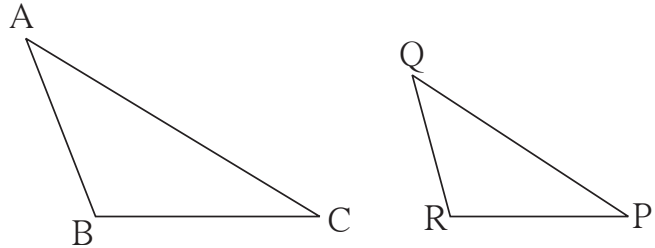
Fig. 1.67

(2) If in Δ DEF and Δ PQR, ∠ D ≅ ∠ Q, ∠ R ≅ ∠ E then which of the following statements is false ?

(A) $\frac{EF}{PR} = \frac{DF}{PQ}$

(B) $\frac{DE}{PQ} = \frac{EF}{RP}$

(C) $\frac{DE}{QR} = \frac{DF}{PQ}$

(D) $\frac{EF}{RP} = \frac{DE}{QR}$
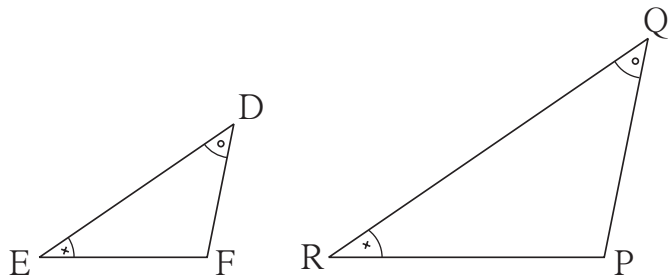
Fig. 1.68

(3) In Δ ABC and Δ DEF ∠ B = ∠ E, ∠ F = ∠C and AB = 3DE then which of the statements regarding the two triangles is true ?

(A) The triangles are not congruent and not similar

(B) The triangles are similar but not congruent.

(C) The triangles are congruent and similar.

(D) None of the statements above is true.

Fig. 1.69

(4) Δ ABC and Δ DEF are equilateral triangles, A(ΔABC):A(ΔDEF)=1:2 If AB = 4 then what is length of DE ?

(A) $2\sqrt{2}$ (B) 4   (C) 8   (D) $4\sqrt{2}$

Fig. 1.70

(5) In figure 1.71, seg XY ∥ seg BC, then which of the following statements is true?

(A) $\dfrac{AB}{AC} = \dfrac{AX}{AY}$ (B) $\dfrac{AX}{XB} = \dfrac{AY}{AC}$

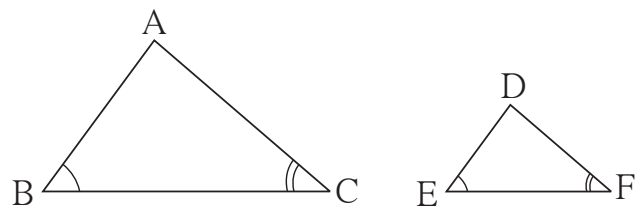(C) $\dfrac{AX}{YC} = \dfrac{AY}{XB}$ (D) $\dfrac{AB}{YC} = \dfrac{AC}{XB}$

Fig. 1.71

2. In Δ ABC, B − D − C and BD = 7, BC = 20 then find following ratios.

    (1) $\dfrac{A(\Delta\ ABD)}{A(\Delta\ ADC)}$

    (2) $\dfrac{A(\Delta\ ABD)}{A(\Delta\ ABC)}$

    (3) $\dfrac{A(\Delta\ ADC)}{A(\Delta\ ABC)}$

Fig. 1.72

3. Ratio of areas of two triangles with equal heights is 2 : 3. If base of the smaller triangle is 6 cm then what is the corresponding base of the bigger triangle ?

4.

Fig. 1.73

In figure 1.73, ∠ABC = ∠DCB = 90°

    AB = 6, DC = 8

    then $\dfrac{A(\Delta\ ABC)}{A(\Delta\ DCB)} = \ ?$

5. In figure 1.74, PM = 10 cm

    A(ΔPQS) = 100 sq.cm

    A(ΔQRS) = 110 sq.cm

    then find NR.

Fig. 1.74

6. Δ MNT ~ Δ QRS. Length of altitude drawn from point T is 5 and length of altitude drawn from point S is 9. Find the ratio $\dfrac{A(\Delta\ MNT)}{A(\Delta\ QRS)}$.

7. In figure 1.75, A−D−C and B−E−C seg DE ∥ side AB If AD = 5, DC = 3, BC = 6.4 then find BE.

Fig. 1.75

Fig. 1.76

8. In the figure 1.76, seg PA, seg QB, seg RC and seg SD are perpendicular to line AD.

AB = 60, BC = 70, CD = 80, PS = 280 then find PQ, QR and RS.

9. In Δ PQR seg PM is a median. Angle bisectors of ∠PMQ and ∠PMR intersect side PQ and side PR in points X and Y respectively. Prove that XY ∥ QR.

Fig. 1.77

Complete the proof by filling in the boxes.

In Δ PMQ, ray MX is bisector of ∠PMQ.

$$\therefore \ \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} \ \ \ \ldots\ldots\ldots \text{(I) theorem of angle bisector.}$$

In Δ PMR, ray MY is bisector of ∠PMR.

$$\therefore \ \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} \ \ \ \ldots\ldots\ldots \text{(II) theorem of angle bisector.}$$

But $\dfrac{MP}{MQ} = \dfrac{MP}{MR}$ ………. M is the midpoint QR, hence MQ = MR.

$$\therefore \ \frac{PX}{XQ} = \frac{PY}{YR}$$

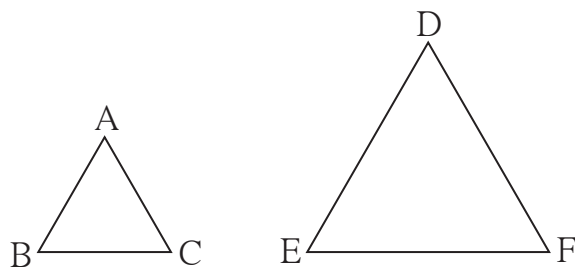∴ XY ∥ QR ………. converse of basic proportionality theorem.

10. In fig 1.78, bisectors of $\angle B$ and $\angle C$ of Δ ABC intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6 then find $\dfrac{AX}{XY}$.
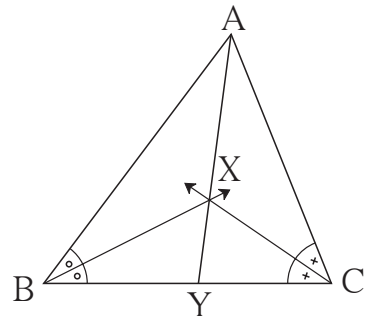

Fig. 1.78

Fig. 1.79

11. In ▢ ABCD, seg AD ∥ seg BC. Diagonal AC and diagonal BD intersect each other in point P. Then show that $\dfrac{AP}{PD} = \dfrac{PC}{BP}$

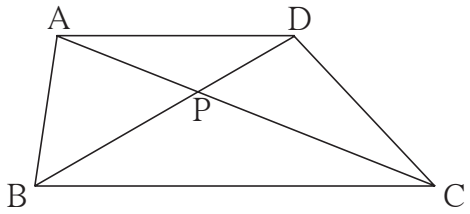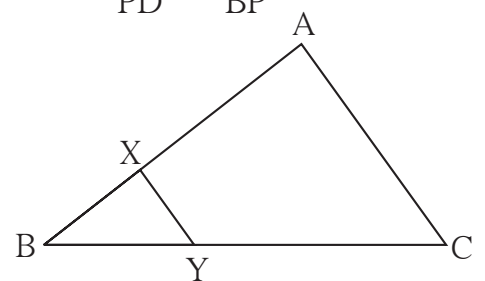**12.** In fig 1.80, XY ∥ seg AC. If 2AX = 3BX and XY = 9. Complete the activity to find the value of AC.

Fig. 1.80

Activity : 2AX = 3BX $\therefore \dfrac{AX}{BX} = \dfrac{\boxed{}}{\boxed{}}$

$\dfrac{AX + BX}{BX} = \dfrac{\boxed{} + \boxed{}}{\boxed{}}$ .... by componendo.

$\dfrac{AB}{BX} = \dfrac{\boxed{}}{\boxed{}}$ .... (I)

Δ BCA ~ Δ BYX .... $\boxed{}$ test of similarity.

$\therefore \dfrac{BA}{BX} = \dfrac{AC}{XY}$ .... corresponding sides of similar triangles.

$\therefore \dfrac{\boxed{}}{\boxed{}} = \dfrac{AC}{9}$ $\therefore$ AC = $\boxed{}$ ...from (I)

13*. In figure 1.81, the vertices of square DEFG are on the sides of Δ ABC. $\angle A = 90°$. Then prove that DE² = BD × EC

(Hint : Show that Δ GBD is similar to Δ CFE. Use GD = FE = DE.)

Fig. 1.81

▢▢▢